

Complementi di Fisica Nucleare I

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Bibliografia

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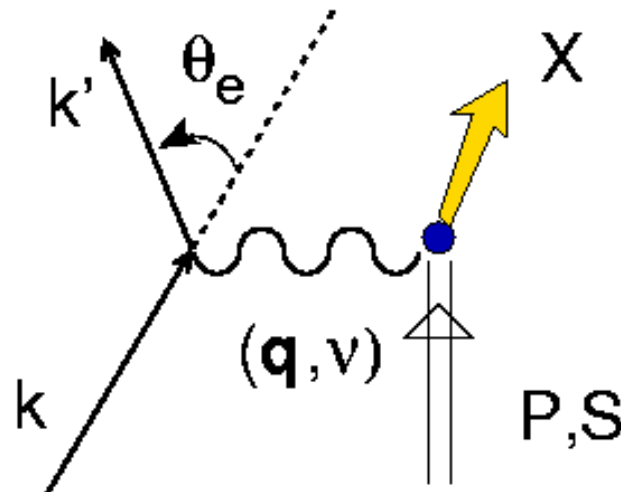
Schema

- scattering leptone-adrone: vantaggi, cinematica, info generali....
- scattering (an)elastico inclusivo su diversi bersagli:
bosone, fermione puntiforme, fermione con struttura interna
- formula generale di Rosenbluth
- definizione del regime di Deep Inelastic Scattering (DIS)
- confronto sezioni d'urto in tale regime -> definizione di scaling
- osservazione sperimentale dello scaling di Bjorken
- modello a partoni (Quark Parton Model, QPM)

Diffusione leptone -- adrone
 (elettrone, neutrino, muone) (nucleone, nucleo, fotone)

- Quantum ElectroDynamics (QED) nota ad ogni ordine
- sonda leptonica esplora tutto il volume del bersaglio
- costante struttura fine piccola -> sviluppo perturbativo possibile
- Born approximation (scambio di un fotone solo) e' accettabile $\alpha = \frac{e^2}{4\pi\hbar c} \sim \frac{1}{137}$
- fotone virtuale (γ^*): (\mathbf{q}, ν) indipendenti, risposta longitudinale e trasversa rispetto alla polarizzazione di γ^*

prototipo
 $e+p \rightarrow e'+X$



4 vettori indipendenti
 k, k', P, S
 θ_e angolo di diffusione

definizioni e cinematica

e^- ultrarelativistico $m_e \ll |\mathbf{k}|, |\mathbf{k}'|$
Target Rest Frame (TRF)

$$P = (M, \mathbf{0})$$

$$k = (\sqrt{m_e^2 + |\mathbf{k}|^2}, 0, 0, |\mathbf{k}|) \\ \sim (E, 0, 0, E)$$

$$k' = (\sqrt{m_e^2 + |\mathbf{k}'|^2}, |\mathbf{k}'| \sin \theta_e, 0, |\mathbf{k}'| \cos \theta_e) \\ \sim (E', E' \sin \theta_e, 0, E' \cos \theta_e)$$

$$q = k - k' = (E - E', \mathbf{k} - \mathbf{k}')$$

Invarianti cinematici

$$P^2 = M^2 \quad ; \quad S^2 = 1 \quad ; \quad P \cdot S = 0$$

$$k^2 = k'^2 = m_e^2 \sim 0$$

$$q^2 \sim -2EE'(1 - \cos \theta_e) = -4EE' \sin^2 \frac{\theta_e}{2} \leq 0$$

$$\implies Q^2 \equiv -q^2 = 4EE' \sin^2 \frac{\theta_e}{2} \geq 0$$

Invarianti cinematici (continua)

$$\nu = \frac{P \cdot q}{M} \stackrel{\text{TRF}}{=} \frac{M(E - E')}{M} = E - E' \quad \text{energia trasferita}$$

$$y = \frac{P \cdot q}{P \cdot k} \stackrel{\text{TRF}}{=} \frac{M(E - E')}{ME} = \frac{E - E'}{E} \quad \text{frazione di energia trasferita}$$

$0 \leq y \leq 1$

$$x_B = \frac{Q^2}{2P \cdot q} \stackrel{\text{TRF}}{=} \frac{Q^2}{2M\nu}$$

massa invariante finale

$0 \leq x_B \leq 1$

← limite elastico

$$W = (P + q)^2 = M^2 + Q^2 \left(\frac{1}{x_B} - 1 \right)$$

← limite anelastico

Scelta alternativa : 3 vettori indipendenti $P, (k+k'), (k-k') \equiv q$

Invarianti : $P^2 = M^2 ; (k-k')^2 = q^2 \equiv -Q^2 ; (k+k')^2 \sim Q^2$

$$P \cdot q \stackrel{\text{TRF}}{=} M\nu$$

$$P \cdot (k+k') = 2P \cdot k - P \cdot q \stackrel{\text{TRF}}{=} \left(\frac{2}{y} - 1\right) M\nu$$

Q e` la “lente di ingrandimento”

Q [GeV]	$\lambda \sim 1/Q$ [fm]	target
0.02	10	nuclei
0.1	2	
0.2	1	mesons / baryons
1	0.2	partoni
.....	??

DIS regime

$$Q^2 \rightarrow \infty$$
$$x_B = \frac{Q^2}{2P \cdot q} \text{ fissato}$$

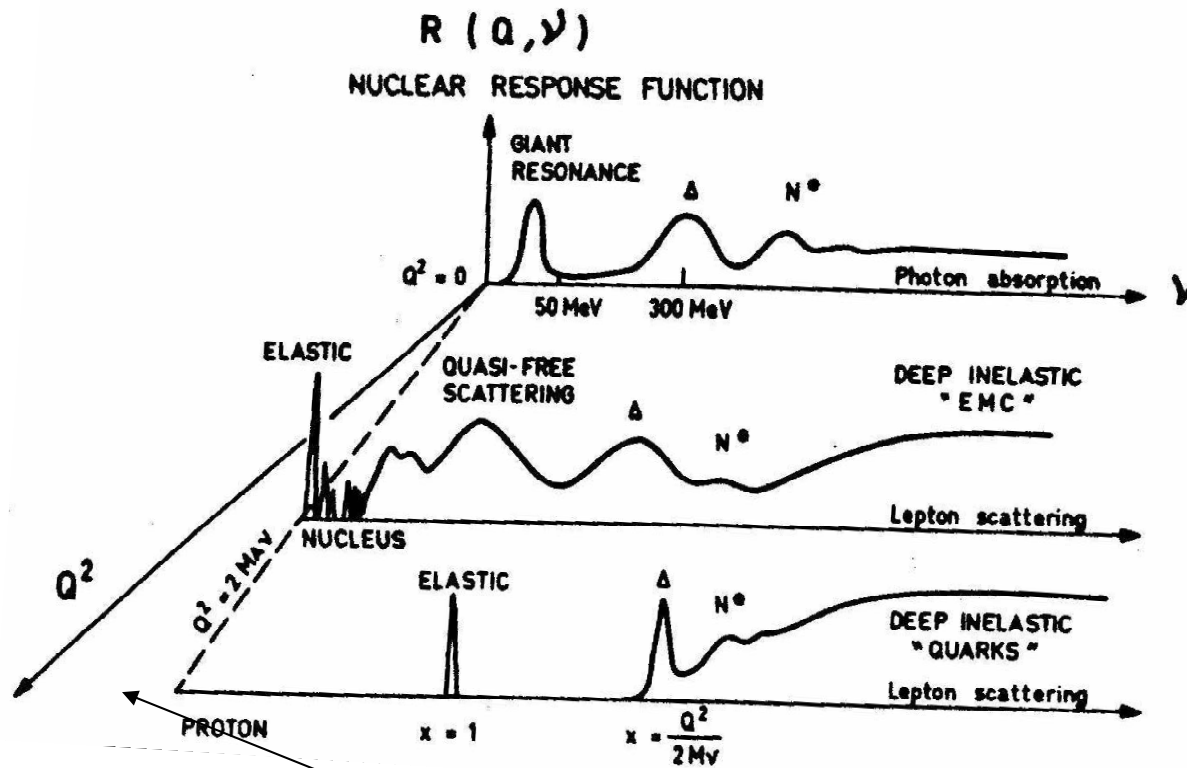
TRF : $v \rightarrow \infty$ velocemente come Q^2

anche $Q^2 = -(q^0)^2 + \mathbf{q}^2 = -v^2 + \mathbf{q}^2$ dunque
 $|\mathbf{q}| \rightarrow \infty$ velocemente come Q^2

dipendente dal frame

indipendente dal frame

Frois, Nucl. Phys. **A434** ('85) 57c



forbidden area

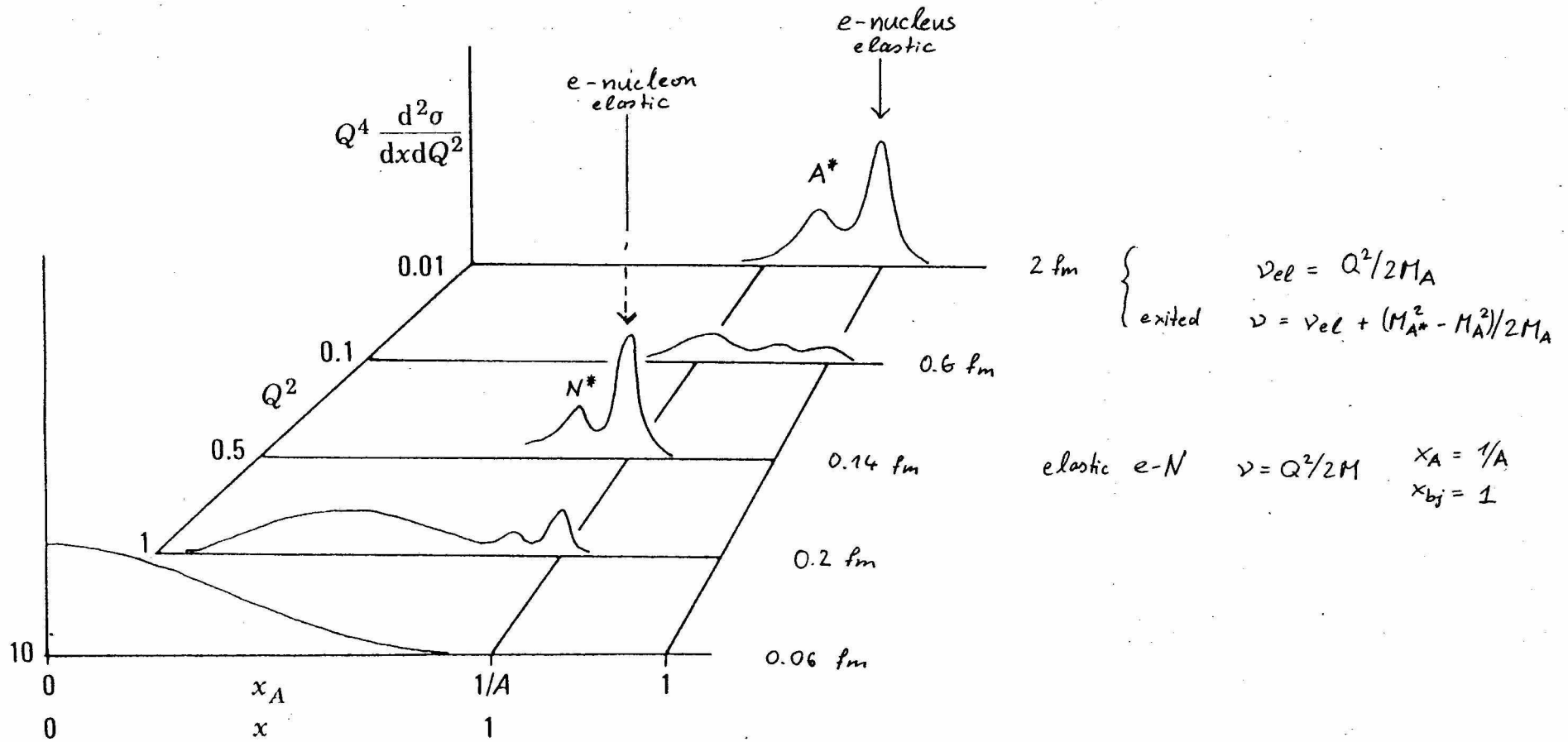


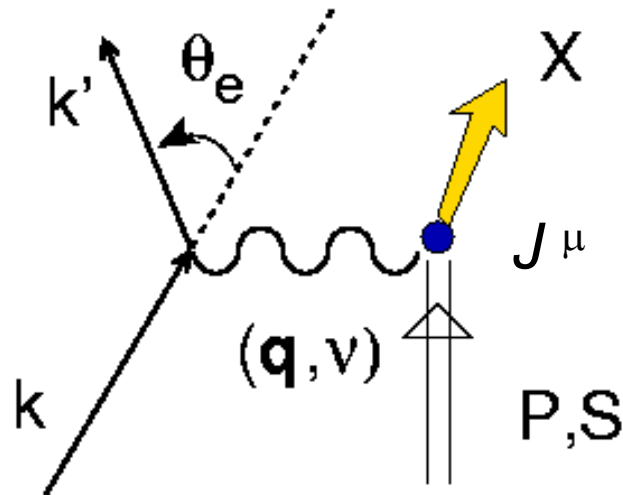
Fig. 1.2 Qualitative features of the cross-section (times Q^4) for electron nucleus scattering.

$$x_A = \frac{Q^2}{2M_A v}, \quad x_{bj} = \frac{Q^2}{2M v}$$

Sezione d'urto

n° eventi per unita` di tempo, diffusore, angolo solido

n° particelle incidenti per unita` di tempo, superficie



$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 dR$$

flusso

$$\mathcal{F} = 4\sqrt{(P \cdot k)^2 - P^2 k^2} \stackrel{\text{TRF}}{=} 4ME$$

spazio fasi

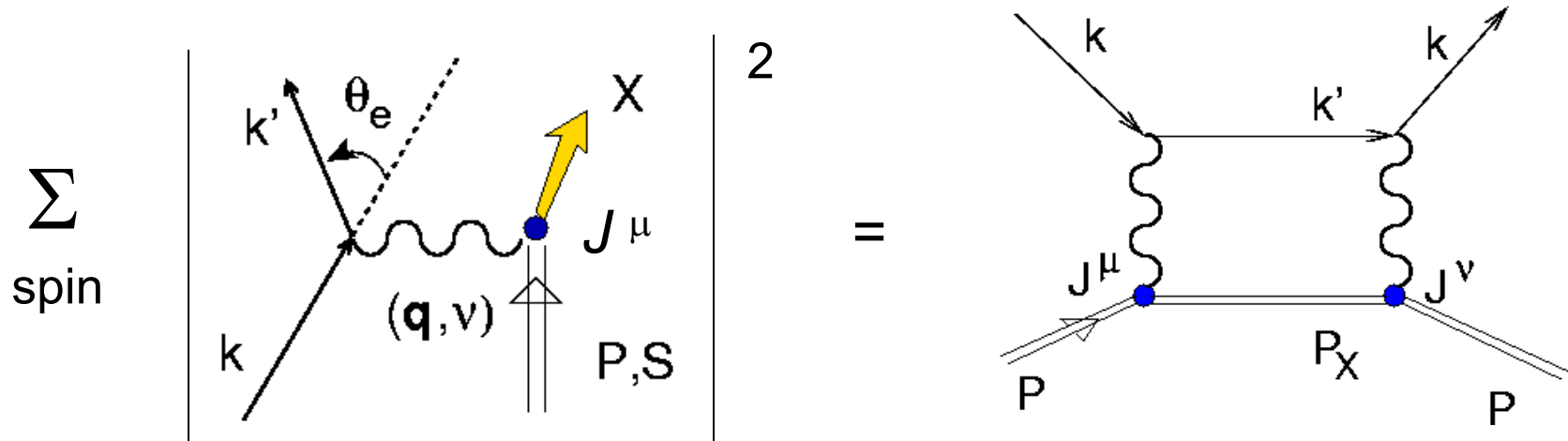
$$dR = (2\pi)^4 \delta(P + q - P_X) \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \frac{d\mathbf{k}'}{(2\pi)^3 2E'}$$

$$\frac{E' dE' d\Omega}{16\pi^3}$$

ampiezza scattering

$$\mathcal{M} = \bar{u}(k') \gamma_\mu u(k) \frac{e^2}{Q^2} \langle P_X | J^\mu(0) | P, S \rangle$$

Tensore adronico

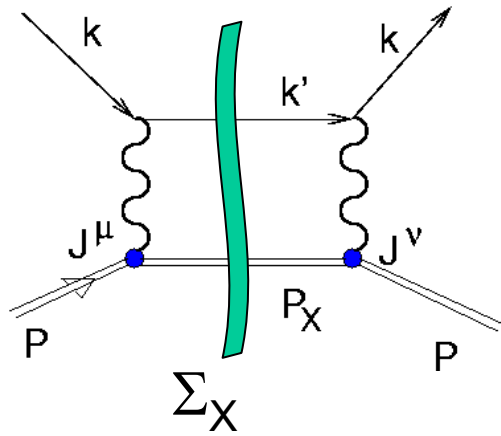


$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu}$$

tensore leptónico $L^{\mu\nu} = 2k^\mu k'^\nu + 2k^\nu k'^\mu - Q^2 g^{\mu\nu}$

tensore adronico $H^{\mu\nu} = \langle P | J^\mu(0) | P_X \rangle \langle P_X | J^\nu(0) | P \rangle$

Scattering inclusivo



tensore adronico

$$2MW^{\mu\nu} = \frac{1}{2\pi} \sum_X \int \frac{dP_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(P+q-P_X) H^{\mu\nu}$$

sezione d'urto per scattering inclusivo (formula generale)

$$\frac{d\sigma}{dE' d\Omega} = \frac{E'}{E} \frac{\alpha^2}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

$$\alpha = \frac{e^2}{4\pi}$$

$$Q^2 = 4EE' \sin^2 \frac{\theta_e}{2} \quad \text{large angles suppressed !}$$

Scattering inclusivo elastico

$$\sum_X \dots |P_X\rangle \langle P_X| \dots = \dots |P'\rangle \langle P'| \dots \quad \text{with } W' = (P+q)^2 = M^2$$

tensore adronico

$$\begin{aligned}
 2MW^{\mu\nu} &= \frac{1}{2\pi} \sum_X \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(P+q-P_X) H^{\mu\nu} \\
 &= \int \frac{d\mathbf{P}'}{2P'^0} \delta(\mathbf{P}+\mathbf{q}-\mathbf{P}') \delta(P^0+\nu-P'^0) H^{\mu\nu} \\
 \delta(f(x)) &= \frac{\delta(x-x_0)}{f'(x_0)} &= \int d\mathbf{P}' \delta(\mathbf{P}+\mathbf{q}-\mathbf{P}') \delta\left((P^0+\nu)^2 - (P'^0)^2\right) H^{\mu\nu} \\
 f(x_0) &= 0 \\
 &= \int d\mathbf{P}' \delta(\mathbf{P}+\mathbf{q}-\mathbf{P}') \delta\left((P^0+\nu)^2 - M^2 - \mathbf{P}'^2\right) H^{\mu\nu} \\
 &= \delta\left((P+q)^2 - M^2\right) H^{\mu\nu} = \delta(2P \cdot q - Q^2) H^{\mu\nu} \\
 \stackrel{\text{TRF}}{=} \delta(2M\nu - Q^2) H^{\mu\nu} &= \frac{1}{2M} \delta\left(\nu - \frac{Q^2}{2M}\right) H^{\mu\nu}
 \end{aligned}$$

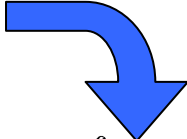
$\nu \leftrightarrow Q$ relation \rightarrow concept of scaling

↑

Sezione d'urto per scattering inclusivo elastico

$$\frac{d\sigma}{dE'd\Omega} = \frac{E'}{E} \frac{\alpha^2}{4M^2Q^4} L_{\mu\nu} H^{\mu\nu} \delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$\int dE' \frac{d\sigma}{dE'd\Omega}$$



$$\int dE' \delta\left(\nu - \frac{Q^2}{2M}\right) = \int dE' \delta\left(E - E' - \frac{4EE'}{2M} \sin^2 \frac{\theta_e}{2}\right)$$

$$= \frac{1}{1 + \frac{2E}{M} \sin^2 \frac{\theta_e}{2}} = \frac{E'}{E' + \frac{4EE'}{2M} \sin^2 \frac{\theta_e}{2}}$$

$$= \frac{E'}{E' + \nu} = \frac{E'}{E}$$

$$\delta(f(x)) = \frac{\delta(x - x_0)}{f'(x_0)}$$

$$f(x_0) = 0$$

vari casi

$$\frac{d\sigma}{d\Omega} = \left(\frac{E'}{E}\right)^2 \frac{\alpha^2}{4M^2Q^4} L_{\mu\nu} H^{\mu\nu}$$

Bersaglio = particella scalare libera

$$H^{\mu\nu} = \langle P | J^\mu | P' \rangle \langle P' | J^\nu | P \rangle$$

2 vettori indipendenti : $R=P+P'$, $q=P-P'$ $\rightarrow J^\mu \sim F_1 R^\mu + F_2 q^\mu$

$$F_{1,2}(q^2, P^2, P'^2) = F_{1,2}(q^2)$$

conservazione della corrente $q_\mu J^\mu = 0 \rightarrow F_2(q^2) = -\frac{R \cdot q}{q^2} F_1(q^2)$

definizione : $\tilde{P}^\mu = R^\mu - \frac{R \cdot q}{q^2} q^\mu \rightarrow J^\mu \sim \tilde{P}^\mu F_1(Q^2)$

$$L_{\mu\nu} H^{\mu\nu} = (2k \cdot \tilde{P} k' \cdot \tilde{P}' - \tilde{P}^2 k \cdot k') |F_1(Q^2)|^2 8EE'$$

$$\stackrel{\text{TRF}}{=} 16EE'M^2 |F_1(Q^2)|^2 \cos^2 \frac{\theta_e}{2}$$

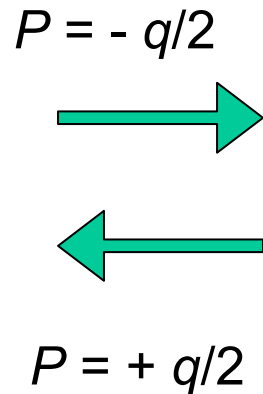
$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2}{Q^4} E'^2 \cos^2 \frac{\theta_e}{2} \frac{E'}{E} |F_1(Q^2)|^2 \equiv \sigma_{\text{Mott}} \frac{E'}{E} |F_1(Q^2)|^2$$

Coulomb scattering elastico da
particella puntiforme

rinculo
bersaglio

struttura
bersaglio

Breit frame \rightarrow form factor



$$v = 0$$

$$R^\mu = (2E, \mathbf{0})$$

$$q^\mu = (0, \mathbf{q})$$

$$J^\mu = (J^0, \mathbf{0}) \sim 2E F_1(Q^2)$$

$$F_1(Q^2) \equiv F_1(|\mathbf{q}|^2) = \int d\mathbf{r} \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

↑
fattore di forma di
carica
materia
.....

↑
distribuzione di
carica
materia
.....

Bersaglio = particella di Dirac libera puntiforme

Esempio: $e^- + \mu^- \rightarrow e'^- + \mu^-$

$$J^\mu \sim \bar{u}(P') \gamma^\mu u(P)$$

$$H^{\mu\nu} \equiv L^{\mu\nu} \quad \text{with } k^{(l)} \leftrightarrow P^{(l)}$$

$$\begin{aligned} \sum_{\text{spin}} H^{\mu\nu} &= \frac{1}{2} \text{Tr} \left[(\not{P}' + M) \gamma^\mu (\not{P} + M) \gamma^\nu \right] \\ &= 2 \left[P'^\mu P^\nu + P'^\nu P^\mu - (P \cdot P' - M^2) g^{\mu\nu} \right] \end{aligned}$$

$$L_{\mu\nu} H^{\mu\nu} \stackrel{\text{TRF}}{=} 16EE'M^2 \cos^2 \frac{\theta_e}{2} \left(1 + \frac{Q^2}{2M^2} \tan^2 \frac{\theta_e}{2} \right)$$

$$\frac{d\sigma}{dE' d\Omega} = \sigma_{\text{Mott}} \left(1 + \frac{Q^2}{2M^2} \tan^2 \frac{\theta_e}{2} \right) \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left(1 + \frac{Q^2}{2M^2} \tan^2 \frac{\theta_e}{2} \right)$$

interazione magnetica di spin con γ^*

Bersaglio = particella di Dirac libera con struttura

3 vettori indipendenti $P^\mu, P'^\mu, \gamma^\mu$ (+ invarianza per time-reversal, parita'..)

$$J^\mu \sim \bar{u}(P') \Gamma^\mu u(P) = \bar{u}(P') \left[\Gamma_1 P^\mu + \Gamma_2 P'^\mu + \Gamma_3 \gamma^\mu \right] u(P)$$

$$\Gamma_i(Q^2, P^2, P'^2) \equiv \Gamma_i(Q^2)$$

conservazione della corrente $q_\mu \Gamma^\mu = 0$

$$\begin{aligned} q_\mu \bar{u}(P') \Gamma^\mu u(P) &= \\ &= \bar{u}(P') \left[\Gamma_1 P \cdot (P' - P) + \Gamma_2 P' \cdot (P' - P) + \Gamma_3 \gamma \cdot (P' - P) \right] u(P) \\ &= \bar{u}(P') \left[(P \cdot P' - M^2) (\Gamma_1 - \Gamma_2) + \Gamma_3 (\not{P}' - \not{P}) \right] u(P) = 0 \\ &\Rightarrow \Gamma_1 = \Gamma_2 \end{aligned}$$

$$J^\mu \sim \bar{u}(P') \left[\Gamma_1(Q^2) \not{P}^\mu + \Gamma_3(Q^2) \gamma^\mu \right] u(P)$$

eq. di Dirac

$$\not{P}u = Mu$$

$$\bar{u} \not{P} = \bar{u}M$$

Decomposizione di Gordon (on-shell)

$$J^\mu \sim \bar{u}(P') \left[\Gamma_1(Q^2) R^\mu + \Gamma_3(Q^2) \gamma^\mu \right] u(P)$$

$$\bar{u} \gamma^\mu u = \bar{u} \left[\frac{R^\mu}{2M} + \frac{i}{2M} \sigma^{\mu\nu} q_\nu \right] u \quad \text{with } \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

cioe' $R^\mu \leftrightarrow 2M \gamma^\mu - i \sigma^{\mu\nu} q_\nu$

proof flow-chart

- da destra, inserire def. di $\sigma^{\mu\nu}$
- usare eq. di Dirac
- usare $\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu}$
- usare eq. Dirac \rightarrow sinistra

$$J^\mu \sim \bar{u}(P') \left[F_1(Q^2) \gamma^\mu + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2(Q^2) \right] u(P)$$

$$F_1 = 2M\Gamma_1 + \Gamma_3 \quad \text{Dirac form factor} \quad F_1(0) = 1$$

$$F_2 = -2M\Gamma_1 \quad \text{Pauli form factor} \quad F_2(0) = \kappa$$

Bersaglio = particella di Dirac libera e composita

Sezione d'urto

$$\sum_{\text{spin}} H^{\mu\nu} = \frac{1}{2} \text{Tr} \left[(\not{P}' + M) \Gamma^\mu (\not{P} + M) \Gamma^\nu \right]$$

$$\Gamma^\mu = F_1 \gamma^\mu + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2$$

.....

$$\frac{d\sigma}{dE' d\Omega} = \sigma_{\text{Mott}} \left[\left(F_1^2 + \frac{Q^2}{4M^2} F_2^2 \right) + \frac{Q^2}{2M^2} (F_1 + F_2)^2 \tan^2 \frac{\theta_e}{2} \right] \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[\left(F_1^2 + \frac{Q^2}{4M^2} F_2^2 \right) + \frac{Q^2}{2M^2} (F_1 + F_2)^2 \tan^2 \frac{\theta_e}{2} \right]$$

internal structure

Formula di Rosenbluth

Definizione fattori di forma di Sachs $G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$ with $\tau = \frac{Q^2}{4M^2}$
(Yennie, 1957) $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

N.B.: infatti, in Breit frame + riduzione nonrel. \rightarrow

$$J^0 \sim G_E$$
$$\mathbf{J} \sim \frac{G_M}{2M} \boldsymbol{\sigma} \times \mathbf{q}$$

distribuzione di carica/magnetica
del bersaglio

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta_e}{2} \right]$$
$$\equiv \sigma_{\text{Mott}} \frac{E'}{E} \left[A(Q^2) + B(Q^2) \tan^2 \frac{\theta_e}{2} \right]$$

Separazione di Rosenbluth

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta_e}{2} \right]$$

- larghi θ_e (larghi Q^2) \rightarrow estrarre G_M
- piccoli θ_e (piccoli Q^2) \rightarrow estrarre G_E per differenza
- Rosenbluth plot

$$\epsilon(1 + \tau) \frac{E}{E'} \frac{1}{\sigma_{\text{Mott}}} \frac{d\sigma}{d\Omega} = \epsilon G_E^2 + \tau G_M^2 \quad \epsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right]^{-1}$$

polarizz. trasversa lineare di γ^*

misure con diverse $(E, \theta_e) \rightarrow$ plot in ϵ a fisso Q^2

intercetta a $\epsilon = 0 \rightarrow G_M$

pendenza in $\epsilon \rightarrow G_E$

Sezione d'urto (an)elastica inclusiva per particella di Dirac composita

Risultato generale :
$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[A(\nu, Q^2) + B(\nu, Q^2) \tan^2 \frac{\theta_e}{2} \right]$$

- Procedura :
- 2 vettori indipendenti P, q
 - base tensoriale: $b_1 = g^{\mu\nu}, b_2 = q^\mu q^\nu, b_3 = P^\mu P^\nu,$
 $b_4 = (P^\mu q^\nu + P^\nu q^\mu), b_5 = (P^\mu q^\nu - P^\nu q^\mu),$
 $b_6 = \varepsilon_{\mu\nu\rho\sigma} q^\rho P^\sigma$
 - tensore adronico $W^{\mu\nu} = \sum_i c_i (q^2, P \cdot q) b_i$
 - invarianza per parita` e time-reversal,
 conservazione della corrente $q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu = 0$
 - sistema lineare con c_6 indeterminato ($=0$), $c_5=0$,
 c_1 e c_3 dipendenti da c_2 e c_4
 - Risultato finale :

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) c_2(q^2, P \cdot q) + \frac{\tilde{P}^\mu \tilde{P}^\nu}{M^2} c_4(q^2, P \cdot q)$$

$$\tilde{P}^\mu = P^\mu - \frac{P \cdot q}{q^2} q^\mu$$

(continua)

- struttura $\varepsilon_{\mu\nu\rho\sigma} q^\rho P^\sigma$ proibita da invarianza per parita`
- struttura $(P^\mu q^\nu - P^\nu q^\mu)$ proibita da invarianza per time-reversal
- strutture $(P^\mu q^\nu + P^\nu q^\mu)$, $q^\mu q^\nu$ trascurabili perche` $\sim m_e^2$, ma non proibite (violazione della conservazione della corrente)
- hermiticity $W^{\mu\nu} = W^{\nu\mu*} \rightarrow c_{2,4}$ funzioni reali
- convenzione : $c_2 \equiv W_1$; $c_4 \equiv W_2$

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2) \tan^2 \frac{\theta_e}{2} \right]$$

N.B. base dei vettori di polarizzazione di γ^*

$$\epsilon_{\pm}^{\mu} = \frac{1}{\sqrt{2}} (0, \mp 1, -i, 0) = \mp \frac{1}{\sqrt{2}} (e_x \pm i e_y) \quad ; \quad \epsilon_0^{\mu} = \frac{1}{\sqrt{Q^2}} (q_z, 0, 0, \nu)$$

$$W_T = W_1 \quad ; \quad W_L = -W_1 + \left(1 + \frac{\nu^2}{Q^2} \right) W_2$$

$$\frac{d\sigma}{dE' d\Omega} = \sigma_{\text{Mott}} \frac{Q^2}{\epsilon^{-1}(\nu^2 + Q^2)} \left[W_L(\nu, Q^2) + W_T(\nu, Q^2) \left(1 + \frac{2(\nu^2 + Q^2)}{Q^2} \right) \right]$$

Riepilogo

Scattering inclusivo su particella di Dirac libera e composita

anelastico

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2) \tan^2 \frac{\theta_e}{2} \right]$$

elastico

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left[(F_1^2 + \tau F_2^2) + 2\tau (F_1 + F_2)^2 \tan^2 \frac{\theta_e}{2} \right] \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$W_2^{\text{el}} \leftrightarrow (F_1^2 + \tau F_2^2) \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$2W_1^{\text{el}} \leftrightarrow 2\tau (F_1 + F_2)^2 \delta \left(\nu - \frac{Q^2}{2M} \right)$$

elastico puntiforme

$$\begin{aligned} F_1 &\rightarrow 1 \\ F_2 &\rightarrow 0 \end{aligned}$$

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left(1 + 2\tau \tan^2 \frac{\theta_e}{2} \right) \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$W_2^{\text{el}} \leftrightarrow \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$W_1^{\text{el}} \leftrightarrow \tau \delta \left(\nu - \frac{Q^2}{2M} \right)$$

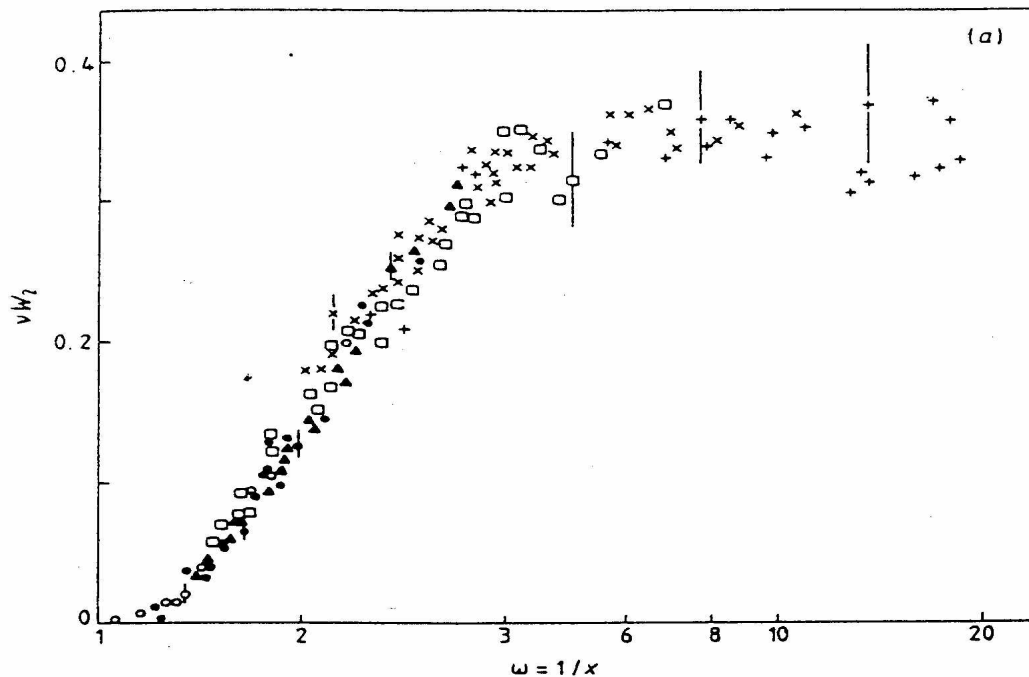
Scaling

$$\begin{aligned} W_2^{\text{el}} &\leftrightarrow \delta\left(\nu - \frac{Q^2}{2M}\right) & \nu W_2^{\text{el}} &\leftrightarrow \delta\left(1 - \frac{Q^2}{2M\nu}\right) \equiv \delta(1 - x_B) \equiv F_2(x_B) \\ W_1^{\text{el}} &\leftrightarrow \tau\delta\left(\nu - \frac{Q^2}{2M}\right) & 2MW_1^{\text{el}} &\leftrightarrow \frac{Q^2}{2M\nu}\delta\left(1 - \frac{Q^2}{2M\nu}\right) \equiv x_B\delta(1 - x_B) \equiv 2F_1(x_B) \end{aligned}$$

Osservazione sperimentale dello scaling = segnale che nella cinematica DIS, cioè $Q^2, \nu \rightarrow \infty$, x_B fixed, lo scattering si può rappresentare come la somma incoerente di scattering elastici da costituenti puntiformi del bersaglio \rightarrow origine del concetto di partone

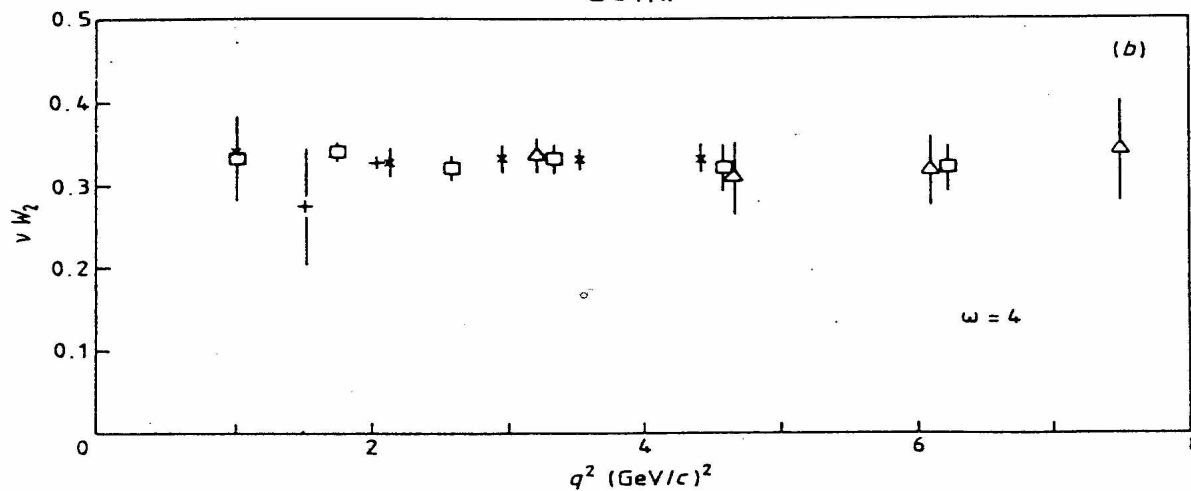
N.B. Analogo dell'esperimento di Rutherford sullo scattering di particelle α da atomi

νW_2



$1/x$

Aitchison
& Hey



Q^2

Figure 4.2 Bjorken scaling: the structure function νW_2 (a) plotted against $\omega = 1/x$ for different q^2 values (Miller *et al* 1972) (b) plotted against q^2 for a single value of $x = 0.25$ ($\omega = 4$) (Friedman and Kendall 1972).

Nachtmann

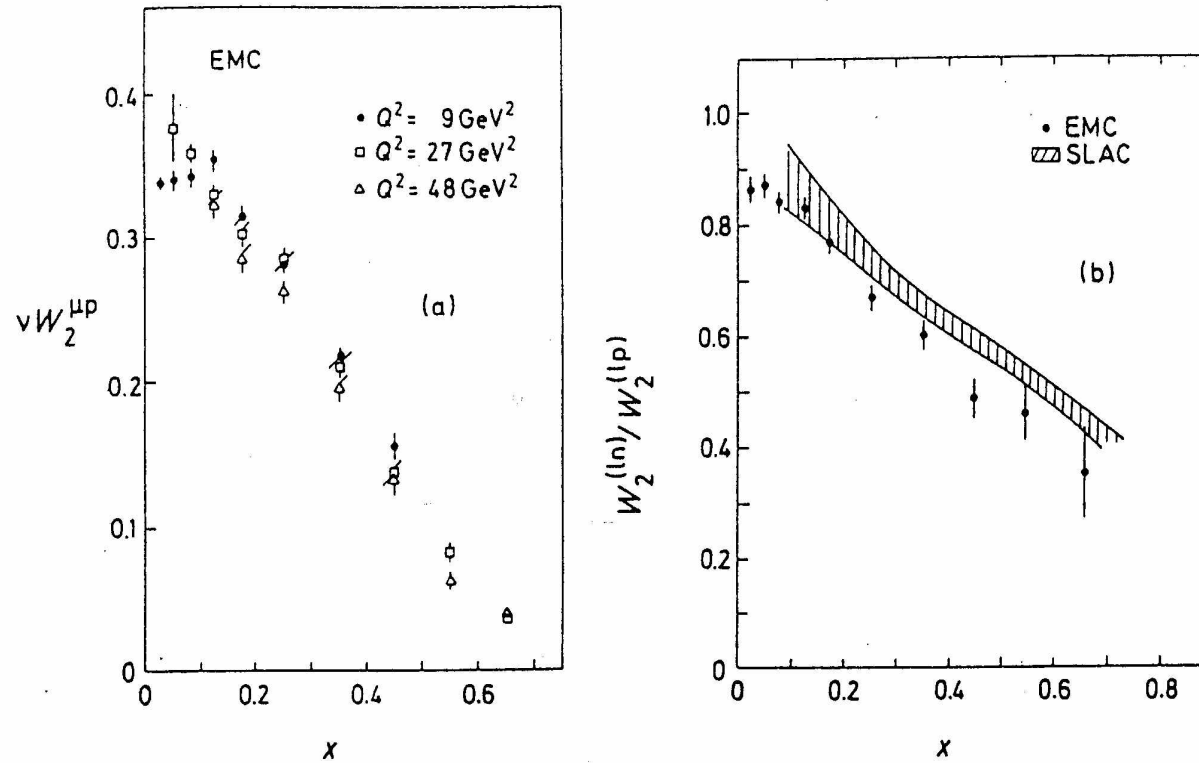


Figure 18.9 Data from the European Muon Collaboration (EMC) for the structure function $vW_2^{(\mu p)}(v, Q^2)$ of the proton as a function of $x = Q^2/(2Mv)$ for various Q^2 values. Exact Bjorken scale invariance would demand that the data points for the same x but different Q^2 should lie on top of one another (a). Part (b) shows the ratio of the neutron and proton structure functions $W_2^{(ln)}(v, Q^2)$ and $W_2^{(lp)}(v, Q^2)$ ($l = e, \mu$) as a function of x . The shaded band represents the SLAC data obtained from electron scattering in the interval $2 \leq Q^2 \leq 20 \text{ GeV}^2$. The points correspond to preliminary EMC data from muon scattering in the interval $10 \leq Q^2 \leq 80 \text{ GeV}^2$ (after Drees 1983 and Dydak 1983).

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