Schema

- riassunto precedente lezione
- correnti neutre nel QPM
- fattorizzazione e universalita` nel QPM: dal DIS al e^+e^-
- e^+e^- semi-inclusivo: funzioni di frammentazione
- confronto tra semi-inclusivo in DIS ed e^+e^-
- sezione d'urto di jet

- cenni sulla genesi del settore elettrodebole del Modello Standard (unificazione delle interazioni e.m. e debole nello scambio di 4 bosoni vettori di gauge; rottura spontanea della simmetria e rinormalizzabilita`; necessita` del quark charm; matrice CKM di flavor mixing; correnti neutre; conferme sperimentali)

- estensione delle formule a DIS inclusivo con interazioni elettrodeboli (interazioni deboli selezionano componenti (right)left-handed dei (anti)fermioni; accoppiamento *V-A* e *V+A* ; terza funzione di struttura F_3 da interferenza *VA*)

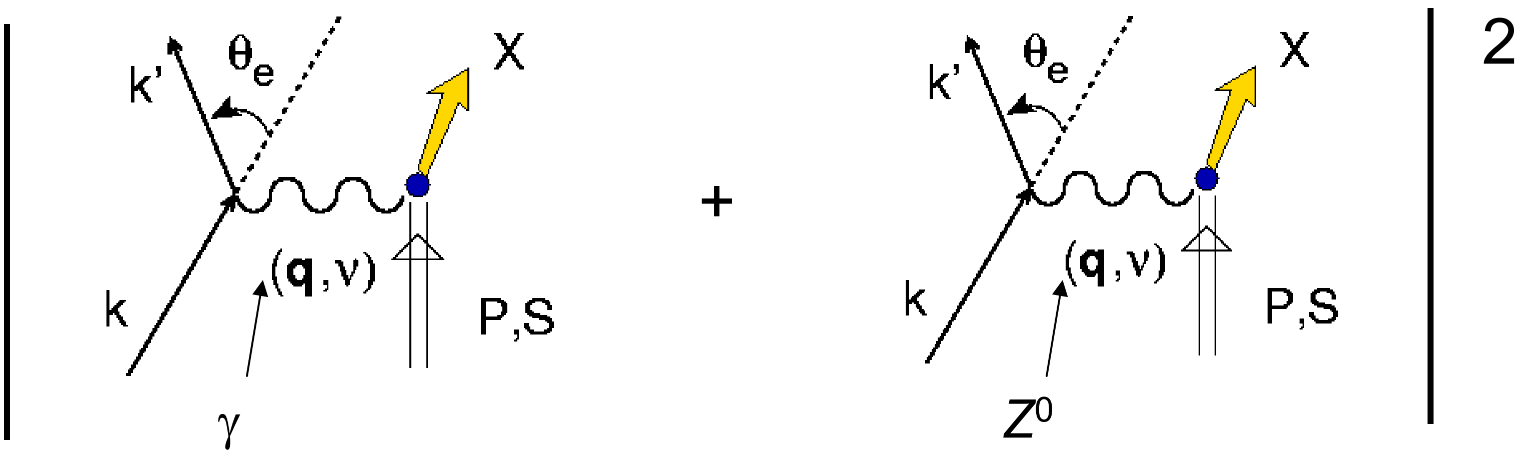
- QPM: la F_3 da` informazioni valence-like non di singoletto; DIS inclusivo elettrodebole (fasci di e^- ed (anti)neutrini) determinano il sistema $\rightarrow$ funzioni di distribuzione per tutti i flavors

- confronto con dati sperimentali e regole di somma:
  quark valenza ("mare") dominano per $x_B \rightarrow 1$ (0)   (coerente con DIS e.m.)
  quark a spin ½ interagiscono come i leptoni; antiquark soppressi
  partoni a carica nulla (i gluoni) portano circa meta` del momento dell'adrone

Correnti neutre

reazioni tipo:

$$\begin{cases} e^{\pm} N \to e'^{\pm} N \\ \mu^{\pm} N \to \mu'^{\pm} N \\ \nu N \to \nu N \\ \bar{\nu} N \to \bar{\nu} N \end{cases}$$



$$\frac{d\sigma}{dE' d\Omega} = \frac{d\sigma^{\gamma\gamma}}{dE' d\Omega} + \frac{d\sigma^{\gamma Z}}{dE' d\Omega} + \frac{d\sigma^{ZZ}}{dE' d\Omega}$$

Settore elettrodebole del Modello Standard: $SU_T(2) \otimes U_Y(1)$

rottura spontanea della simmetria $SU_T(2) \rightarrow$

$$\begin{aligned} A &= \cos\theta_W B + \sin\theta_W W_3 \\ Z^0 &= -\sin\theta_W B + \cos\theta_W W_3 \end{aligned}$$

$$\mathcal{L}_{weak} \rightarrow \begin{cases} J^\mu_\gamma = & e\bar{u}\,\gamma^\mu\, u \\ J^\mu_{Z^0} = & e\bar{u}\left[\frac{\frac{1}{2}T_3 - Q\sin^2\theta_W}{2\sin\theta_W\cos\theta_W}\gamma^\mu(1-\gamma_5) - \frac{Q\sin^2\theta_W}{2\sin\theta_W\cos\theta_W}\gamma^\mu(1+\gamma_5)\right] u \end{cases}$$

$\gamma\,\gamma$

media iniziale su spin del leptone

$$L_{\mu\nu} = \frac{1}{2}e^2\,\mathrm{Tr}\,\left[\gamma_\mu\, \not{k}'\gamma_\nu\, \not{k}\right] \equiv e^2\, L^{(S)}_{\mu\nu}$$

$$\frac{d\sigma^{\gamma\gamma}}{dE'd\Omega} = \frac{E'}{E}\,\frac{\alpha^2}{Q^4}\,L^{(S)}_{\mu\nu}\,W^{\mu\nu}_{\gamma\gamma}$$

ZZ

media iniziale su spin del leptone

$$L_{\mu\nu}^{ZZ} = \frac{1}{2}\frac{e^2}{4\sin^2\theta_W\cos^2\theta_W}\mathrm{Tr}\left[\left[\left(T_3 - Q\sin^2\theta_W\right)\gamma_\mu(1-\gamma_5) - Q\sin^2\theta_W\gamma_\mu(1+\gamma_5)\right]\not{k}'\right.$$
$$\left.\times\left[\left(T_3 - Q\sin^2\theta_W\right)\gamma_\nu(1-\gamma_5) - Q\sin^2\theta_W\gamma_\nu(1+\gamma_5)\right]\not{k}\right]$$
$$= \frac{1}{2}\frac{e^2}{4\sin^2\theta_W\cos^2\theta_W}\left\{\left(T_3 - Q\sin^2\theta_W\right)^2\mathrm{Tr}\left[\gamma_\mu(1-\gamma_5)\not{k}'\gamma_\nu(1-\gamma_5)\not{k}\right]\right.$$
$$\left.+ Q^2\sin^4\theta_W\,\mathrm{Tr}\left[\gamma_\mu(1+\gamma_5)\not{k}'\gamma_\nu(1+\gamma_5)\not{k}\right] + 0\right\}$$
$$\equiv \frac{1}{2}\frac{e^2}{4\sin^2\theta_W\cos^2\theta_W}\left\{\left(T_3 - Q\sin^2\theta_W\right)^2\left(L_{\mu\nu}^{(S)} - L_{\mu\nu}^{(A)}\right) + Q^2\sin^4\theta_W\left(L_{\mu\nu}^{(S)} + L_{\mu\nu}^{(A)}\right)\right\}$$
$$T_3 = \frac{1}{2}; \quad Q = 1$$
$$= \frac{1}{2}\frac{e^2}{4\sin^2\theta_W\cos^2\theta_W}\left\{\left(1 - 4\sin^2\theta_W + 8\sin^4\theta_W\right)L_{\mu\nu}^{(S)} - \left(1 - 4\sin^2\theta_W\right)L_{\mu\nu}^{(A)}\right\}$$

$$\frac{d\sigma^{ZZ}}{dE'd\Omega} = \frac{e^4}{16\sin^4\theta_W\cos^4\theta_W}\frac{E'}{E}\frac{1}{16\pi^2}\frac{1}{(Q^2+M_Z^2)^2}L_{\mu\nu}^{ZZ}W_{ZZ}^{\mu\nu}$$
$$= \frac{G_F^2}{16\pi^2}\left(\frac{M_Z^2}{Q^2+M_Z^2}\right)^2\frac{E'}{E}L_{\mu\nu}^{ZZ}W_{ZZ}^{\mu\nu}$$

$\gamma\, Z$

media iniziale su spin del leptone

$$L_{\mu\nu}^{\gamma Z} = \frac{1}{2}\frac{e^2}{2\sin\theta_W\cos\theta_W}\,\mathrm{Tr}\left[\gamma_\mu\, \not{k}'\left[\left(T_3 - Q\sin^2\theta_W\right)\gamma_\nu(1-\gamma_5) - Q\sin^2\theta_W\gamma_\nu(1+\gamma_5)\right]\ \not{k}\right]$$

$$= \frac{1}{2}\frac{e^2}{2\sin\theta_W\cos\theta_W}\left\{\left(T_3 - Q\sin^2\theta_W\right)\mathrm{Tr}\left[\gamma_\mu\not{k}'\gamma_\nu(1-\gamma_5)\not{k}\right] - Q\sin^2\theta_W\,\mathrm{Tr}\left[\gamma_\mu\not{k}'\gamma_\nu(1+\gamma_5)\not{k}\right]\right\}$$

$$\equiv \frac{1}{2}\frac{e^2}{2\sin\theta_W\cos\theta_W}\left\{\left(T_3 - Q\sin^2\theta_W\right)\left(L_{\mu\nu}^{(S)} - L_{\mu\nu}^{(A)}\right) - Q\sin^2\theta_W\left(L_{\mu\nu}^{(S)} + L_{\mu\nu}^{(A)}\right)\right\}$$

$$T_3 = \frac{1}{2}; \qquad Q = 1$$

$$= \frac{1}{2}\frac{e^2}{2\sin\theta_W\cos\theta_W}\left\{\left(1 - 4\sin^2\theta_W\right)L_{\mu\nu}^{(S)} - L_{\mu\nu}^{(A)}\right\}$$

$$\frac{d\sigma^{\gamma Z}}{dE'd\Omega} = 2\frac{e^4}{4\sin^2\theta_W\cos^2\theta_W}\frac{E'}{E}\frac{1}{16\pi^2}\frac{1}{Q^2(Q^2+M_Z^2)^2}L_{\mu\nu}^{\gamma Z}W_{\gamma Z}^{\mu\nu}$$

$$= \frac{G_F\alpha}{2\pi\sqrt{2}}\frac{M_Z^2}{Q^2(Q^2+M_Z^2)}\frac{E'}{E}L_{\mu\nu}^{\gamma Z}W_{\gamma Z}^{\mu\nu}$$

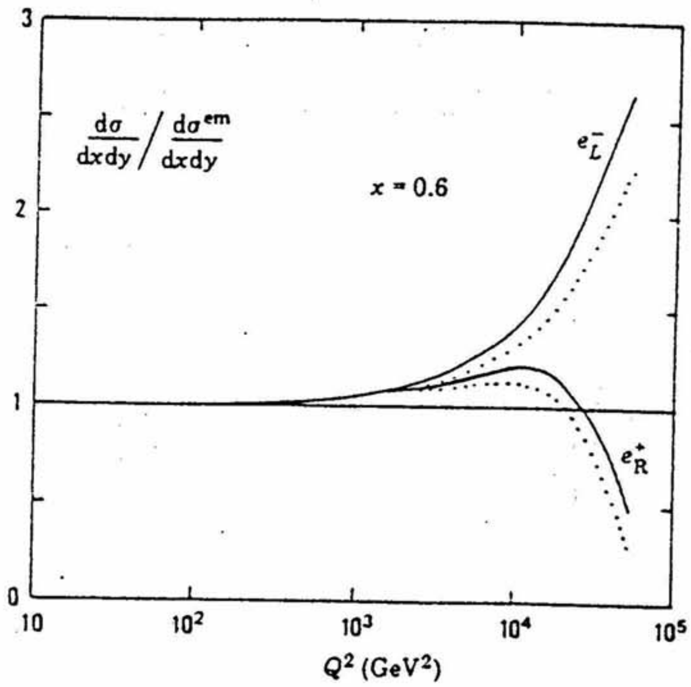


Fig. 3.3 The ratio of the full neutral current cross-section to the electromagnetic contribution at $x = 0.6$ for $e^-_L p \to e^-_L X$ and $e^+_R p \to e^+_R X$ as a function of Q^2. The dotted lines correspond to $\gamma - Z$ interference only.

$$\frac{d\sigma}{dE'd\Omega} = \frac{d\sigma^{\gamma\gamma}}{dE'd\Omega} + \frac{d\sigma^{\gamma Z}}{dE'd\Omega} + \frac{d\sigma^{ZZ}}{dE'd\Omega}$$

Correnti neutre con fasci di (anti)neutrini

$$J_Z^\mu = \frac{2e}{2\sin\theta_W\cos\theta_W}\,\bar{\chi}\left\{\begin{array}{c} T_3 - e_f\sin^2\theta_W \\ -e_f\sin^2\theta_W \end{array}\right\}\chi \;=\; \sum_f C_f^V\,\bar{q}_f\,\gamma^\mu q_f + C_f^A\,\bar{q}_f\,\gamma^\mu\gamma_5 q_f$$

$$= \sum_f \bar{q}_f\left[C_f^R\gamma^\mu(1+\gamma_5) + C_f^L\gamma^\mu(1-\gamma_5)\right] q_f$$

$$\chi_{L/R} = \binom{u}{d_\theta}_{L/R}$$

$$\begin{array}{rcl} \frac{1}{2} - \frac{2}{3}\sin^2\theta_W\,u_L & \equiv & C_u^L \\ -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\,d_{\theta_L} & \equiv & C_d^L \\ -\frac{2}{3}\sin^2\theta_W\,u_R & \equiv & C_u^R \\ \frac{1}{3}\sin^2\theta_W\,d_{\theta_R} & \equiv & C_d^R \\ \cdots\cdots\cdots & \cdots & \cdots \end{array}$$

Ricorda: $\nu \to \mu^-\,;\quad \bar{\nu} \to \mu^+$

$$\frac{d\sigma^{\mathrm{el}\nu}}{dx_B\,dy} = \frac{G_F^2 e_f^2}{8\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 s\left[1|_{q(x)} + (1-y)^2|_{\bar{q}(x)}\right] x\,\delta(x-x_B)$$

$$\frac{d\sigma^{\mathrm{el}\bar{\nu}}}{dx_B\,dy} = \frac{G_F^2 e_f^2}{8\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 s\left[1|_{\bar{q}(x)} + (1-y)^2|_{q(x)}\right] x\,\delta(x-x_B)$$

Approssimazioni: *u,d* quarks, no antiquarks, *N* "medio" (n_u = n_d)

$$R^{\nu} \equiv \frac{\int_0^1 dy\, d\sigma(\nu \to \nu)}{\int_0^1 dy\, d\sigma(\nu \to \mu^-)} = \frac{K q(x_B) \int_0^1 dy \sum_f (C_f^{L\,2} + C_f^{R\,2}(1-y)^2)}{K q(x_B)}$$

$$= \sum_f \left(C_f^{L\,2} + \frac{1}{3} C_f^{R\,2} \right) = \frac{1}{2} - \sin^2\theta_w + \frac{20}{27} \sin^4\theta_W$$

$$R^{\bar{\nu}} \equiv \frac{\int_0^1 dy\, d\sigma(\bar{\nu} \to \bar{\nu})}{\int_0^1 dy\, d\sigma(\bar{\nu} \to \mu^+)} = \frac{K q(x_B) \int_0^1 dy \sum_f (C_f^{R\,2} + C_f^{L\,2}(1-y)^2)}{K q(x_B) \int_0^1 dy\, (1-y)^2}$$

$$= \sum_f \left(C_f^{L\,2} + 3\, C_f^{R\,2} \right) = \frac{1}{2} - \sin^2\theta_w + \frac{20}{9} \sin^4\theta_W$$

$0.2 \leq \sin^2 \theta_W \leq 0.4$

dashed = 15% antiquark

Blietschau *et al.,* N.P. **B114** 169 (76)
Benvenuti et al., Int. ν Conf. Aachen (76)

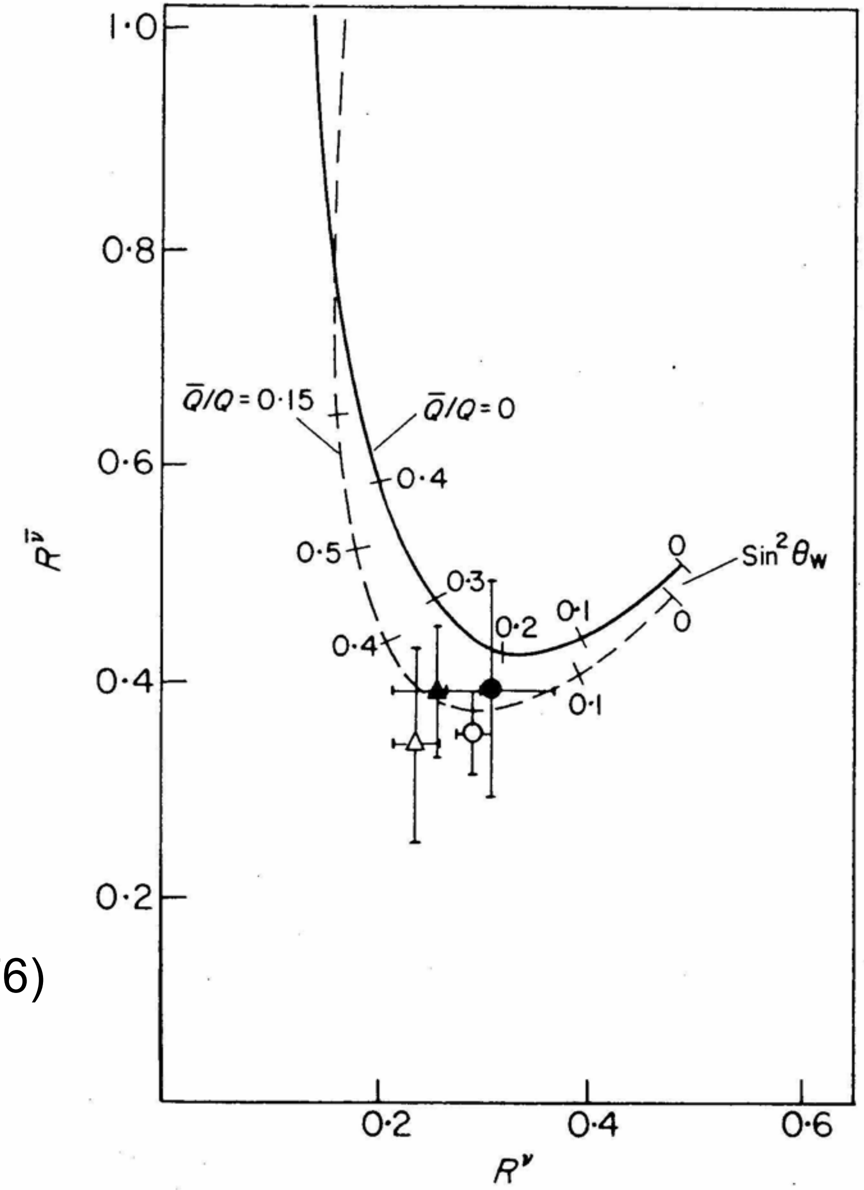


FIG. 11.19. R^{ν} and $R^{\bar{\nu}}$ in Weinberg–Salam model as a function of θ_W.

Quark Parton Model

sezione d'urto per
processo fondamentale

= sezione d'urto elastica su partoni puntiformi a spin ½ $\otimes$ probabilita` di distribuzione dei partoni nell'adrone

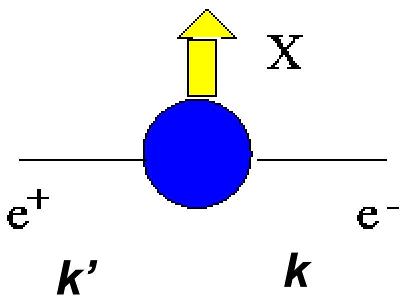
- partoni prevalenti su antipartoni
- partoni interagiscono come leptoni
  $\rightarrow$ sez. d'urto calcolabile all'ordine voluto in QED
- sez. d'urto indipendente dal processo; cinematica hard (high Q)

- distribuzione partonica contiene effetti nonperturbativi per formare adroni da partoni; cinematica soft (low p_T)
- dipende dall'adrone ma non dal processo $\rightarrow$ "universale"
- estraibile da confronto con dati dopo aver calcolato sez. d'urto elementare

QPM

fenomeni ad alta energia =
{processi hard calcolabili in QED} + {distribuzioni partoniche universali estraibili da un set di dati}

e^+e^- inclusivo



$q = k+k'$ time-like $q^2 \equiv Q^2 = s \geq 0$

$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 \, dR$$

$$\mathcal{F} = 4\sqrt{(k \cdot k')^2 - k^2 k'^2} \overset{\text{TRF}}{=} 2Q^2 \equiv 2s$$

$$dR = (2\pi)^4 \delta(k + k' - P_X) \, \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}$$

$$\mathcal{M} = \bar{v}(k')\gamma_\mu u(k) \, \frac{e^2}{Q^2} \langle P_X | J^\mu(0) | 0 \rangle$$

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} \, H^{\mu\nu}$$

$$L_{\mu\nu} = 2k_\mu k'_\nu + 2k'_\mu k_\nu - Q^2 g_{\mu\nu}$$

$$H^{\mu\nu} = \sum\nolimits_{S_X} \langle 0 | J^\mu | P_X \rangle \langle P_X | J^\nu | 0 \rangle$$

$$\sigma = \frac{1}{2} \frac{1}{2Q^2} \frac{e^4}{Q^4} L_{\mu\nu} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \, \delta(q - P_X) \, H^{\mu\nu} = \frac{4\pi\alpha^2}{Q^6} L_{\mu\nu} \, W^{\mu\nu}$$

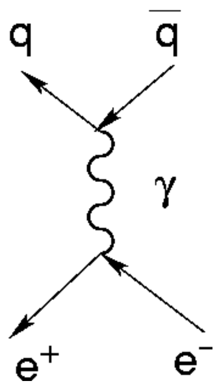
media su polarizzazioni iniziali

e^+e^- inclusivo (continua)

Metodo alternativo

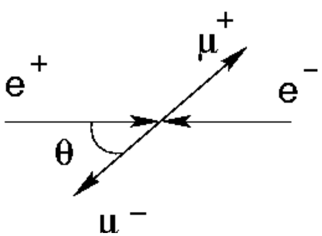
→ no adroni in stato iniziale e finale

σ in QPM $\equiv$ σ elementare $e^+e^- \to q\bar{q}$ ← solo N_c modi di creare la coppia conservando il colore nel vertice



$Q^2 = s$ tale da avere solo produzione di γ

$$\sigma(e^+e^- \to q\bar{q}) \equiv \sigma(e^+e^- \to \mu^+\mu^-)$$



$$\begin{aligned}\sigma(e^+e^- \to X) &= N_c \sum_f e_f^2 \sigma(e^+e^- \to q\bar{q}) \\ &= N_c \sum_f e_f^2 \int d\Omega \frac{d\sigma}{d\Omega}(e^+e^- \to \mu^+\mu^-) \\ &= N_c \sum_f e_f^2 \int d\Omega \frac{\alpha^2}{4Q^2}(1+\cos^2\theta) = N_c \sum_f e_f^2 \frac{4\pi\alpha^2}{3Q^2}\end{aligned}$$

$Q^2 \sigma (e^+e^- \to X)$ scala !

Quindi

evidenza di N_c
test di strutture
SU_c (3) e SU_f (3)

$$R = \frac{\sigma(e^+e^-\to \text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)} = N_c \sum_f e_f^2$$

sotto soglia del *c*
$R = 3\ (4/9 + 2/9) = 2$
vicino soglia
risonanze J/ψ, ψ'
sopra soglia
R = 2 + 3 4/9 = 3+1/3

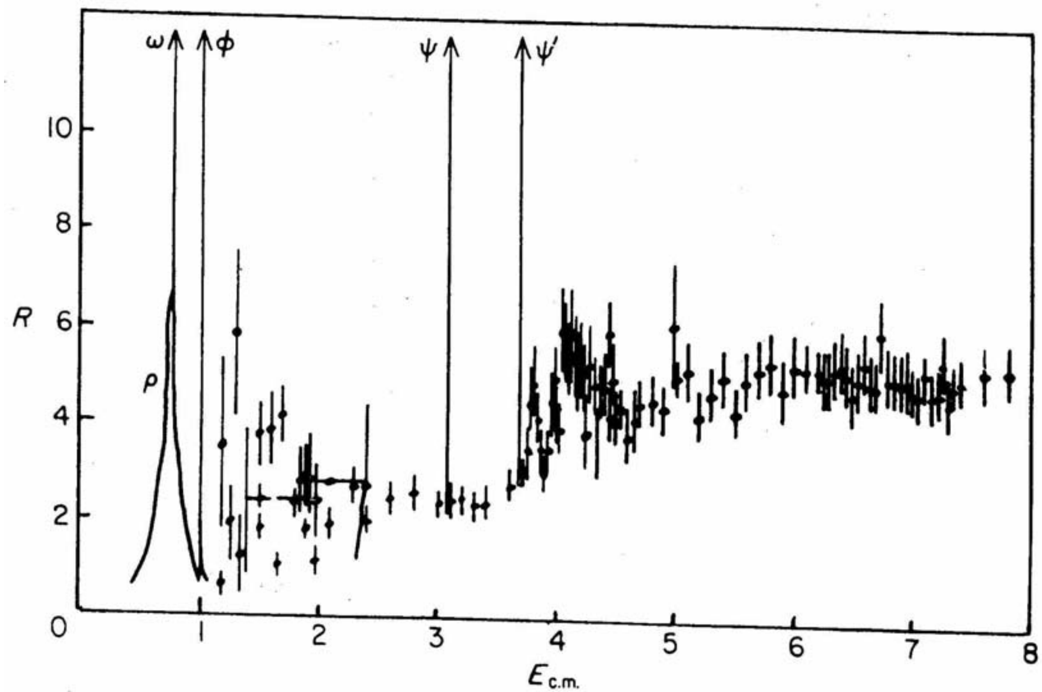
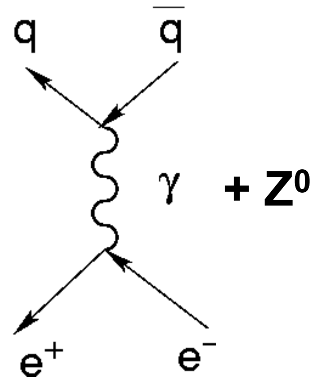


FIG. 11.15. Data on $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$.

Wu, Phys.Rep. **C107** 59 (84)



Per $Q^2 = s \gtrsim M_Z$

$$\sigma(e^+e^- \to X) = \frac{4\pi\alpha^2}{3Q^2} N_c \sum_f e_f^2 \ \{ \ 1$$

← γ

γ Z →

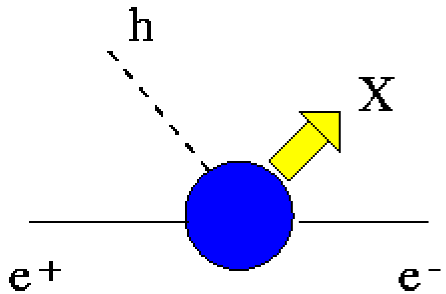
$$-2T_3^2 \frac{Q^2}{Q^2 + M_Z^2} \frac{1}{4 \sin\theta_W \cos\theta_W}$$

Z →

$$+ \left[T_3^2 + \left(T_3 - 2Q\sin^2\theta_W\right)^2\right] \left(\frac{Q^2}{Q^2 + M_Z^2}\right)^2 \times \frac{1}{16\sin^2\theta_W \cos^2\theta_W} \Big\}$$

$$T_3 = \tfrac{1}{2}; \quad Q = 1$$

e^+e^- semi-inclusivo



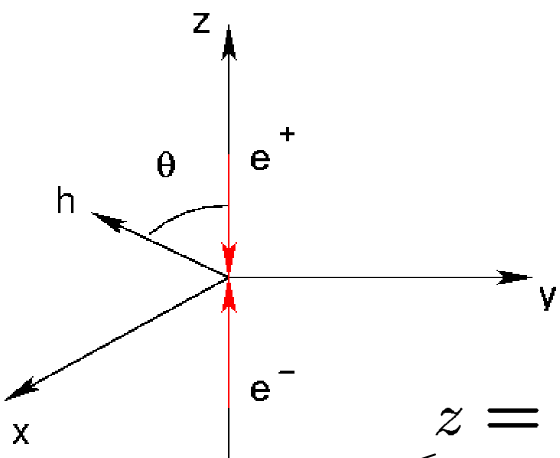
crossing con DIS inclusivo

particelle ultrarelativistiche

$e^{\pm\,\mu} = (E, 0\ , 0, \mp E)$

$q^\mu = (2E, \mathbf{0}\ )$

$P_h^\mu = (E_h, E_h \sin\theta, 0\ , E_h \cos\theta)$



invarianti

$$z = \frac{2P_h\cdot q}{q^2} \sim \frac{4EE_h}{4E^2} = \frac{E_h}{E}$$

$E_h \sim 0 \quad z \to 0$

$E_h \sim E \quad z \to 1$

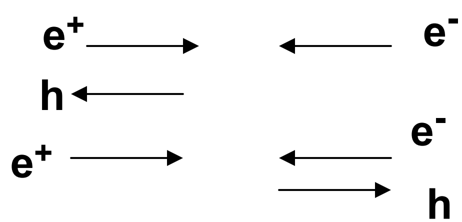
processo elastico

$\sim$ "x_B^{-1}", misura comunque elasticita`

$$y = \frac{P_h\cdot k'}{P_h\cdot q} \sim \frac{EE_h}{2EE_h}(1-\cos\theta) = \frac{1}{2}(1-\cos\theta)$$ $\sim$ rapidity

$y = 0 \to \theta = 0$

$y = 1 \to \theta = \pi$



e^+e^- semi-inclusivo (continua)

regime DIS : $Q^2 \to \infty$ con $z = \frac{2 P_h \cdot q}{Q^2}$ finito

$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 \, dR \qquad \mathcal{F} = 4\sqrt{(k\cdot k')^2 - k^2 k'^2} \overset{\text{TRF}}{=} 2Q^2 \equiv 2s$$

$$dR = (2\pi)^4 \delta(q - P_X - P_h) \, \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \, \frac{d\mathbf{P}_h}{(2\pi)^3 2E_h}$$

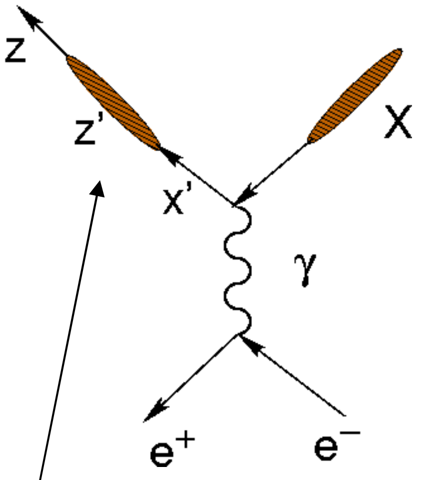
$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu}$$

$$\begin{aligned} 2E_h \frac{d\sigma}{d\mathbf{P}_h} &= \frac{1}{2Q^2} \frac{e^4}{Q^4} \frac{1}{(2\pi)^2} L_{\mu\nu} \frac{1}{2\pi} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \, \delta(q - P_X - P_h) \, H^{\mu\nu} \\ &= \frac{\alpha^2}{Q^6} L_{\mu\nu} W^{\mu\nu} \end{aligned}$$

cross-check

$$\begin{aligned} \sigma &= \int \frac{d\mathbf{P}_h}{(2\pi)^3 2E_h} 2E_h \frac{d\sigma}{d\mathbf{P}_h} \\ &= \frac{\alpha^2}{Q^6} L_{\mu\nu} (2\pi)^2 \int \frac{d\mathbf{P}_{X'}}{(2\pi)^3 2P_{X'}^0} (2\pi)^4 \delta(q - P_{X'} - P_h) \int \frac{d\mathbf{P}_h}{(2\pi)^3 2E_h} \langle 0|J^\mu|P_{X'} P_h\rangle \langle P_{X'} P_h|J^\nu|0\rangle \\ &= \frac{\alpha^2}{Q^6} L_{\mu\nu} (2\pi)^2 \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(q - P_X) \int \frac{d\mathbf{P}_h}{(2\pi)^3 2E_h} \langle 0|J^\mu|P_X - P_h, P_h\rangle \langle P_X - P_h, P_h|J^\nu|0\rangle \\ &= \frac{4\pi^2\alpha^2}{Q^6} L_{\mu\nu} W^{\mu\nu} \end{aligned}$$

In alternativa, QPM :



$$\frac{d\sigma}{dy\,dz} = N_c \sum_f \int_0^1 dx'dz' \, \frac{d\sigma^{el}}{dx'dy}(e^+e^- \to q\bar{q}) \times D_f(z')\,\delta\left(z'x' - z\right)$$

$$= N_c \sum_f e_f^2 \int_0^1 dx'dz' \, \frac{d\sigma^{el}}{dy}(e^+e^- \to \mu^+\mu^-) \times \delta(1 - x')\, D_f(z')\,\delta\left(z'x' - z\right)$$

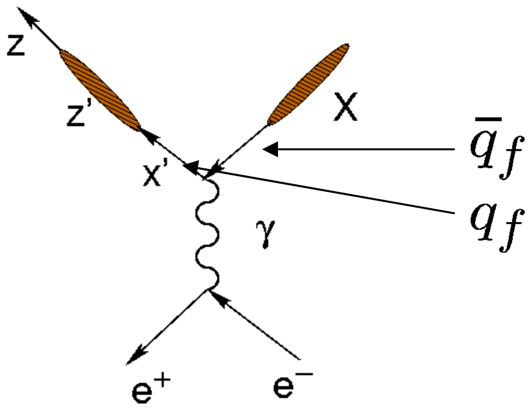
elastico

probabilita` di trovare adrone con frazione *z'* del momento *x'* del partone.

somma su tutte le combinazioni *x' z'* → adrone con frazione *z* dell'energia disponibile

$$= N_c \frac{\pi\alpha^2}{Q^2}(1 + \cos^2\theta) \sum_f e_f^2 \, D_f(z)$$

commenti



conservazione del sapore nel vertice
→ stesso *f* per (anti)quark ⇐ Born approx.

correzioni ad ordini superiori da, e.g., *g* che decade in coppie → *f* ' di antiquark ≠ *f* di quark

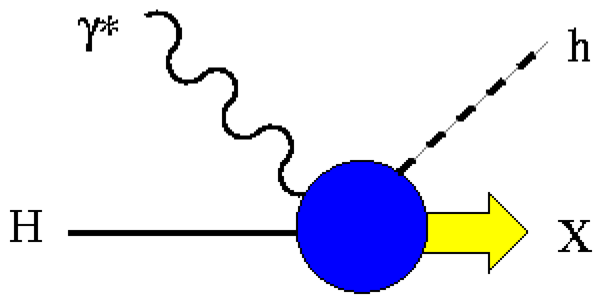
$$\frac{d\sigma}{dydz} = N_c \frac{\pi\alpha^2}{Q^2} (1 + \cos^2\theta) \sum_f e_f^2 D_f(z)$$

evidenza del colore

distribuzione angolare dell'adrone e\` data da una sez. d'urto di QED ($e^+e^- \to \mu^+\mu^-$) !

nuova incognita
fattorizzazione → estrarre info da confronto con I dati anche in altri processi
⇒ DIS semi-inclusivo (SIDIS)

Semi-inclusive DIS (SIDIS)



stesse definizioni del caso inclusivo per cinematica e invarianti con in piu`

$$z = \frac{P \cdot P_h}{P \cdot q} \sim -\frac{2 P_h \cdot q}{Q^2}$$

$z < 0$ $\quad P_h$ opposto a $P \rightarrow$ h viene da frammentazione del bersaglio H

target fragmentation region

$z > 0$ $\quad P_h$ equiverso a $P \rightarrow$ h viene da frammentazione di partone

current fragmentation region

SIDIS

$$d\sigma = \frac{1}{\mathcal{F}}|\mathcal{M}|^2\, dR \qquad \mathcal{F} = 4\sqrt{(P\cdot k)^2 - P^2k^2} \overset{\text{TRF}}{=} 4ME \equiv 2s$$

$$dR = (2\pi)^4\delta(k+P-k'-P_X-P_h)\;\frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}\,\frac{d\mathbf{k}}{(2\pi)^3 2E'}\,\frac{d\mathbf{P}_h}{(2\pi)^3 2E_h}$$

$$L_{\mu\nu} = \quad 2k_\mu k'_\nu + 2k'_\mu k_\nu - Q^2 g_{\mu\nu}$$

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4}\,L_{\mu\nu}\,H^{\mu\nu}$$

$$H^{\mu\nu} = \; \tfrac{1}{2}\textstyle\sum_{SS_h}\langle PS|J^\mu|P_X,P_hS_h\rangle\langle P_X,P_hS_h|J^\nu|PS\rangle$$

$$\begin{aligned} 2E_h\,\frac{d\sigma}{d\mathbf{P}_h dE' d\Omega} &= \frac{1}{4ME}\,\frac{e^4}{Q^4}\,\frac{E'}{16\pi^3}\,\frac{1}{(2\pi)^3}\,L_{\mu\nu}\int\frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}\,(2\pi)^4\,\delta(k+P-k'-P_X-P_h)\,H^{\mu\nu} \\ &= \frac{\alpha^2}{Q^4}\,\frac{E'}{E}\,L_{\mu\nu}\,W^{\mu\nu} \end{aligned}$$

cross-check

$$\begin{aligned} \frac{d\sigma}{dE'd\Omega} &= \int\frac{d\mathbf{P}_h}{(2\pi)^3 2E_h}\,2E_h\,\frac{d\sigma}{d\mathbf{P}_h dE' d\Omega} \\ &= \frac{\alpha^2}{Q^4}\,\frac{E'}{E}\,\frac{1}{4M\pi}\,L_{\mu\nu}\int\frac{d\mathbf{P}_{X'}}{(2\pi)^3 2P_{X'}^0}\,(2\pi)^4\delta(q+P-P_{X'}-P_h)\int\frac{d\mathbf{P}_h}{(2\pi)^3 2E_h}\,\langle P|J^\mu|P_{X'}P_h\rangle\langle P_{X'}P_h|J^\nu|P\rangle \\ &= \frac{\alpha^2}{Q^4}\,\frac{E'}{E}\,\frac{1}{4M\pi}\,L_{\mu\nu}\int\frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}\,(2\pi)^4\delta(q+P-P_X)\int\frac{d\mathbf{P}_h}{(2\pi)^3 2E_h}\,\langle P|J^\mu|P_X-P_h,P_h\rangle\langle P_X-P_h,P_h|J^\nu|P\rangle \\ &= \frac{\alpha^2}{Q^4}\,\frac{E'}{E}\,L_{\mu\nu}\,W^{\mu\nu} \end{aligned}$$

SIDIS (continua)

$$\frac{d\mathbf{k}'}{(2\pi)^3 2E'} = \frac{E' dE' d\Omega}{16\pi^3} = \frac{\pi y s dx_B dy}{16\pi^3}$$

$$\frac{d\mathbf{P}_h}{2E_h} \sim d\mathbf{P}_{h\perp} \frac{dE_h}{2E_h} \sim d\mathbf{P}_{h\perp} \frac{dz}{z}$$

$$\frac{d\sigma}{d\mathbf{P}_{h\perp} dx_B dy dz} = \frac{\pi y s}{zE'} 2E_h \frac{d\sigma}{d\mathbf{P}_h dE' d\Omega} = \frac{\pi\alpha^2}{Q^4} \frac{y}{z} L_{\mu\nu} W^{\mu\nu}$$

$$d\mathbf{P}_{h\perp} = z^2 d\mathbf{q}_T$$

$$\frac{d\sigma}{dx_B dy dz} = \frac{\pi\alpha^2}{Q^4} yz\, L_{\mu\nu} \int d\mathbf{q}_T\, W^{\mu\nu}$$

cruciale per fattorizzazione tra distribuzione e frammentazione

Metodo alternativo : QPM

$$
\begin{aligned}
\frac{d\sigma}{dx_B dy dz} &= \sum_f \int_0^1 dx dz' \, \frac{d\sigma^{el}}{dx dy}(e^- q \to e^- q) \, \phi_f(x) \, \delta\left(1 - \frac{x}{x_B}\right) \, D_f\left(\frac{z'}{x_B}\right) \, \delta(z' - zx) \\
&= \sum_f e_f^2 \int_0^1 dx dz' \, \frac{d\sigma^{el}}{dx dy}(e^- \mu^+ \to e^- \mu^+) \, \phi_f(x) \, \delta\left(1 - \frac{x}{x_B}\right) \, D_f\left(\frac{z'}{x_B}\right) \, \delta(z' - zx) \\
&= \sum_f e_f^2 \int_0^1 dx dz' \, \frac{4\pi\alpha^2 s}{Q^4} \left(\frac{y^2}{2} + 1 - y\right) \, \phi_f(x) \, \delta\left(1 - \frac{x}{x_B}\right) \, D_f\left(\frac{z'}{x_B}\right) \, \delta(z' - zx) \\
&= \frac{4\pi\alpha^2 s}{Q^4} \left(\frac{y^2}{2} + 1 - y\right) \sum_f e_f^2 \, x_B \, \phi_f(x_B) \, D_f(z)
\end{aligned}
$$

cross-check

Callan-Gross

$F_2 = 2\, x_B\, F_1$

$$
\begin{aligned}
\frac{d\sigma}{dx_B dy} &= \int dz \, \frac{d\sigma}{dx_B dy dz}\Big|_{D_f(z)=\delta(1-z)} \\
&= \frac{4\pi\alpha^2 s}{Q^4} \left(\frac{y^2}{2} + 1 - y\right) x_B \sum_f e_f^2 \, \phi_f(x_B) = \frac{4\pi\alpha^2 s}{Q^4} \left(x_B y^2 F_1 + (1 - y) F_2\right)
\end{aligned}
$$

Fenomenologia di SIDIS

fasci di neutrini $\nu p \to \mu^- h X$

$$\frac{1}{\sigma_{incl}} \frac{d\sigma^\nu}{dx_B dz} \sim \frac{d\, D_u^h + \frac{1}{3}\bar{u}\, D_{\bar{d}}^h}{d + \frac{1}{3}\bar{u}} \sim D_u^h$$

fasci di antineutrini $\bar{\nu} p \to \mu^+ h X$

$$\frac{1}{\sigma_{incl}} \frac{d\sigma^{\bar{\nu}}}{dx_B dz} \sim \frac{u\, D_d^h + \frac{1}{3}\bar{d}\, D_{\bar{u}}^h}{u + \frac{1}{3}\bar{d}} \sim D_d^h = D_u^{\bar{h}}$$

no *c* e $\theta_C \sim 0$

isospin & *C*-conjug. symmetry

$$D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+}$$
$$D_d^{\pi^+} = D_u^{\pi^-} = D_{\bar{d}}^{\pi^-} = D_{\bar{u}}^{\pi^+}$$
$$D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^-} = D_{\bar{s}}^{\pi^+}$$

dati sperimentali mostrano che

$$\frac{\frac{1}{\sigma_{incl}} \frac{d\sigma^\nu}{dx_B dz}(\nu p \to \mu^- \pi^+ X)}{\frac{1}{\sigma_{incl}} \frac{d\sigma^{\bar{\nu}}}{dx_B dz}(\bar{\nu} p \to \mu^+ \pi^- X)} \sim 2.6$$

e indipendente da x_B in $0.7 \gtrsim z \gtrsim 0.3$

Quindi
$$\frac{D_u^{\pi^+}}{D_u^{\pi^-}} \equiv \eta(z) \sim 2.6$$

N.B. per z $\gtrsim$ 0.7 dominio quark valenza, ma $\pi^- = d\bar{u}$ quindi η(z) cresce

Fenomenologia di SIDIS (continua)

fasci di elettroni $e^- p \to e'^- \pi^+ X$

approssimazioni

$$s = \bar{s} = \bar{u} = \bar{d} \equiv K(x_B)$$

$$D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_u^{\pi^-}$$

$$\frac{\sum_f e_f^2 \phi_f(x_B)}{\sigma_{incl}} \frac{d\sigma}{dx_B dz} = \sum_f e_f^2 \phi_f(x_B) D_f(z) = \frac{D_u^{\pi^-}}{9} [4u\eta + d + 4K + K\eta + 2K]$$

$$= \frac{D_u^{\pi^-}}{9} [4u_V\eta + d_V + 5K\eta + 7K]$$

u* = *u*$_V$ + *K

per $e^- p \to e'^- \pi^- X$

$$\frac{\sum_f e_f^2 \phi_f(x_B)}{\sigma_{incl}} \frac{d\sigma}{dx_B dz} = \sum_f e_f^2 \phi_f(x_B) D_f(z) = ... = \frac{D_u^{\pi^-}}{9} [4u_V + d_V\eta + 5K\eta + 7K]$$

$$R = \left[\frac{\sum_f e_f^2 \phi_f(x_B)}{\sigma_{incl}} \frac{d\sigma^{\pi^+}}{dx_B dz}\right] \times \left[\frac{\sum_f e_f^2 \phi_f(x_B)}{\sigma_{incl}} \frac{d\sigma^{\pi^-}}{dx_B dz}\right]^{-1} = \frac{4u_V\eta + d_V + (5\eta + 7)K}{4u_V + d_V\eta + (5\eta + 7)K}$$

η da SIDIS con ν
u_V, d_V, K da DIS inclusivo
} confronto con i dati

$$R_p = \frac{4u_V\eta + d_V + (5\eta+7)K}{4u_V + d_V\eta + (5\eta+7)K}$$

$$\eta \overset{x_B\to 1}{\longleftarrow} R_p \overset{x_B\to 0}{\longrightarrow} 1$$

valence **sea**

$$R_n = R_p(u_V \leftrightarrow d_V)$$

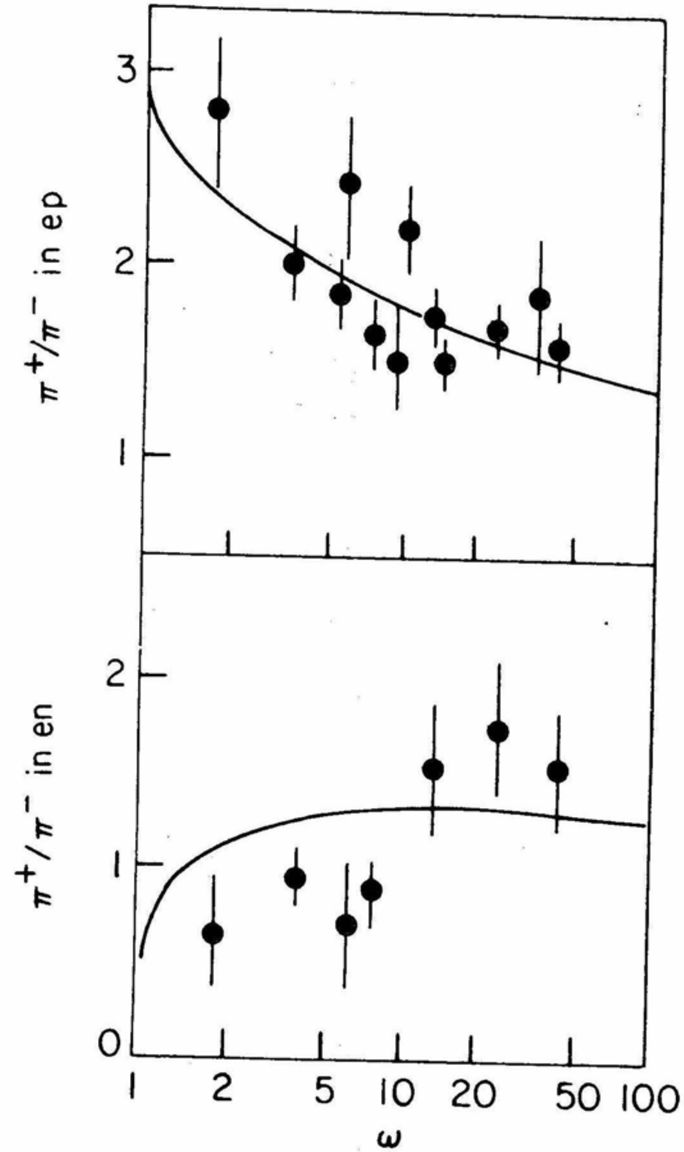
$$\frac{1}{\eta} \overset{x_B\to 1}{\longleftarrow} R_n \overset{x_B\to 0}{\longrightarrow} 1$$

valence **sea**

$\omega \equiv \nu$

$x_B \to 0$

per $\nu \to \infty$



12.7. Omega dependence of the π^+/π^- ratio in ep and en interactions compared with quark–parton model expectations.

Fenomenologia e$^+$e$^-$ semi-inclusivo

$$\sigma_{incl} = \sigma(e^+e^- \to X) = N_c \frac{4\pi\alpha^2}{3Q^2} \sum_f e_f^2$$

$$y = \tfrac{1}{2}(1 - \cos\theta)$$

$$\sigma_{\mu\mu} = \int_0^1 dy \frac{d\sigma}{dy}(e^+e^- \to \mu^+\mu^-) = \frac{\pi\alpha^2}{Q^2} \int_0^1 dy (1 + \cos^2\theta) = \frac{4\pi\alpha^2}{3Q^2}$$

$$\frac{d\sigma}{dz} = \int_0^1 dy \frac{d\sigma}{dz\,dy}(e^+e^- \to hX) = N_c \frac{\pi\alpha^2}{Q^2} \sum_f e_f^2 D_f(z) \int_0^1 dy\,(1 + \cos^2\theta)$$

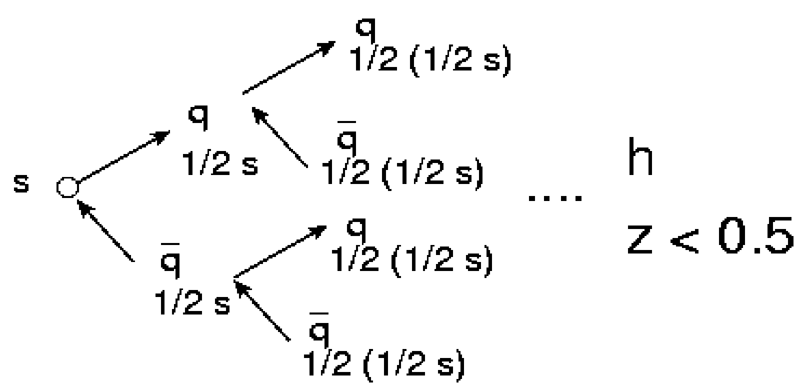
$$= N_c \frac{4\pi\alpha^2}{3Q^2} \sum_f e_f^2 D_f(z)$$

Quindi $$Q^2 \frac{1}{\sigma_{\mu\mu}} \frac{d\sigma}{dz} = N_c \sum_f e_f^2 \left(D_f(z) + D_{\bar{f}}(z)\right)$$ info su frammentazione

Inoltre $$Q^2 \frac{d\sigma}{dz} = \frac{4\pi\alpha^2}{3} N_c \sum_f e_f^2 \left(D_f(z) + D_{\bar{f}}(z)\right)$$

scaling in z $\forall$ $Q^2 \equiv s$

Ma



violazione per $0.5 \gtrsim z$

$s\,\frac{d\sigma}{dz}(e^+e^- \to hX)$

Schwitters *et al.*,
P.R.L.. **35** 1320 (75)

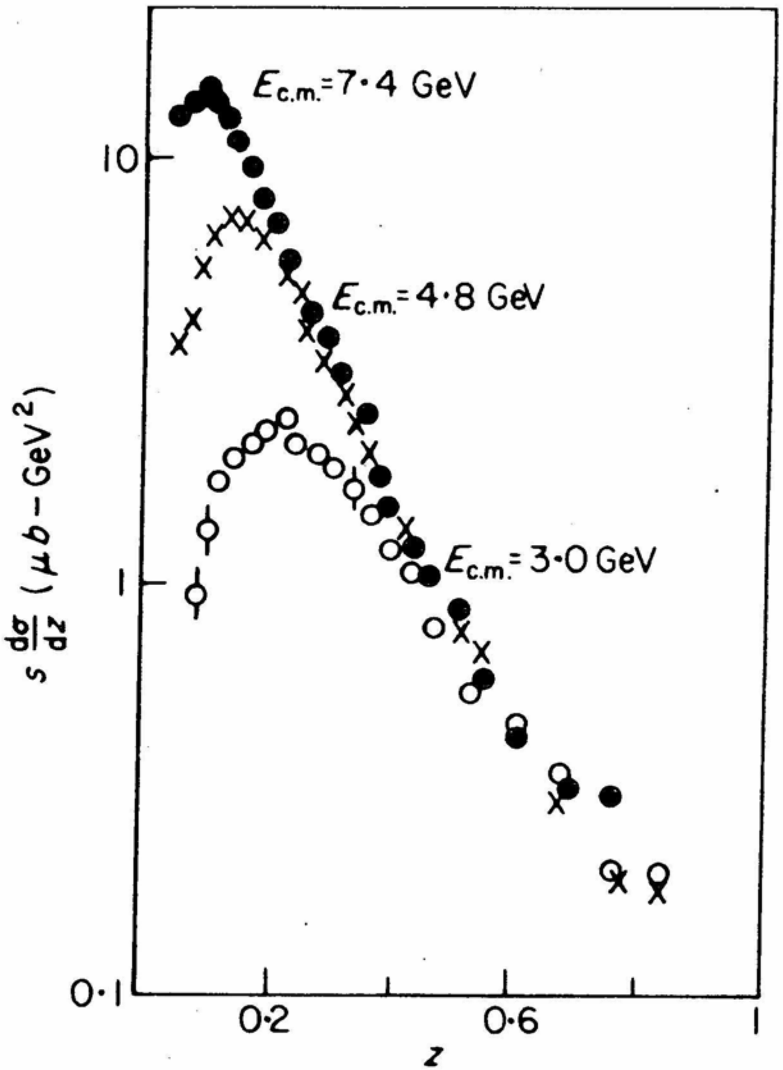


FIG. 12.9. $s\,(d\sigma/dz)$ at 3, 4·8, and 7·4 GeV e^+e^- centre of mass energies.

Wu, Phys.Rep. **C107** 59 (84)

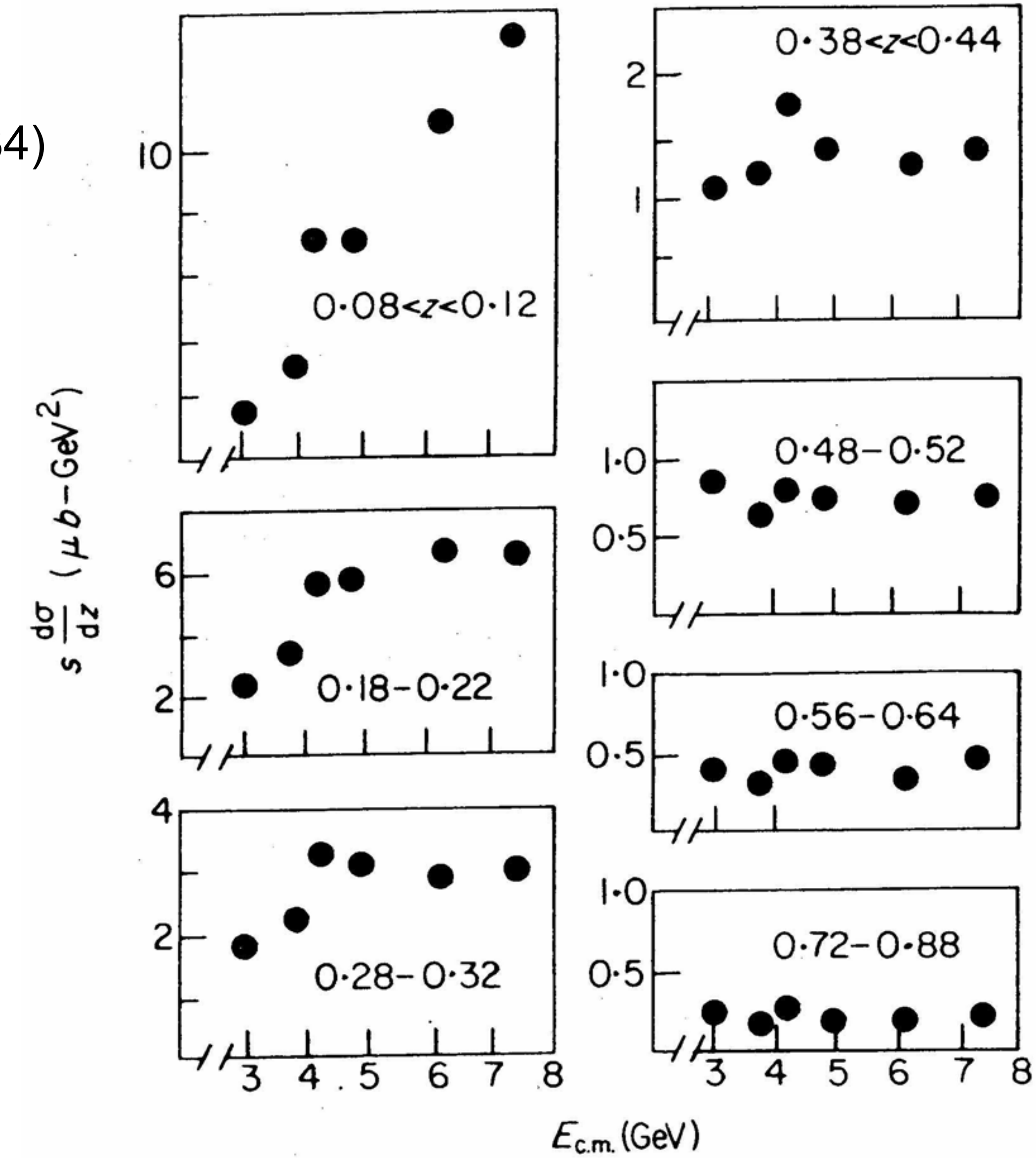


FIG. 12.10. $s\,(d\sigma/dz)$ versus $E_{c.m.}$ for various z intervals.

SIDIS

$$e^- p \to e'^- h^\pm X$$

$$\frac{1}{\sigma_{incl}} \frac{d\sigma}{dx_B dz} = \frac{\sum_f e_f^2 \phi_f(x_B) D_f(z)}{\sum_f e_f^2 \phi_f(x_B)}$$

$$\sim D_u^h$$

e+e- semi-inclusivo

$$e^+ e^- \to h^\pm X$$

$$\frac{1}{\sigma_{incl}} \frac{d\sigma}{dz} = \frac{\sum_f e_f^2 D_f(z)}{\sum_f e_f^2}$$

$$\sim D_u^h + D_{\bar{u}}^h$$

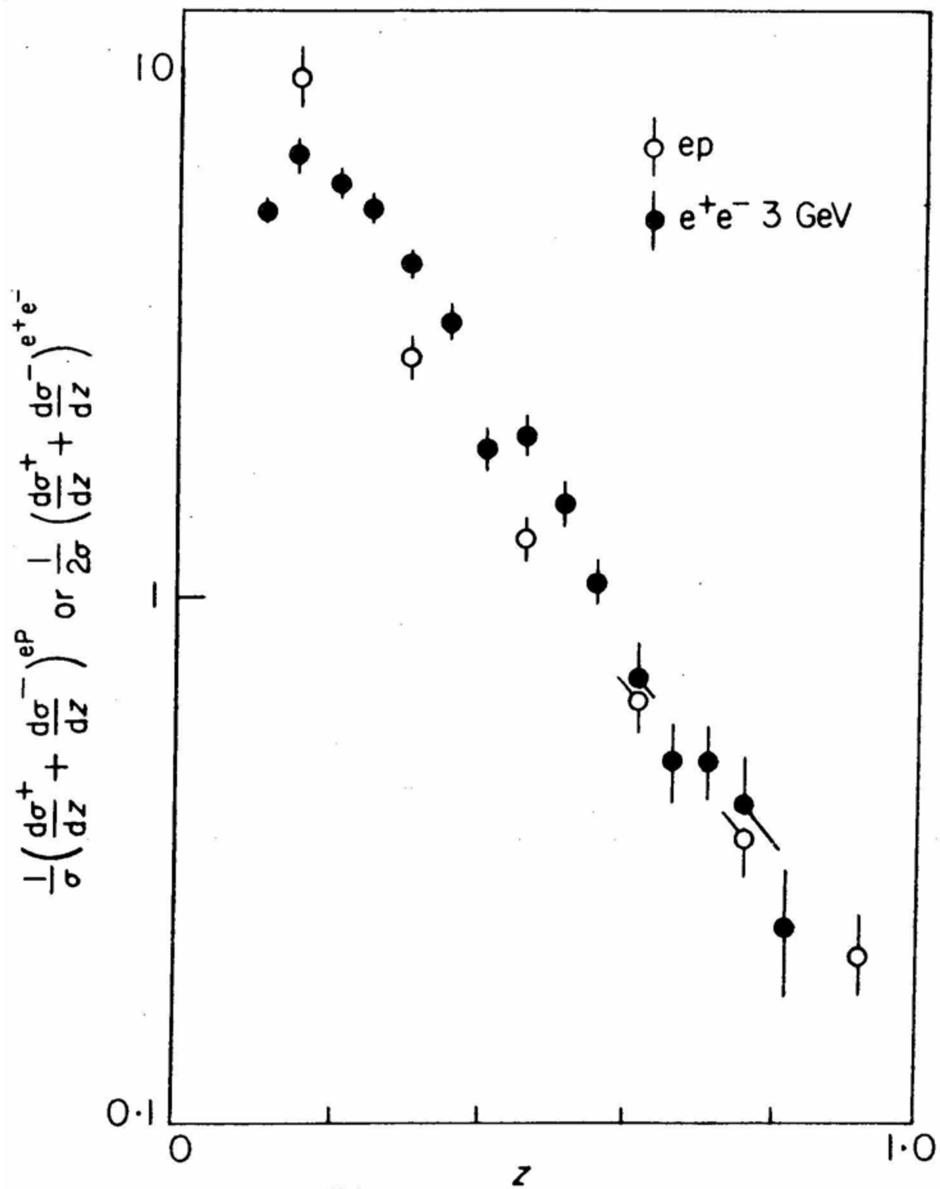
dominanza *u* quark

quindi

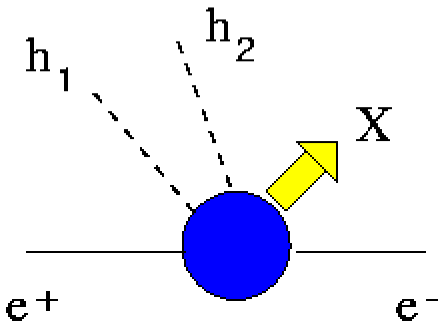
$$\frac{1}{\sigma_{incl}} \left(\frac{d\sigma^{h^+}}{dx_B dz} + \frac{d\sigma^{h^-}}{dx_B dz} \right) \Leftrightarrow \frac{1}{\sigma_{incl}} \frac{1}{2} \left(\frac{d\sigma^{h^+}}{dz} + \frac{d\sigma^{h^-}}{dz} \right)$$

$$\sim D_u^{h^+} + D_u^{h^-}$$

Gilman, Int. Symp. on lepton and photon interactions at high energies, SLAC (75)



e^+e^- semi-inclusivo in due adroni



$$z_1 = \frac{2P_1\cdot q}{q^2} \qquad z_2 = \frac{2P_2\cdot q}{q^2}$$

$$d\sigma = \frac{1}{\mathcal{F}}|\mathcal{M}|^2\, dR \qquad |\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu}$$

$$\mathcal{F} \stackrel{\text{TRF}}{=} 2Q^2 \equiv 2s \qquad dR = (2\pi)^4\delta(q-P_X-P_1-P_2)\,\frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}\,\frac{d\mathbf{P}_1}{(2\pi)^3 2E_1}\,\frac{d\mathbf{P}_2}{(2\pi)^3 2E_2}$$

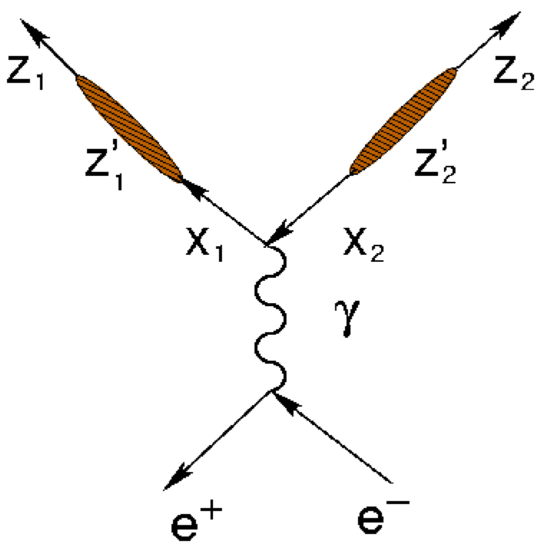
$$\begin{aligned} 2E_1 2E_2\,\frac{d\sigma}{d\mathbf{P}_1 d\mathbf{P}_2} &= \frac{1}{2Q^2}\,\frac{e^4}{Q^4}\,\frac{1}{(2\pi)^2}\,L_{\mu\nu} \\ &\quad \times \frac{1}{(2\pi)^4}\int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}\,(2\pi)^4\,\delta(q-P_X-P_1-P_2)\,H^{\mu\nu} \\ &= \frac{\alpha^2}{Q^6}\,L_{\mu\nu}\,W^{\mu\nu} \end{aligned}$$

cross-check

$$\int \frac{d\mathbf{P}_2}{2E_2}\,2E_1\,2E_2\,\frac{d\sigma}{d\mathbf{P}_1 d\mathbf{P}_2} = 2E_1\,\frac{d\sigma}{d\mathbf{P}_1} = \ldots . = \frac{\alpha^2}{Q^6}\,L_{\mu\nu}\,W^{\mu\nu}$$

In alternativa, QPM :

$$\begin{aligned}
\frac{d\sigma}{dy\,dz_1\,dz_2} &= N_c \sum_f \int_0^1 dx_1 dx_2 dz_1' dz_2' \,\frac{d\sigma^{el}}{dx_1 dx_2 dy}(e^+e^- \to q\bar{q}) \\
&\quad \times D_f(z_1')\,\delta\left(z_1' x_1 - z_1\right) D_f(z_2')\,\delta\left(z_2' x_2 - z_2\right) \\
&= N_c \sum_f e_f^2 \int_0^1 dx_1 dx_2 dz_1' dz_2' \,\frac{d\sigma^{el}}{dy}(e^+e^- \to \mu^+\mu^-) \\
&\quad \times \delta(1-x_1)\, D_f(z_1')\,\delta\left(z_1' x_1 - z_1\right) \\
&\quad \times \delta(1-x_2)\, D_f(z_2')\,\delta\left(z_2' x_2 - z_2\right) \\
&= N_c \frac{\pi\alpha^2}{Q^2}(1+\cos^2\theta) \sum_f e_f^2\, D_f(z_1)\, D_f(z_2)
\end{aligned}$$



cross-check $D_f(z_2) = \delta(1-z_2)$ secondo adrone $\equiv$ jet adronico

$$\int dz_2 \, \frac{d\sigma}{dy dz_1 dz_2}\bigg|_{D_f(z_2)=\delta(1-z_2)} = \frac{d\sigma}{dy dz} = N_c \frac{\pi\alpha^2}{Q^2} (1+\cos^2\theta) \sum_f e_f^2 \, D_f(z)$$

e^+e^- semi-inclusivo

Adesso $D_f(z_1) = \delta(1-z_1)$ anche primo adrone $\equiv$ jet adronico

$$\int dz \, \frac{d\sigma}{dy dz}\bigg|_{D_f(z)=\delta(1-z)} = \frac{d\sigma^{jet}}{dy} = N_c \frac{\pi\alpha^2}{Q^2} (1+\cos^2\theta) \sum_f e_f^2$$

distribuzione angolare di tutti gli adroni nello stato finale = sez. d'urto di jet

$$\frac{d\sigma^{el}}{dy}(e^+e^- \to q\bar{q}) = \frac{\pi\alpha^2}{Q^2}\underline{(1+\cos^2\theta)}\, e_f^2$$

$$\frac{d\sigma}{dydz}(e^+e^- \to hX) = N_c \frac{\pi\alpha^2}{Q^2}\underline{(1+\cos^2\theta)} \sum_f e_f^2 D_f(z)$$

$$\frac{d\sigma}{dy}(e^+e^- \to \text{jets}) = N_c \frac{\pi\alpha^2}{Q^2}\underline{(1+\cos^2\theta)} \sum_f e_f^2$$

gli adroni sono “frammenti” dei partoni a spin ½ del processo elementare

eventi a molti adroni = gruppi di adroni con p_T limitato rispetto ad un certo asse

dato asse θ, sfericita` $S = \frac{3\sum_i p_{Ti}^2}{2\sum_i p_i^2}$ $\begin{cases} S = 1 & \text{sfera} \\ S = 0 & \text{jet} \end{cases}$ $S \overset{s\to\infty}{\to} 0$

adrone in stato finale con $1 \geq z \geq 0$ si muove in un jet che rappresenta la direzione θ del quark di frammentazione rispetto all’asse z

la direzione del jet e` data processo elementare di QED !

Wu, Phys. Rep. **C107** 59 (84)