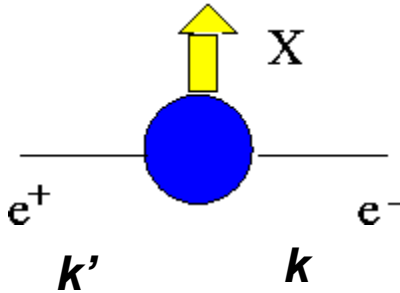


Riassunto della lezione precedente

- verifica sperimentale di QPM in reazioni elettrodeboli :
 1. DIS con (anti)neutrini su nuclei isoscalari
 - sez. d'urto di neutrino domina quella per antineutrino
 - antipartoni soppressi in mezzo nucleare
 2. DIS con (anti)neutrini su protone
 - dominanza di quark valenza / mare per $x_B \rightarrow 1/0$
 3. rapporto di carica per nuclei isoscalari
- regole di somma :
 1. Gottfried sum rule → $SU_f(3)$ rotta per i quark del mare
 2. Momentum sum rule → gluoni portano metà del momento dell'adrone
- verifica fattorizzazione → esplorare nuovi processi fondamentali

e^+e^- inclusivo



$$q = k+k' \text{ time-like} \quad q^2 \equiv Q^2 = s \geq 0$$

$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 dR$$

$$\mathcal{F} = 4\sqrt{(k \cdot k')^2 - k^2 k'^2} \stackrel{\text{TRF}}{=} 2Q^2 \equiv 2s$$



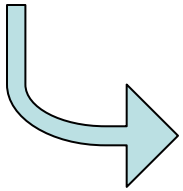
$$\mathcal{M} = \bar{v}(k') \gamma_\mu u(k) \frac{e^2}{Q^2} \langle P_X | J^\mu(0) | 0 \rangle \quad dR = (2\pi)^4 \delta(k + k' - P_X) \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}$$

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu} \begin{cases} L_{\mu\nu} = 2(k_\mu k'_\nu + k'_\mu k_\nu - k \cdot k' g_{\mu\nu}) \\ H^{\mu\nu} = \sum_{S_X} \langle 0 | J^\mu | P_X \rangle \langle P_X | J^\nu | 0 \rangle \end{cases}$$

$$\sigma = \frac{1}{2} \frac{1}{2Q^2} \frac{e^4}{Q^4} L_{\mu\nu} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(q - P_X) H^{\mu\nu} = \frac{4\pi^2 \alpha^2}{Q^6} L_{\mu\nu} W^{\mu\nu}$$

media su polarizzazioni iniziali

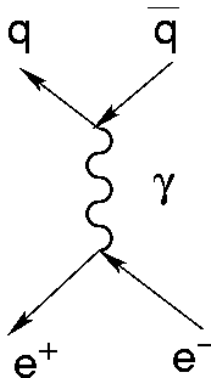
QPM picture



no adroni in stato iniziale e finale

σ in QPM $\equiv \sigma$ elementare $e^+e^- \rightarrow q\bar{q}$

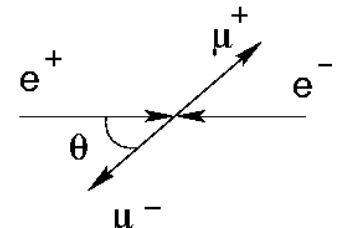
solo N_c modi di creare la coppia conservando il colore nel vertice



$Q^2 = s$ tale da avere solo produzione di γ

$$\sigma(e^+e^- \rightarrow q\bar{q}) \equiv \sigma(e^+e^- \rightarrow \mu^+\mu^-)$$

$$\begin{aligned}\sigma(e^+e^- \rightarrow X) &= N_c \sum_f e_f^2 \sigma(e^+e^- \rightarrow q\bar{q}) \\ &= N_c \sum_f e_f^2 \int d\Omega \frac{d\sigma}{d\Omega}(e^+e^- \rightarrow \mu^+\mu^-) \\ &= N_c \sum_f e_f^2 \int d\Omega \frac{\alpha^2}{4Q^2} (1 + \cos^2 \theta) = N_c \sum_f e_f^2 \frac{4\pi\alpha^2}{3Q^2}\end{aligned}$$



$$Q^2 \sigma(e^+e^- \rightarrow X) \text{ scala !}$$



Quindi

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum_f e_f^2$$

evidenza di N_c
test di strutture
 $SU_c(3)$ e $SU_f(3)$

sotto soglia del c

$$R = 3 \left(\frac{4}{9} + \frac{2}{9} \right) = 2$$

vicino soglia

risonanze J/ψ , ψ'

sopra soglia

$$R = 2 + 3 \frac{4}{9} = 3 + \frac{1}{3}$$

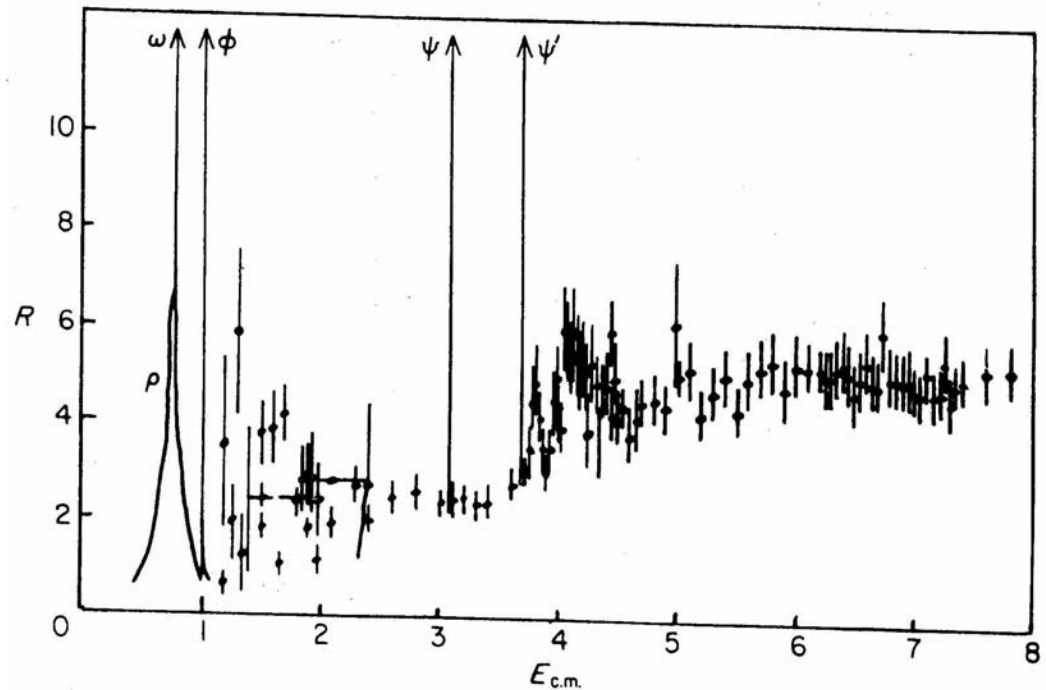
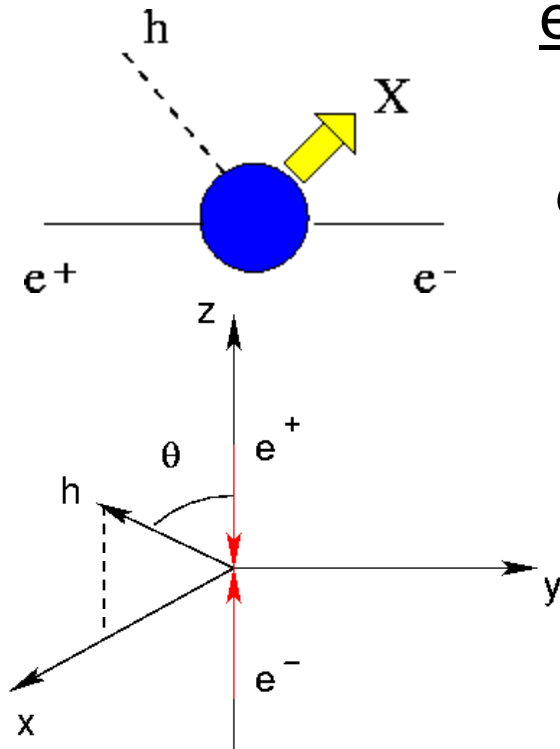


FIG. 11.15. Data on $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$.

Wu, Phys.Rep. **C107** 59 (84)

e^+e^- semi-inclusivo

crossing con DIS inclusivo



particelle ultrarelativistiche

$$k^\pm{}^\mu = (E, 0, 0, \mp E)$$

$$q^\mu = (2E, \mathbf{0})$$

$$P_h^\mu = (E_h, E_h \sin \theta, 0, E_h \cos \theta)$$

invarianti

$$z = \frac{2P_h \cdot q}{q^2} \sim \frac{4EE_h}{4E^2} = \frac{E_h}{E}$$

$\sim "x_B^{-1}"$, misura elasticita`

$$E_h \sim 0 \quad z \rightarrow 0$$

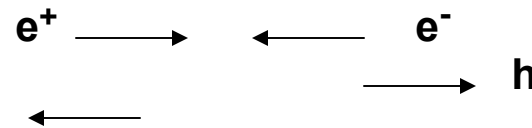
$$E_h \sim E \quad z \rightarrow 1$$

$$y = \frac{P_h \cdot k'}{P_h \cdot q} \sim \frac{EE_h}{2EE_h} (1 + \cos \theta) = \frac{1}{2} (1 + \cos \theta) \sim \text{rapidity}$$

**processo
elastico**

$$y = 0 \rightarrow \theta = \pi$$

$$y = 1 \rightarrow \theta = 0$$



e^+e^- semi-inclusivo (continua)

$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 dR \quad \mathcal{F} = 4\sqrt{(k \cdot k')^2 - k^2 k'^2} \stackrel{\text{TRF}}{=} 2Q^2 \equiv 2s$$

$$dR = (2\pi)^4 \delta(q - P_X - P_h) \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \frac{d\mathbf{P}_h}{(2\pi)^3 2E_h}$$

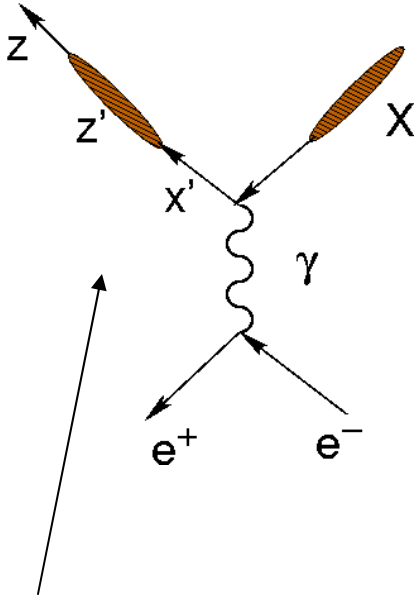
$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H_{1PI}^{\mu\nu}$$

$$\begin{aligned} 2E_h \frac{d\sigma}{d\mathbf{P}_h} &= \frac{1}{2} \frac{1}{2Q^2} \frac{1}{(2\pi)^2} \frac{e^4}{Q^4} L_{\mu\nu} \frac{1}{2\pi} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(q - P_X - P_h) H_{1PI}^{\mu\nu} \\ &= \frac{\alpha^2}{Q^6} L_{\mu\nu} W_{1PI}^{\mu\nu} \end{aligned}$$

N.B. cross-check

$$\sigma = \int \frac{d\mathbf{P}_h}{2E_h} 2E_h \frac{d\sigma}{d\mathbf{P}_h} = \frac{4\pi^2 \alpha^2}{Q^6} L_{\mu\nu} W^{\mu\nu}$$

QPM picture



regime DIS : $Q^2 \rightarrow \infty$ con $z = \frac{2P_h \cdot q}{Q^2}$ finito

$$\begin{aligned} \frac{d\sigma}{dydz} &= N_c \sum_{f=q,\bar{q}} \int_0^1 dx' dz' \frac{d\sigma^{el}}{dx' dy} (e^+ e^- \rightarrow q \bar{q}) \\ &\quad \times D_f(z') \delta(z' x' - z) \\ &= N_c \sum_{f=q,\bar{q}} e_f^2 \int_0^1 dx' dz' \frac{d\sigma^{el}}{dy} (e^+ e^- \rightarrow \mu^+ \mu^-) \\ &\quad \times \delta(1 - x') D_f(z') \delta(z' x' - z) \end{aligned}$$

elastico

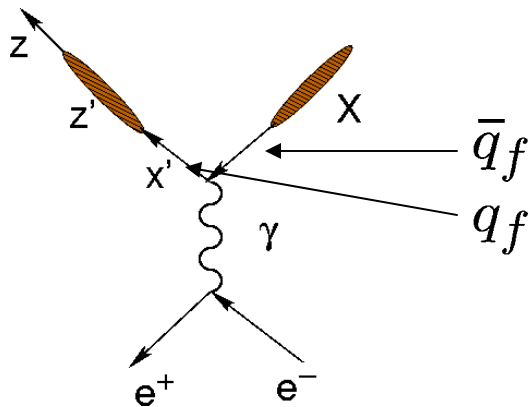


probabilità di trovare adrone
con frazione z' del momento x'
del partone.
somma su tutte le combinazioni
 $x' z' \rightarrow$ adrone con frazione z
dell'energia disponibile

$$= N_c \frac{\pi \alpha^2}{Q^2} (1 + \cos^2 \theta) \sum_{f=q,\bar{q}} e_f^2 D_f(z)$$

commenti





conservazione del sapore nel vertice
 \rightarrow stesso f per (anti)quark \Leftarrow Born approx.

correzioni ad ordini superiori da, e.g., g che decade in coppie $\rightarrow f'$ di antiquark $\neq f$ di quark

$$\frac{d\sigma}{dydz} = N_c \frac{\pi\alpha^2}{Q^2} (1 + \cos^2 \theta) \sum_{f=q,\bar{q}} e_f^2 D_f(z)$$

evidenza del colore

distribuzione angolare
 dell'adrone e' data da
 una sez. d'urto di QED
 $(e^+e^- \rightarrow \mu^+\mu^-)$!

nuova incognita
 fattorizzazione
 \rightarrow estrarre info da
 confronto con i dati

Fenomenologia e^+e^- semi-inclusivo

$$\sigma_{incl} = \sigma(e^+e^- \rightarrow X) = N_c \frac{4\pi\alpha^2}{3Q^2} \sum_f e_f^2$$

$$\sigma_{\mu\mu} = \int_0^1 dy \frac{d\sigma}{dy}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{\pi\alpha^2}{Q^2} \int_0^1 dy (1 + \cos^2 \theta) = \frac{4\pi\alpha^2}{3Q^2}$$

$$\frac{d\sigma}{dz} = \int_0^1 dy \frac{d\sigma}{dzdy}(e^+e^- \rightarrow hX) = N_c \frac{4\pi\alpha^2}{3Q^2} \sum_{f=q,\bar{q}} e_f^2 D_f(z)$$

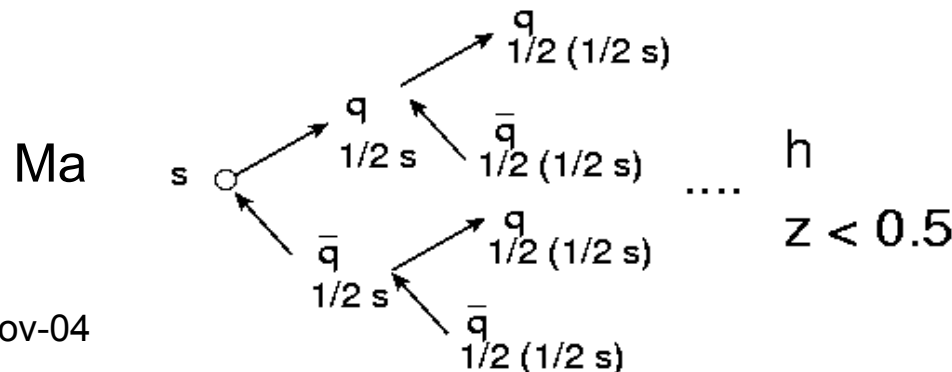


$$\frac{1}{\sigma_{\mu\mu}} \frac{d\sigma}{dz} = N_c \sum_f e_f^2 (D_f(z) + D_{\bar{f}}(z))$$

info su frammentazione

$$Q^2 \frac{d\sigma}{dz} = \frac{4\pi\alpha^2}{3} N_c \sum_f e_f^2 (D_f(z) + D_{\bar{f}}(z))$$

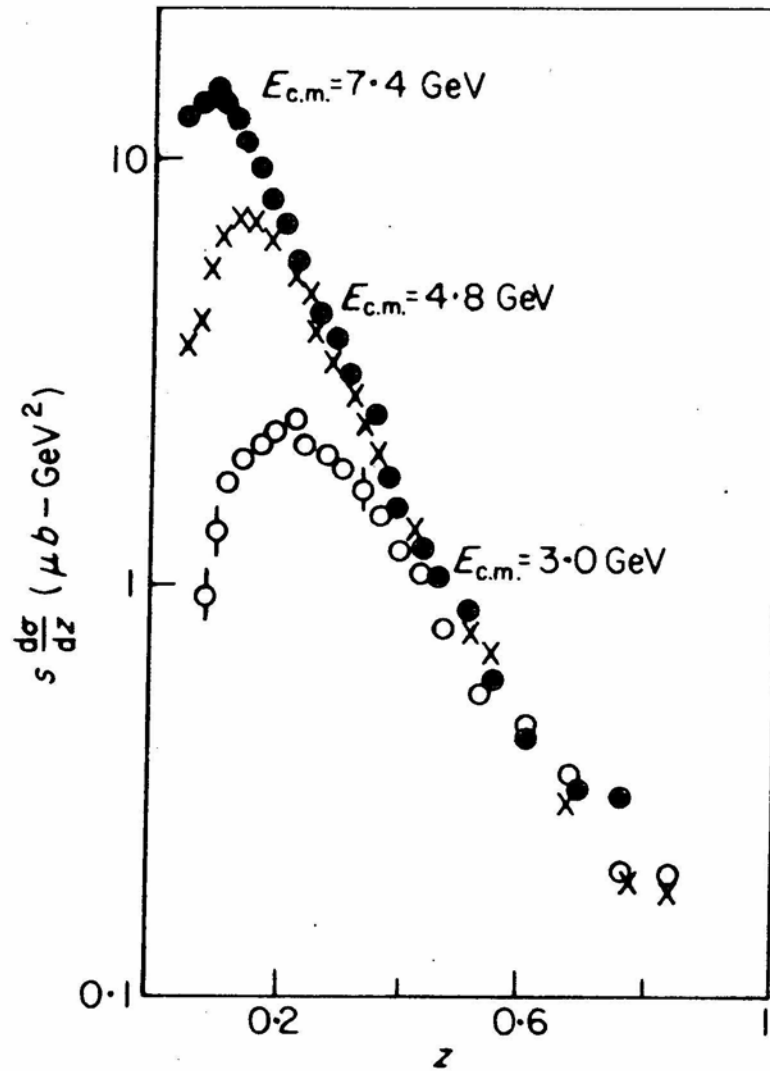
scaling in $z \forall Q^2 \equiv s$



violazione per $z \lesssim 0.5$



$$s \frac{d\sigma}{dz}(e^+e^- \rightarrow hX)$$



Schwitters *et al.*,
P.R.L. **35** 1320 (75)

FIG. 12.9. $s (d\sigma/dz)$ at 3, 4.8, and 7.4 GeV e^+e^- centre of mass energies.

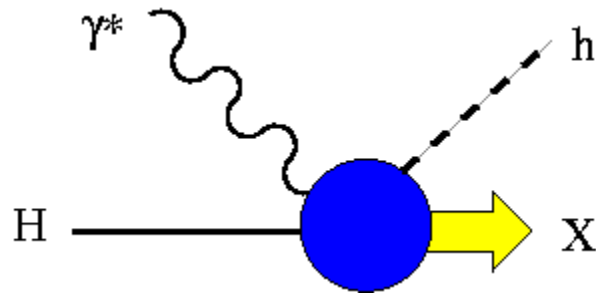
e^+e^- semi-inclusivo \leftarrow fattorizzazione \rightarrow DIS ?

$$\frac{d\sigma}{dydz} = N_c \frac{\pi\alpha^2}{Q^2} (1+\cos^2\theta) \sum_f e_f^2 (D_f(z) + D_{\bar{f}}(z))$$



nuova incognita
fattorizzazione \rightarrow estrarre info da confronto con i
dati anche in altri processi
 \Rightarrow DIS semi-inclusivo (SIDIS)

Semi-inclusive DIS (SIDIS)



stesse definizioni del caso inclusivo per cinematica e invarianti con in più`

$$z_h = \frac{P \cdot P_h}{P \cdot q} \sim \frac{2P_h \cdot q}{q^2} \equiv z \quad e^+e^- \text{ semi-inclusivo}$$



$z > 0$ P_h equiverso a $P \rightarrow h$ viene da frammentazione di partone
 ↑
 current fragmentation region

$z < 0$ P_h opposto a $P \rightarrow h$ viene da frammentazione del bersaglio H
 target fragmentation region

N.B. in TRF $z \sim \frac{ME_h}{M\nu} = \frac{E_h}{\nu}$

SIDIS

$$\mathcal{F} = 4\sqrt{(P \cdot k)^2 - P^2 k^2} \stackrel{\text{TRF}}{=} 4ME \equiv 2s \qquad d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 dR$$

$$dR = (2\pi)^4 \delta(k + P - k' - P_X - P_h) \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \frac{d\mathbf{k}}{(2\pi)^3 2E'} \frac{d\mathbf{P}_h}{(2\pi)^3 2E_h}$$

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H_{1PI}^{\mu\nu} \begin{cases} L_{\mu\nu} = 2(k_\mu k'_\nu + k'_\mu k_\nu - k \cdot k' g_{\mu\nu}) \\ H_{1PI}^{\mu\nu} = \frac{1}{2} \sum_{SS_h} \langle PS | J^\mu | P_X, P_h S_h \rangle \langle P_X, P_h S_h | J^\nu | PS \rangle \end{cases}$$

$$\begin{aligned} 2E_h \frac{d\sigma}{d\mathbf{P}_h dE' d\Omega} &= \frac{1}{4ME} \frac{E'}{16\pi^3} \frac{e^4}{Q^4} L_{\mu\nu} \frac{1}{(2\pi)^3} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(k + P - k' - P_X - P_h) H_{1PI}^{\mu\nu} \\ &= \frac{\alpha^2}{Q^4} \frac{E'}{E} L_{\mu\nu} \frac{1}{2M} 2M W_{1PI}^{\mu\nu} \end{aligned}$$

N.B. cross-check $\frac{d\sigma}{dE' d\Omega} = \int \frac{d\mathbf{P}_h}{2E_h} 2E_h \frac{d\sigma}{d\mathbf{P}_h dE' d\Omega} = \frac{\alpha^2}{Q^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}$

SIDIS (continua)

$$\frac{d\mathbf{k}'}{(2\pi)^3 2E'} = \frac{E' dE' d\Omega}{16\pi^3} = \frac{\pi y s dx_B dy}{16\pi^3}$$

$$\frac{d\mathbf{P}_h}{2E_h} \sim d\mathbf{P}_{h\perp} \frac{dE_h}{2E_h} \sim d\mathbf{P}_{h\perp} \frac{dz}{2z}$$

$$\frac{d\sigma}{d\mathbf{P}_{h\perp} dx_B dy dz} = \frac{\pi\alpha^2}{Q^4} \frac{y}{2z} L_{\mu\nu} 2M W_{1PI}^{\mu\nu}$$



$$\frac{d\sigma}{dx_B dy dz} = \frac{\pi\alpha^2}{Q^4} \frac{y}{2z} L_{\mu\nu} 2M \int d\mathbf{P}_{h\perp} W_{1PI}^{\mu\nu}$$



cruciale per fattorizzazione tra
distribuzione e frammentazione

QPM picture

$$\begin{aligned}\frac{d\sigma}{dx_B dy dz} &= \sum_{f=q,\bar{q}} \int_0^1 dx' dz' \phi_f(x') \frac{d\sigma^{(e^- q \rightarrow e^- q)}}{dx' dy} \left(\frac{x'}{x_B} \right) D_f(z') \delta \left(z - z' \frac{x'}{x_B} \right) \\ &= \frac{4\pi\alpha_s^2}{Q^4} \left(\frac{y^2}{2} + 1 - y \right) x_B \sum_{f=q,\bar{q}} e_f^2 \phi_f(x_B) D_f(z)\end{aligned}$$



N.B. cross-check

$$\begin{aligned}\frac{d\sigma}{dx_B dy} &= \int dz \frac{d\sigma}{dx_B dy dz} \Big|_{D_f(z)=\delta(1-z)} \\ &= \frac{4\pi\alpha_s^2}{Q^4} \left(\frac{y^2}{2} + 1 - y \right) \left(x_B \sum_{f=q,\bar{q}} e_f^2 \phi_f(x_B) \right) = \frac{4\pi\alpha_s^2}{Q^4} (x_B y^2 F_1 + (1-y) F_2)\end{aligned}$$

Callan-Gross
 $F_2 = 2 x_B F_1$

DIS inclusivo

SIDIS

$$e^- p \rightarrow e'^- h^\pm X$$

$$\frac{1}{\sigma_{incl}} \frac{d\sigma}{dx_B dz} = \frac{\sum_f e_f^2 \phi_f(x_B) D_f(z)}{\sum_f e_f^2 \phi_f(x_B)} \sim D_u^h$$

e^+e^- semi-inclusivo

$$e^+ e^- \rightarrow h^\pm X$$

$$\frac{1}{\sigma_{incl}} \frac{d\sigma}{dz} = \frac{\sum_f e_f^2 D_f(z)}{\sum_f e_f^2} \sim D_u^h + D_{\bar{u}}^h$$

dominanza u quark

quindi

$$\frac{1}{\sigma_{incl}} \left(\frac{d\sigma^{h^+}}{dx_B dz} + \frac{d\sigma^{h^-}}{dx_B dz} \right) \Leftrightarrow \frac{1}{\sigma_{incl}} \frac{1}{2} \left(\frac{d\sigma^{h^+}}{dz} + \frac{d\sigma^{h^-}}{dz} \right) \sim D_u^{h^+} + D_{\bar{u}}^{h^-}$$



Gilman, Int. Symp. on lepton
and photon interactions
at high energies,
SLAC (75)

