

# Riassunto della lezione precedente

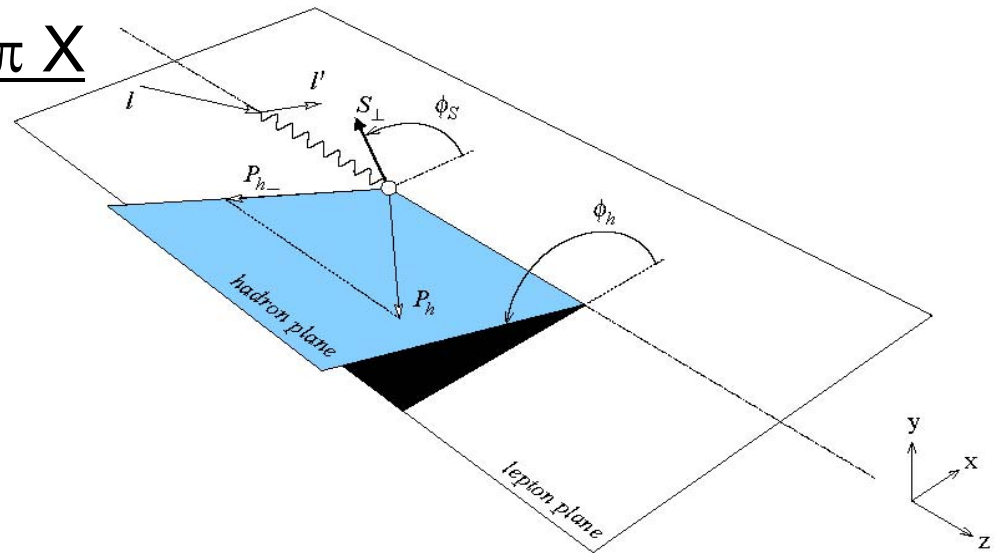
- importanza del momento trasverso intrinseco dei partoni
  - sezione d'urto più ricca → nuove possibilità
  - effetto Collins : meccanismo non perturbativo che lega spin del quark a moto orbitale dell'adrone finale

N.B. Funzione di Collins è naïve T-odd  
per FSI tra adrone e jet residuo  
→ nuova classe di funzioni

- effetto Collins = prodotto di due incognite; universalità necessaria per estrarre info su funzione di Collins da corrispondente  $e^+e^-$
- problema della dipendenza esplicita da momento trasverso intrinseco dei partoni (no teoremi di fattorizzazione generali)
  - funzioni di frammentazione di interferenza (per due adroni)
- color-gauge invariance dei correlatori bilocali → operatore di gauge link
  - funzioni naïve T-odd permesse anche nello stato iniziale
  - struttura più ricca al leading twist → diverse sorgenti di SSA

# Riconsideriamo SIDIS e $p^\uparrow \rightarrow e' \pi X$

sezione d'urto  
a twist=2



$$\frac{d^6\sigma_{OT}}{dx\,dy\,dz\,d\phi_S\,d\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2} \sum_{f\bar{f}} e_f^2 \left\{ \right.$$

$$A(y) \mathcal{F} \left[ f_1^f(x, \mathbf{p}_T^2) D_1^f(z, \mathbf{k}_T^2) \right]$$

$$-B(y) \cos 2\phi_h \mathcal{F} \left[ \frac{2\hat{\mathbf{P}}_{h\perp} \cdot \mathbf{p}_T \hat{\mathbf{P}}_{h\perp} \cdot \mathbf{k}_T - \mathbf{p}_T \cdot \mathbf{k}_T}{M M_h} h_1^\perp f(x, \mathbf{p}_T^2) H_1^\perp f(z, \mathbf{k}_T^2) \right]$$

$$+|S_T| B(y) \sin(\phi_h + \phi_S) \mathcal{F} \left[ \frac{\mathbf{k}_T \cdot \hat{\mathbf{P}}_{h\perp}}{M_h} h_1^f(x, \mathbf{p}_T^2) H_1^\perp f(z, \mathbf{k}_T^2) \right] \quad \leftarrow \text{effetto Collins}$$

$$+|S_T| B(y) \sin(\phi_h - \phi_S) \mathcal{F} \left[ \frac{\mathbf{p}_T \cdot \hat{\mathbf{P}}_{h\perp}}{M} f_{1T}^\perp f(x, \mathbf{p}_T^2) D_1^f(z, \mathbf{k}_T^2) \right] \quad \leftarrow \text{effetto Sivers}$$

$$+|S_T| B(y) \sin(3\phi_h - \phi_S) \mathcal{F} \left[ \frac{4(\hat{\mathbf{P}}_{h\perp} \cdot \mathbf{p}_T)^2 \hat{\mathbf{P}}_{h\perp} \cdot \mathbf{k}_T - 2\hat{\mathbf{P}}_{h\perp} \cdot \mathbf{p}_T \mathbf{p}_T \cdot \mathbf{k}_T - \mathbf{p}_T^2 \hat{\mathbf{P}}_{h\perp} \cdot \mathbf{k}_T}{M^2 M_h} \right.$$

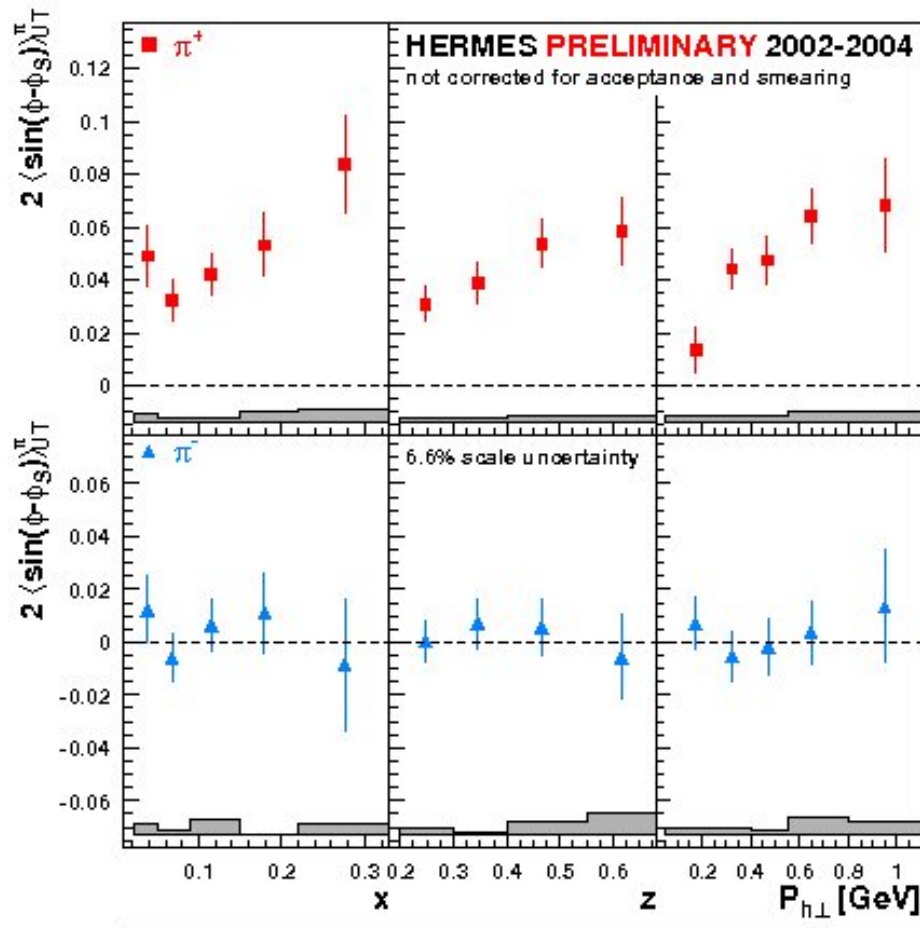
$$\left. \times h_{1T}^\perp f(x, \mathbf{p}_T^2) H_1^\perp f(z, \mathbf{k}_T^2) \right] \left. \right\}$$

$$\phi_S \neq 0, \pi$$

i due effetti sono separabili in linea di principio ; operativamente ci sono difficoltà

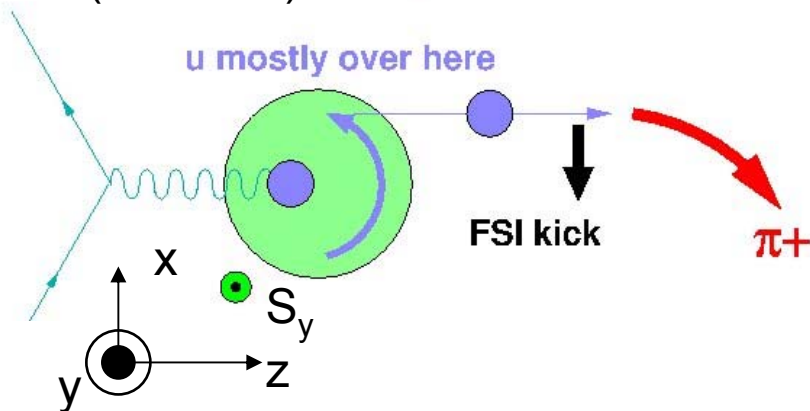
# Effetto Sivers e relativa SSA

$$\frac{\int d\phi_S d\mathbf{P}_{h\perp} \frac{|\mathbf{P}_{h\perp}|}{M_h} \sin(\phi_h - \phi_S) (d\sigma^\uparrow - d\sigma^\downarrow)}{\int d\phi_S d\mathbf{P}_{h\perp} (d\sigma^\uparrow + d\sigma^\downarrow)} \propto -|\mathbf{S}_T| \frac{B(y)}{A(y)} \frac{\sum_{f\bar{f}} e_f^2 f_{1T}^{\perp f(1)}(x) D_1^f(z)}{\sum_{f\bar{f}} e_f^2 f_1^f(x) D_1^f(z)}$$



$\pi^+$  positivo; se  $\sin(\phi_h - \phi_S) > 0$   
 $\rightarrow f_{1T}^{\perp u}$  negativa  
 $f_{1T}^{\perp d}$  positiva ? (piccola)

(continua)



$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = \pi \end{array} \right\} \sin(\phi - \phi_S) > 0$$

deflessione quark  $d$  a  $x > 0$

possibile interpretazione:

$N^\uparrow \rightarrow$  distribuzione asimmetrica  
nel piano trasverso:  $u$  va  
a  $x > 0$  e  $d$  va a  $x < 0$

**perché  $S_y \neq 0 \rightarrow L_q \neq 0$**

$\gamma$  colpisce  $u$  che viene deflesso  
a  $x < 0$  per confinamento (forza  
colore attrattiva); opposto per  $d$   
(Burkardt, Phys. Rev. **D66**('02) 114005)

effetto diretto del momento  
angolare orbitale dei quark



K. Bruhwal - TJNAF - 2003



(continua)

distorsione nel piano trasverso della distribuzione di partoni provocata da polarizzazione trasversa del protone può essere descritta da

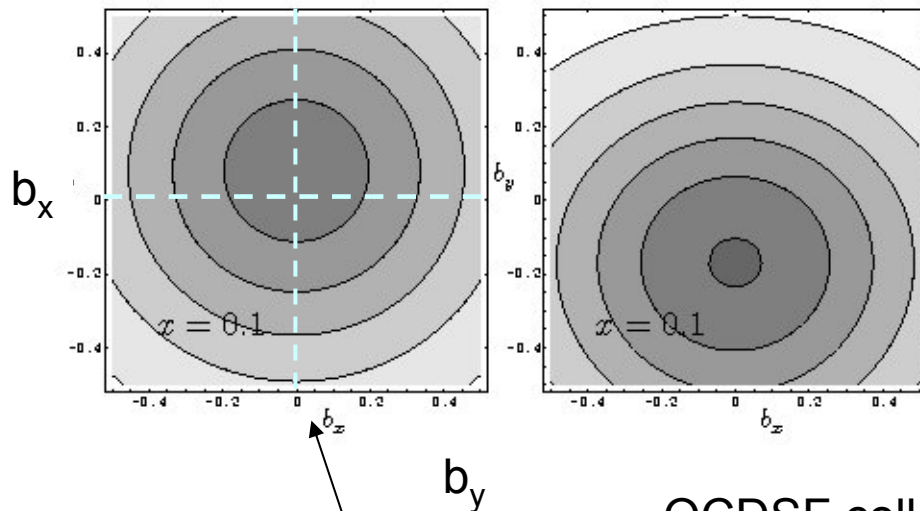
Generalized Parton Distribution  $E$

→ legame PDF  $f_{1T}^\perp(x, p_T) \leftrightarrow$  GPD  $E(x, 0, -\Delta_T^2)$

$$f_{q/p\uparrow}(x, \vec{b}_T) = f_1^q(x, \vec{b}_T^2) - \frac{1}{M} \frac{\partial}{\partial b_x} \int \frac{d\vec{\Delta}_T}{(2\pi)^2} E^q(x, 0, -\vec{\Delta}_T^2) e^{-\vec{b}_T \cdot \vec{\Delta}_T}$$

$u_X(x, \mathbf{b}_\perp)$

$d_X(x, \mathbf{b}_\perp)$



$$\int dx E^q(x, 0, 0) = \kappa^q$$

calcolo su reticolo dà  
 $\partial \hat{E}^u(x, \mathbf{b}_T) / \partial b_x > 0$  cioè  
 $f_{1T}^{\perp u} < 0$   
 opposto per quark d  
 $f_{1T}^{\perp d} > 0$

$S_y$

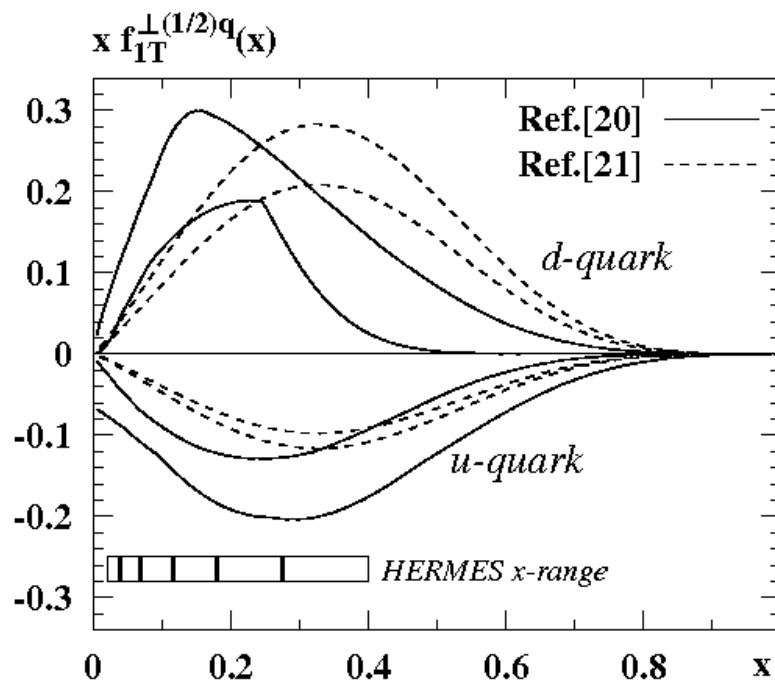
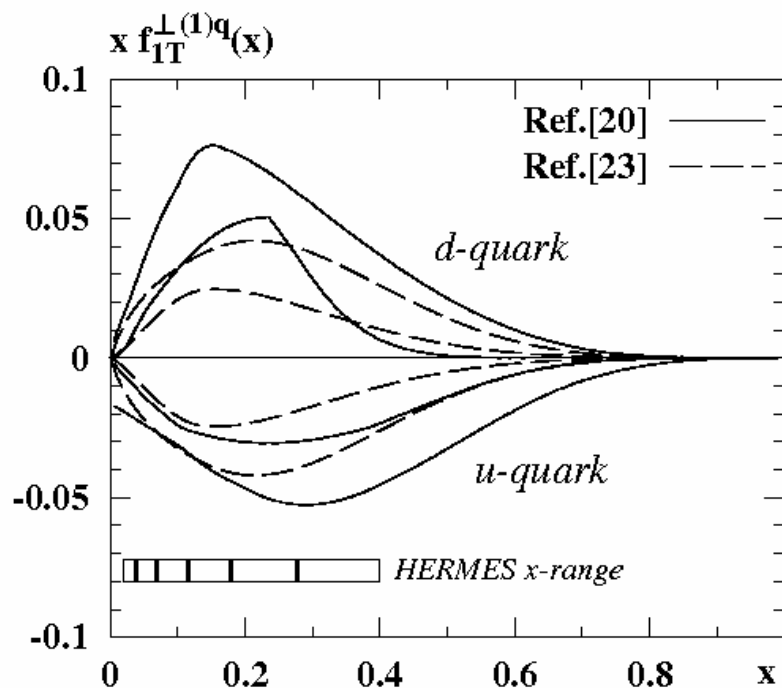
QCDSF coll. hep-ph/0511032

10-Gen-07

$$\vec{R}_\perp = \frac{\sum_i x_i \vec{r}_{i\perp}}{\sum_i x_i}$$

Diversi modelli per calcolare la funzione di Sivers, ma soprattutto nuove parametrizzazioni da analisi di dati di HERMES (dipendenti dal flavor), di BNL e di COMPASS

collezione di risultati in Anselmino *et al.* hep-ph/0511017



[20] Anselmino et al., P.R. D72 (05) 094007

[21] Vogelsang, Yuan, P.R. D72 (05) 054028

[23] Collins et al., hep-ph/0510342

# Teorema di universalità per funzione di Sivers

## Teorema di fattorizzazione

(Ji, Ma, Yuan, P.R. D71 (05) 034005;  
Collins, Metz, P.R.L. **93** (04) 252001)

$$\frac{d^6\sigma_{OT}}{dx dy dz d\phi_S d\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2} \sum_{f\bar{f}} e_f^2 \left\{ \right.$$

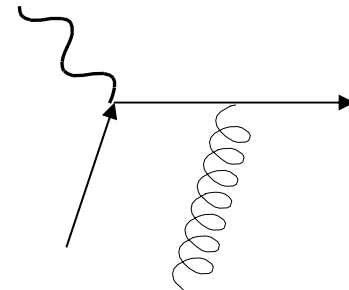
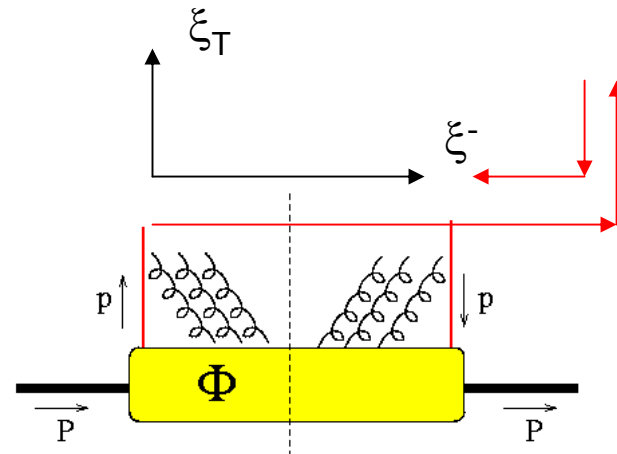
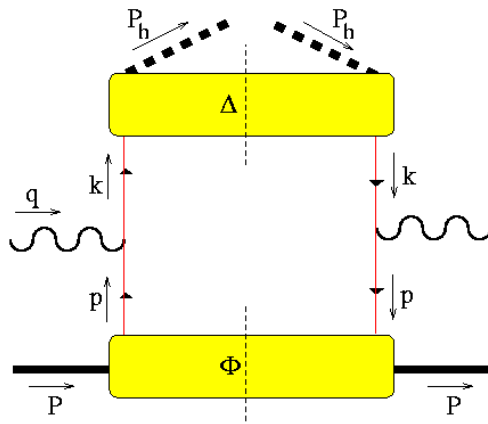
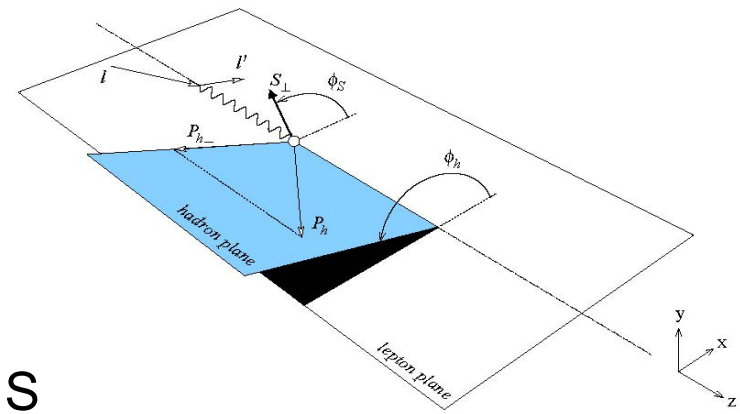
$$A(y) \mathcal{F} \left[ f_1^f(x, \mathbf{p}_T^2) D_1^f(z, \mathbf{k}_T^2) \right]$$

$$+ \dots$$

$$+ |\mathbf{S}_T| B(y) \sin(\phi_h - \phi_S) \mathcal{F} \left[ \frac{\mathbf{p}_T \cdot \hat{\mathbf{P}}_{h\perp}}{M} f_{1T}^\perp(x, \mathbf{p}_T^2) D_1^f(z, \mathbf{k}_T^2) \right] + \dots \left. \right\}$$

SIDIS

$e p^\uparrow \rightarrow e' \pi X$



(continua)

## Teorema di fattorizzazione

(Collins, Soper, Sterman, N.P. **B250** (85) 199 )

$$\frac{d\sigma}{dx_1 dx_2 d\Omega d\mathbf{q}_T} = \frac{\alpha^2}{3Q^2} \sum_{f\bar{f}} e_f^2 \left\{ \right.$$



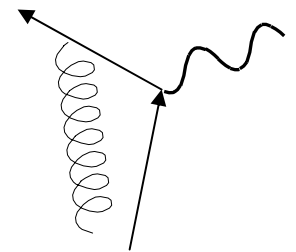
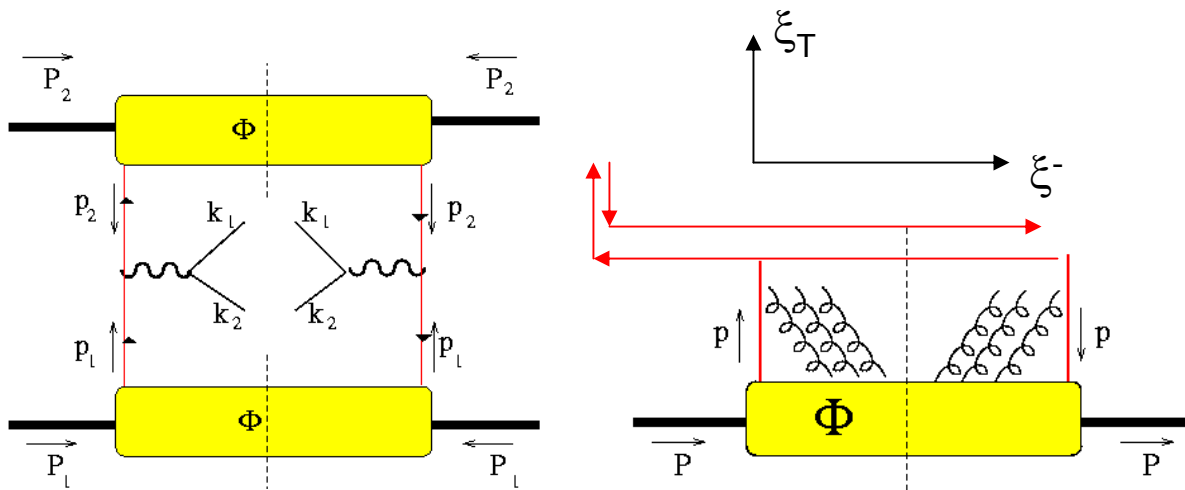
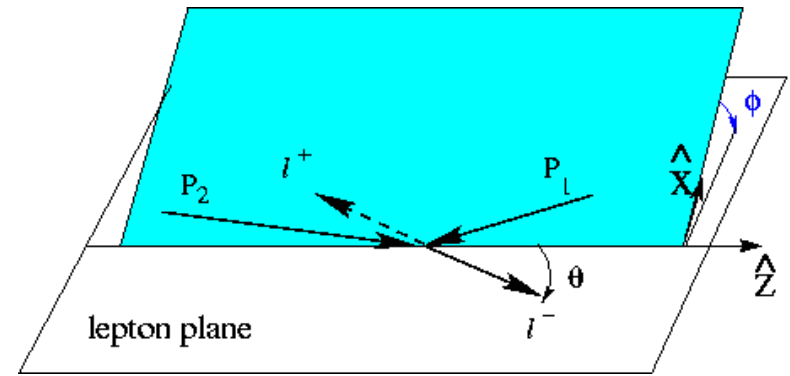
$$A(y) \mathcal{F} \left[ f_1^f(x_1, \mathbf{p}_{1T}^2) \bar{f}_1^f(x_2, \mathbf{p}_{2T}^2) \right]$$

+.....

$$+ |S_{1T}| A(y) \sin(\phi - \phi_{s1}) \mathcal{F} \left[ \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_{1T}}{M_1} f_{1T}^\perp(x_1, \mathbf{p}_{1T}^2) \bar{f}_1^f(x_2, \mathbf{p}_{2T}^2) \right] + \dots \left\{ \right.$$

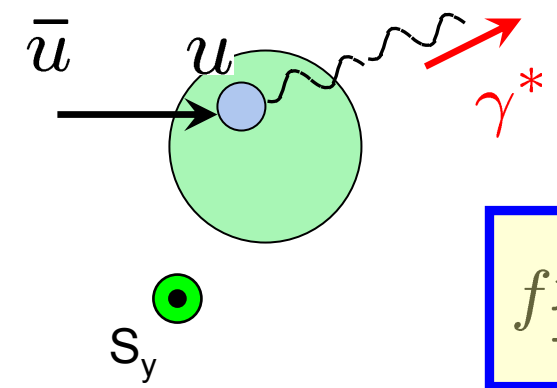
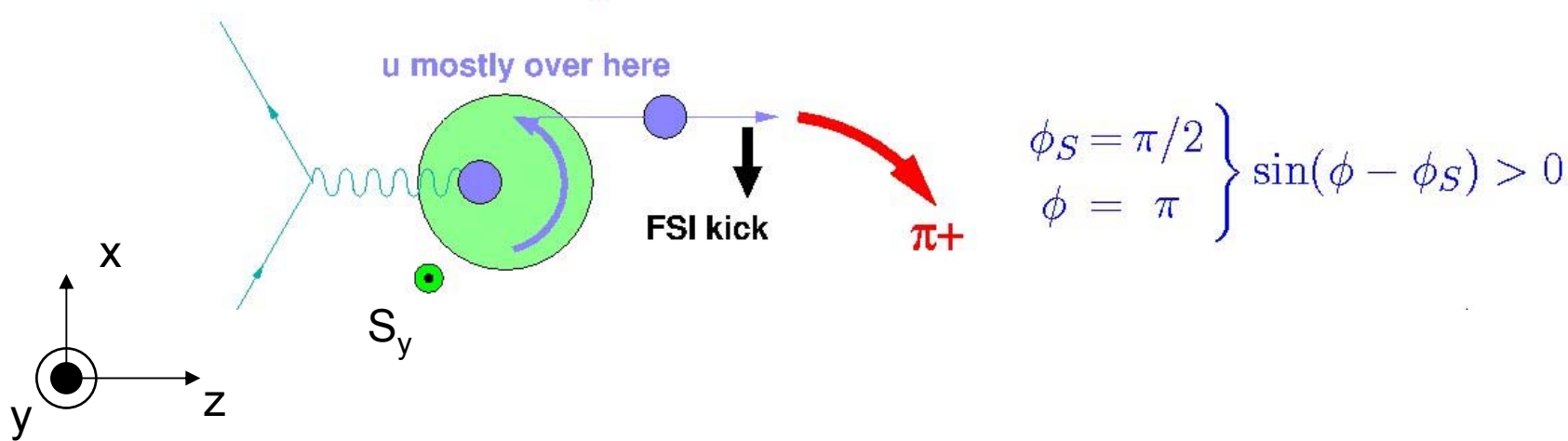
Drell-Yan

$$p p^\uparrow \rightarrow \mu^+ \mu^- X$$



cambio di segno !

(continua)

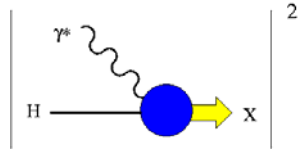


$$f_{1T}^{\perp q} \Big|_{SIDIS} = -f_{1T}^{\perp q} \Big|_{Drell-Yan}$$

STAR & PHENIX @ RHIC } conferma sperimentale ? Collins P.L. **B536** (02) 43  
PAX @ GSI }

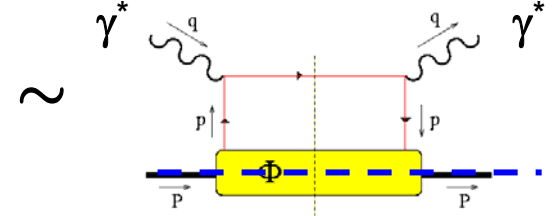
# Generalized Parton Distributions (GPD)

DIS inclusivo    teorema ottico    forward Compton



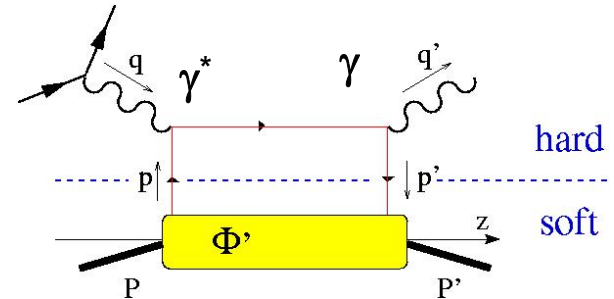
$$\sim \text{Im} \left\{ \begin{array}{c} \gamma^* \quad q \\ \text{H} \quad \text{P} \quad \text{P} \quad \text{H} \\ \gamma^* \quad q \end{array} \right\} \quad Q^2 \rightarrow \infty \quad x_B \text{ fisso}$$

dominante



nonforward Compton

$$\begin{array}{c} \gamma^* \quad q \quad q' \quad \gamma \\ \text{H} \quad \text{P} \quad \text{P}' \quad \text{H}' \end{array} \quad Q^2 \rightarrow \infty \quad x_B \text{ fisso} \quad t = (P-P')^2 \text{ piccolo}$$



**Deeply Virtual Compton Scatt. (DVCS)**

correlatore

locale  $\Phi(p, P, S) = \int \frac{d^4 z}{(2\pi)^4} e^{ip \cdot z} \langle PS | \bar{\psi}(0) \psi(z) | PS \rangle$



non locale  $\Phi'(\bar{p}, P, P', S', S) = \int \frac{d^4 z}{(2\pi)^4} e^{i\bar{p} \cdot z} \langle P' S' | \bar{\psi} \left( -\frac{z}{2} \right) \psi \left( \frac{z}{2} \right) | PS \rangle$

$$\bar{p} = \frac{p + p'}{2}$$

teorema fattorizzazione a twist 2  $\forall \alpha_S$

(Collins e Freund, P.R.D59 ('99) 074009)

Ji, P.R.L. **78** ('97) 610; Radyushkin, P.L. **B380** (96) 417

Diehl, Phys. Rept. **388** (2003) 41

# Cinematica del DVCS

$$\bar{P} = \frac{P + P'}{2} = (\bar{P}^0, 0, 0, \bar{P}^z) = (\bar{P}^+, \bar{P}^-, 0_\perp)$$

skewness  $\xi = \frac{(P - P')^+}{(P + P')^+}$

$$P = \left( (1 + \xi)\bar{P}^+, \frac{M^2 + \Delta_\perp^2/4}{2(1 + \xi)\bar{P}^+}, -\frac{\Delta_\perp}{2} \right)$$



$$t = \Delta^2 = -\frac{4\xi^2 M^2 + \Delta_\perp^2}{1 - \xi^2}$$

$$P' = \left( (1 - \xi)\bar{P}^+, \frac{M^2 + \Delta_\perp^2/4}{2(1 - \xi)\bar{P}^+}, +\frac{\Delta_\perp}{2} \right)$$

dato  $t \rightarrow \max \xi_t \leq -\frac{4\xi_t^2 M^2}{1 - \xi_t^2}$

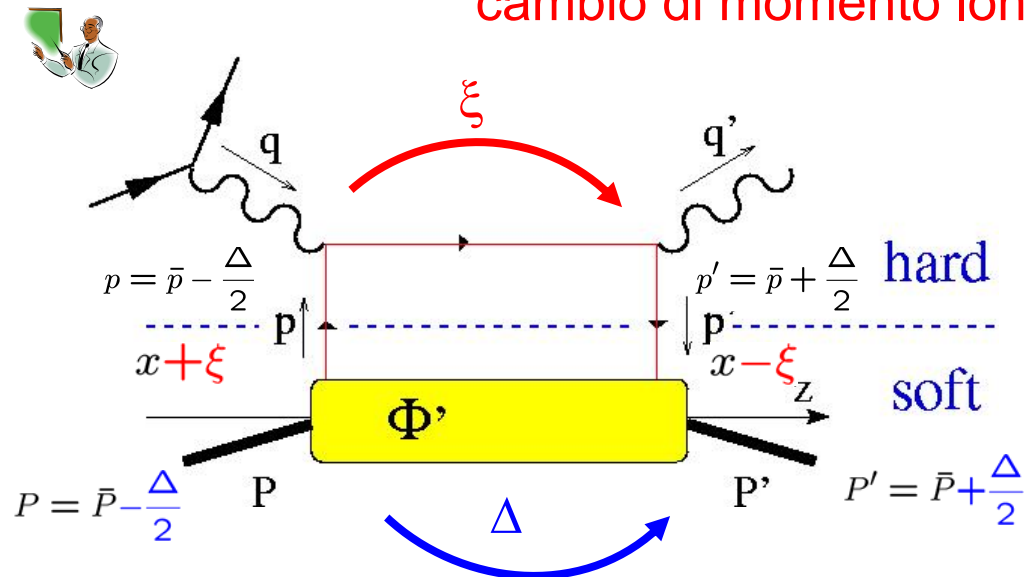
$$\Delta = P' - P = \left( -2\xi\bar{P}^+, \frac{\xi(M^2 + \Delta_\perp^2/4)}{(1 - \xi^2)\bar{P}^+}, \Delta_\perp \right)$$

$$\bar{p} = \frac{p + p'}{2} \quad x = \frac{\bar{p}^+}{\bar{P}^+}$$

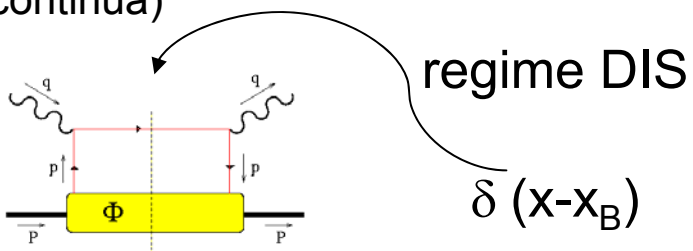
$$\frac{p^+}{\bar{P}^+} = x + \xi$$

$$\frac{p'^+}{\bar{P}^+} = x - \xi$$

cambio di momento longitudinale



(continua)



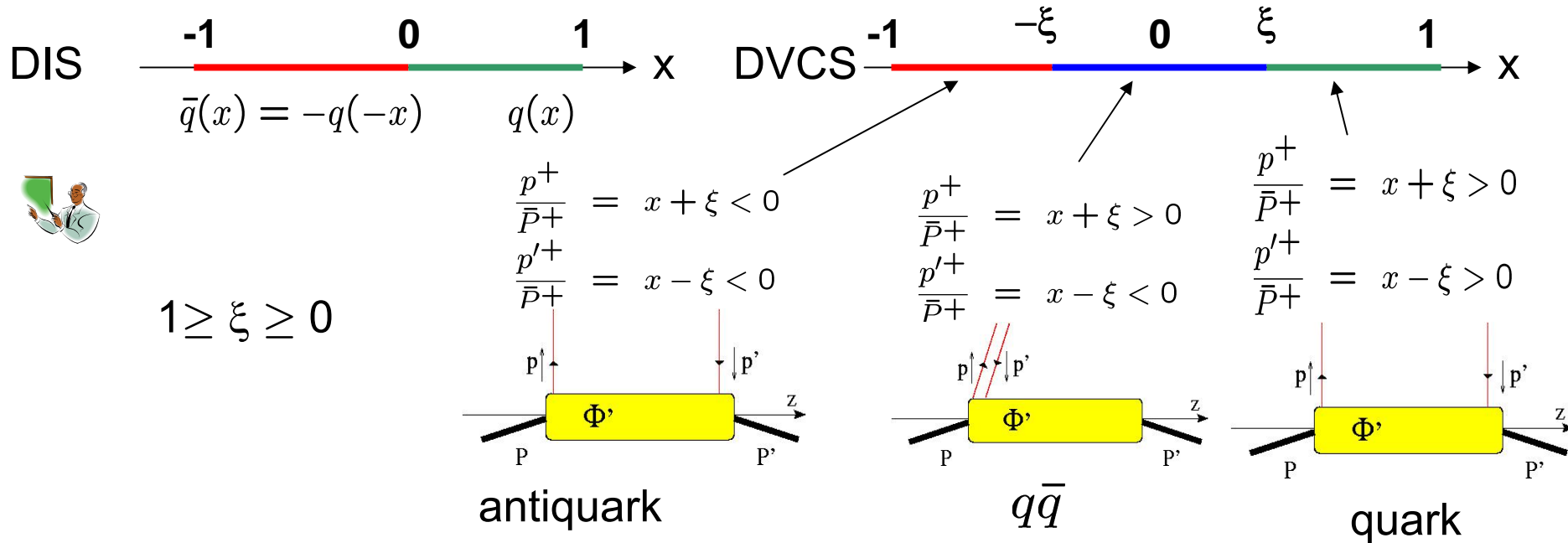
regime DVCS

$$\begin{aligned}
 q^+ &= P'^+ - P^+ \\
 &= \Delta^+ \equiv -2\xi \bar{P}^+ \\
 \frac{x_B}{2-x_B} &= \frac{-q^2}{4P \cdot q + q^2} \\
 &\sim \frac{-q^+}{2P^+ + q^+} = -\frac{q^+}{2\bar{P}^+} = \xi
 \end{aligned}$$

hard

soft

- interpretazione intuitiva della cinematica





## Correlatore non locale → GPD

Invarianza per trasf. Lorentz, gauge + decomposizione di Gordon

$$\begin{aligned}\langle P' | J^\mu | P \rangle &\equiv \langle P' | \bar{q}(0) \gamma^\mu q(0) | P \rangle \\ &= \bar{u}(P') \gamma^\mu u(P) F_1(t) + \bar{u}(P') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} u(P) F_2(t)\end{aligned}$$

caso diagonale :

$$t = \Delta^2 = (P' - P)^2$$

$$\begin{aligned}\langle P | \bar{q}(0) \gamma \cdot n q(0) | P \rangle &= \bar{u}_P \gamma \cdot n u_P F_1(0) \\ &= \int dx \int \frac{d\lambda}{2\pi} e^{i\lambda \cdot x} \langle P | \bar{q}(0) \gamma \cdot n U_{[0,\lambda]} q(\lambda n) | P \rangle\end{aligned}$$

$$\text{momentum sum rule} \equiv P \cdot n \int dx f_1(x) + \text{twist} > 2 \dots \quad n^\mu = (0, 1, 0_T)$$

caso non diagonale

$$\langle P' | \bar{q}(0) \gamma \cdot n q(0) | P \rangle = \bar{u}_{P'} \gamma \cdot n u_P F_1(t) + \bar{u}_{P'} \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} u_P F_2(t)$$

$$= \int dx \int \frac{d\lambda}{2\pi} e^{i\lambda \cdot x} \langle P' | \bar{q}(-\lambda n/2) \gamma \cdot n U_{[-\lambda/2, \lambda/2]} q(\lambda n/2) | P \rangle$$

$$\equiv \bar{u}_{P'} \gamma \cdot n u_P \int dx H(x, \xi, t) + \bar{u}_{P'} \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} u_P \int dx E(x, \xi, t) + \dots$$

## Commenti

- H,E funzioni di invarianti costruiti con  $P^\mu, P'^\mu, n^\mu \rightarrow P \cdot n, P' \cdot n, t=(P'-P)^2 \equiv \Delta^2$
- boost lungo z modifica  $n^\mu$ , ma H,E definite in modo invariante
- dipendenza da  $P' \cdot n / P \cdot n \leftrightarrow \Delta \cdot n / \overline{P} \cdot n = -2 \xi$ , oltre che da x,t
- Momentum sum rule
 
$$F_1(t) = \int dx H(x, \xi, t)$$

$$F_2(t) = \int dx E(x, \xi, t)$$

$\int dx$  cancella ogni riferimento a direzione “+”  $\rightarrow$  anche  $\xi$
- limite diagonale (“forward”) :  $t, \Delta \rightarrow 0$      $H(x,0,0) = f_1(x)$  ; E non determinata
- $f_1(x)$  da proiezione  $\Phi^{[\gamma^+]}(p,P,S)$
- H, E da proiezione  $\Phi'^{[\gamma^+]}(\overline{p},P,P',S,S')$
- analisi a twist=2 di  $\Phi'$

# Analisi in twist del correlatore non diagonale

estrazione del contributo al leading twist procede come per PDF:

- proiezione delle componenti LC “good” con operatori  $\gamma^+$ ,  $\gamma^+\gamma_5$ ,  $i\sigma^{i+}\gamma_5$  integrando su direzione “-” soppressa



$$\begin{aligned}\Phi'[\gamma^+] &= \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P'S' | \bar{\psi} \left( -\frac{z^-}{2} \right) \gamma^+ \psi \left( \frac{z^-}{2} \right) | PS \rangle \Big|_{z^+=z_\perp=0} \\ &= \frac{1}{\bar{P}^+} \bar{u}(P'S') \left\{ \gamma^+ H(x, \xi, t) + \frac{i\sigma^{+\nu}\Delta_\nu}{2M} E(x, \xi, t) \right\} u(PS)\end{aligned}$$

q non polarizzato  
non helicity flip

$$\begin{aligned}\Phi'[\gamma^+\gamma_5] &= \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P'S' | \bar{\psi} \left( -\frac{z^-}{2} \right) \gamma^+\gamma_5 \psi \left( \frac{z^-}{2} \right) | PS \rangle \Big|_{z^+=z_\perp=0} \\ &= \frac{1}{\bar{P}^+} \bar{u}(P'S') \left\{ \gamma^+\gamma_5 \tilde{H}(x, \xi, t) + \frac{\gamma_5\Delta^+}{2M} \tilde{E}(x, \xi, t) \right\} u(PS)\end{aligned}$$

q polarizzato long.  
non helicity flip

$$\begin{aligned}\Phi'[i\sigma^{i+}\gamma_5] &= \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P'S' | \bar{\psi} \left( -\frac{z^-}{2} \right) i\sigma^{i+}\gamma_5 \psi \left( \frac{z^-}{2} \right) | PS \rangle \Big|_{z^+=z_\perp=0} \\ &= \frac{i}{\bar{P}^+} \bar{u}(P'S') \left\{ \sigma^{i+}\gamma_5 H_T(x, \xi, t) + \frac{\epsilon^{i+\alpha\beta}\Delta_\alpha\bar{P}_\beta}{M^2} \tilde{H}_T(x, \xi, t) \right. \\ &\quad \left. + \frac{\epsilon^{i+\alpha\beta}\Delta_\alpha\gamma_\beta}{2M} E_T(x, \xi, t) + \frac{\epsilon^{i+\alpha\beta}\bar{P}_\alpha\gamma_\beta}{M} \tilde{E}_T(x, \xi, t) \right\} u(PS)\end{aligned}$$

helicity flip