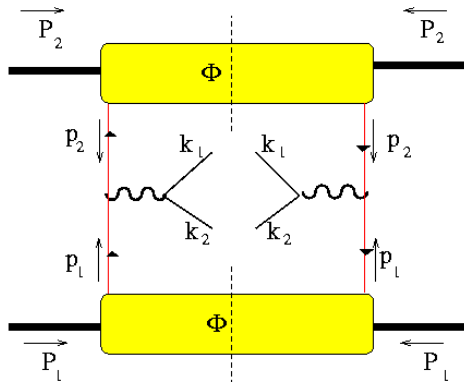


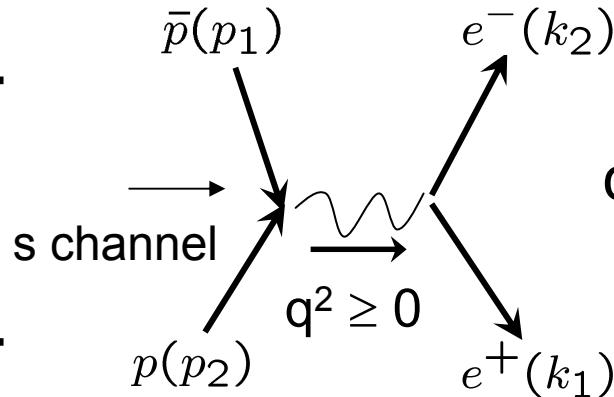
# Riassunto della lezione precedente

- GPD a twist=2: proprietà, interpretazione intuitiva, trasformata di Fourier nel parametro di impatto  $\mathbf{b}_\perp$  e localizzazione dei partoni nel piano  $\perp$
- generalizzazione della momentum sum rule per operatori “tower” di tipo vettoriale ( $\gamma^\mu$ ), assiale-vettoriale ( $\gamma^\mu\gamma_5$ ), etc..
- polinomialità delle GPD non polarizzate H,E
- def. gauge-invariante di operatori momento (da tensore energia-impulso  $T^{\mu\nu}$ ) e momento angolare (da tensore momento angolare  $M^{\lambda\mu\nu}$ )
- polinomialità : fattori di forma generalizzati  $\leftrightarrow$  regola di somma di momento e di spin
- scomposizione gauge-invariante di operatore momento angolare: definizione di momento angolare orbitale
- regola di somma di spin e sua evoluzione con scala  $\mu^2$

# “exclusive” Drell-Yan

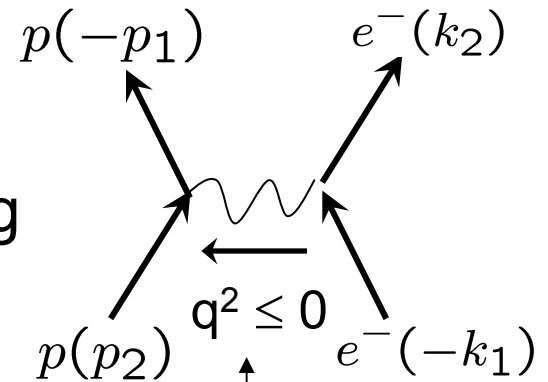


$$\tau = \frac{M^2}{s} = 1$$



$$\begin{aligned} q^2 &= (k_1 + k_2)^2 \\ &= (p_1 + p_2)^2 = s \\ t &= (k_1 - p_2)^2 \rightarrow \theta \end{aligned}$$

crossing



$$\begin{aligned} q^2 &= (k_1 - k_2)^2 \\ &= (p_1 - p_2)^2 = t \\ s &= (k_1 + p_2)^2 \end{aligned}$$

4 vectors  $(k_1 + k_2 \rightarrow p_1 + p_2) \rightarrow 8$  elements in  $J^\mu$ :  $(\gamma^\mu, k_1^\mu, k_2^\mu, p^\mu) \otimes (1, \not{q})$

$$p = (p_1 + p_2)/2$$

$$q = (p_1 - p_2)$$

$q \cdot J = 0 \rightarrow 6$  elements  $\rightarrow$  **3 complex form factors**

a possible parametrization:

$$J_\mu^v = \bar{v}(p_1) \Gamma^\mu [G_E(q^2), G_M(q^2)] u(p_2)$$

$$J_\mu^a = \bar{v}(p_1) \gamma^\mu \gamma_5 A(q^2) u(p_2)$$

N.B. eliminate  $p^\mu$  and  $\not{q} p^\mu$

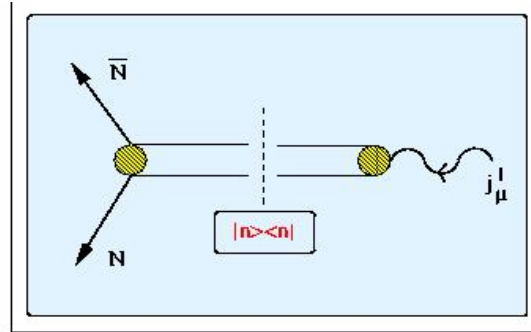
$G_E(q^2), G_M(q^2)$   
 $A(q^2)$   
 $q^2 \geq 0$       complex

analytic  
 continuation

$G_E(Q^2=q^2 e^{-i\pi}), G_M(Q^2=q^2 e^{-i\pi})$   
 0  
 $q^2 \leq 0$       real

$$\begin{aligned}
 & \text{Im} \langle N(p) \bar{N}(p') | J^\mu | 0 \rangle \\
 & \sim \sum_n \langle N(p) \bar{N}(p') | n \rangle \langle n | J^\mu | 0 \rangle \\
 & \Rightarrow \text{Im} F(q^2)
 \end{aligned}$$

vector-meson poles and  
multi-hadron continuum



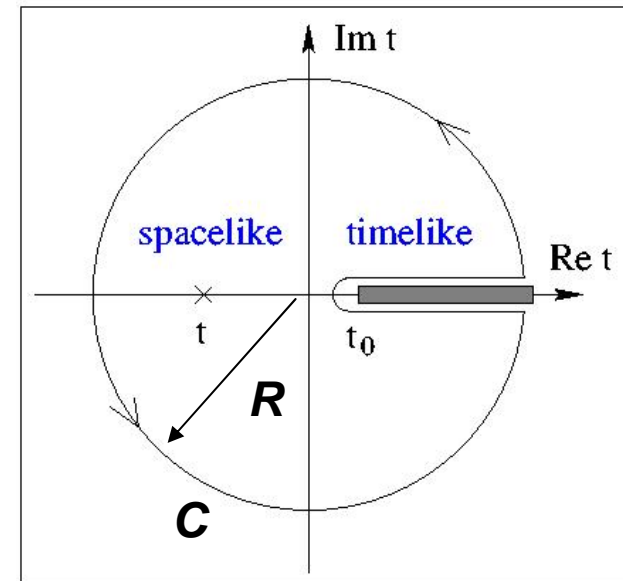
hermitean Hamiltonian

## Dispersion Relation

$F(t)$  analytic function in  $t \in \mathbb{C}$  with  
 cut  $[t_0 = 4m_\pi^2, \infty)$

$$F(t) = \lim_{R \rightarrow \infty} \frac{1}{2\pi i} \oint_C dz \frac{F(z)}{z - t} = \int_{t_0}^{\infty} dt' \frac{\text{Im} F(t')}{t' - t}$$

$\text{Im} F(t') \neq 0$  only in  $[t_0 = 4m_\pi^2, \infty)$



# question in space-like domain

elastic scattering  
cross section

$$N(e, e')$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E' \cos^2 \frac{\theta}{2}}{4E^3 \sin^4 \frac{\theta}{2}} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right]$$

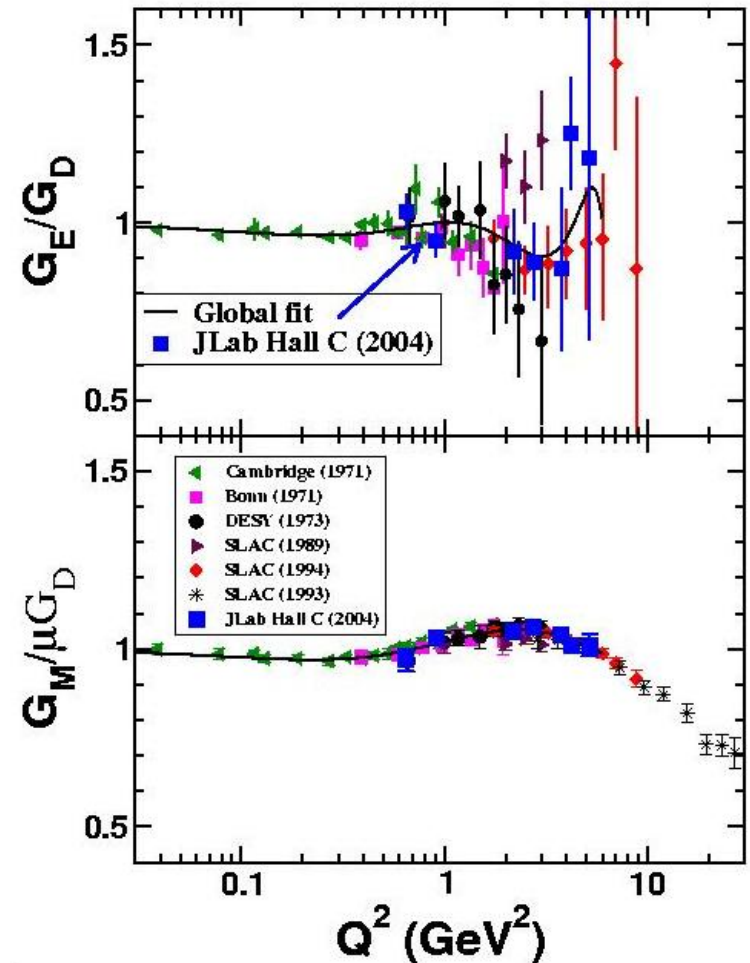
Rosenbluth separation



$$\sigma_R = \frac{\epsilon(1 + \tau)}{\tau \sigma_{\text{Mott}}} \frac{d\sigma}{d\Omega} = \frac{\epsilon}{\tau} G_E^2 + G_M^2$$

fixed  $Q^2$ , vary  $\epsilon = \left[ 1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right]^{-1}$   
 $G_E$  slope,  $G_M$  intercept

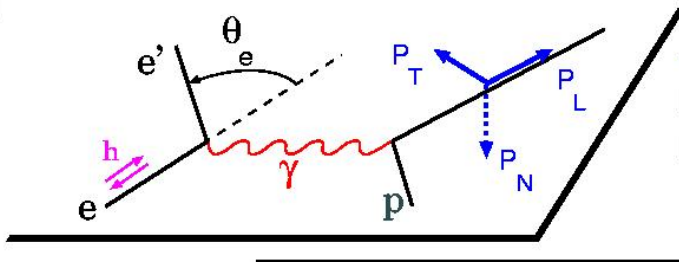
$$\tau = \frac{Q^2}{4M^2} \Rightarrow \frac{\epsilon}{\tau} \sim \frac{1}{Q^2} + \dots \quad \text{large errors in } G_E$$



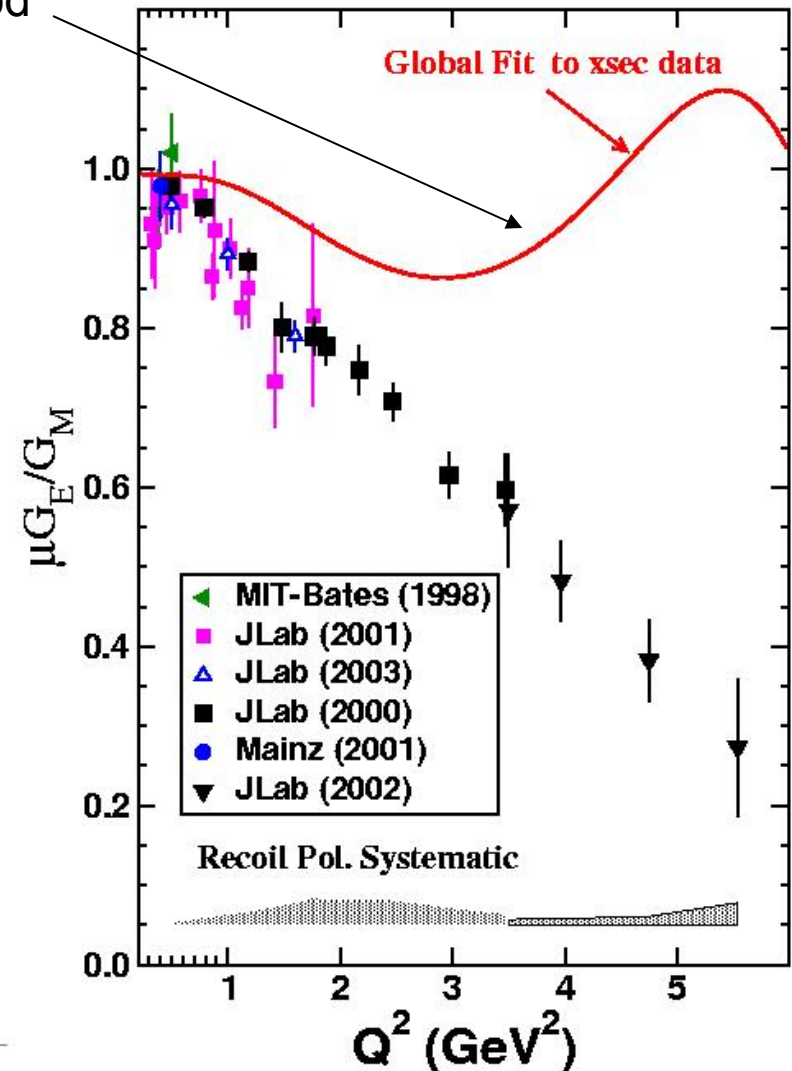
# polarization transfer method

Rosenbluth method

$$^1H(\vec{e}, e' \vec{p})$$



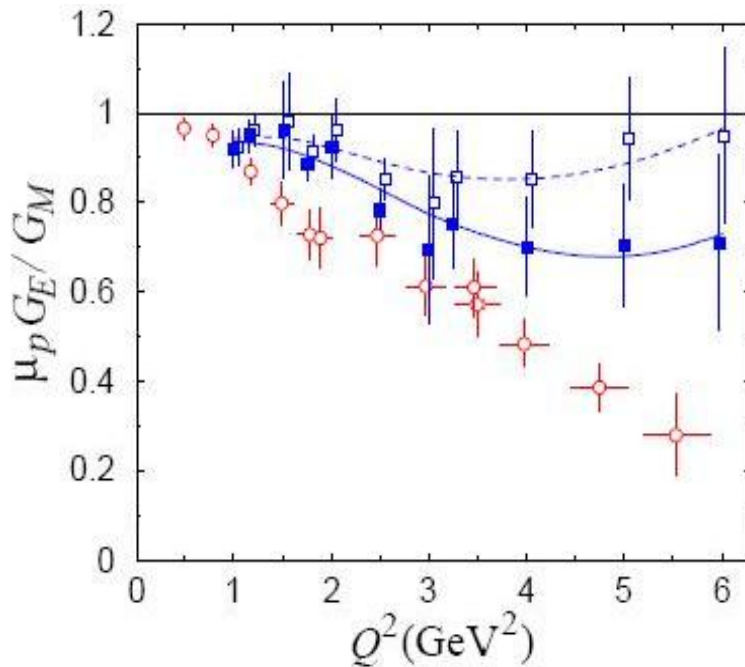
$$\frac{G_E}{G_M} = -\frac{P_T}{P_L} \sqrt{\frac{\tau(1+\epsilon)}{2\epsilon}}$$



# space-like form factor puzzle

Blunden *et al.*

P.R. C72 (05) 034612



1. Rosenbluth method

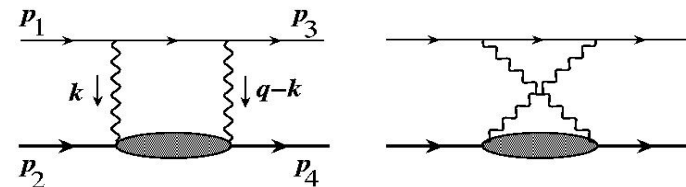
2. after correction for nonBorn ( $2\gamma$ )

3. polarization transfer method

exp's repeated and checked for consistency; see also

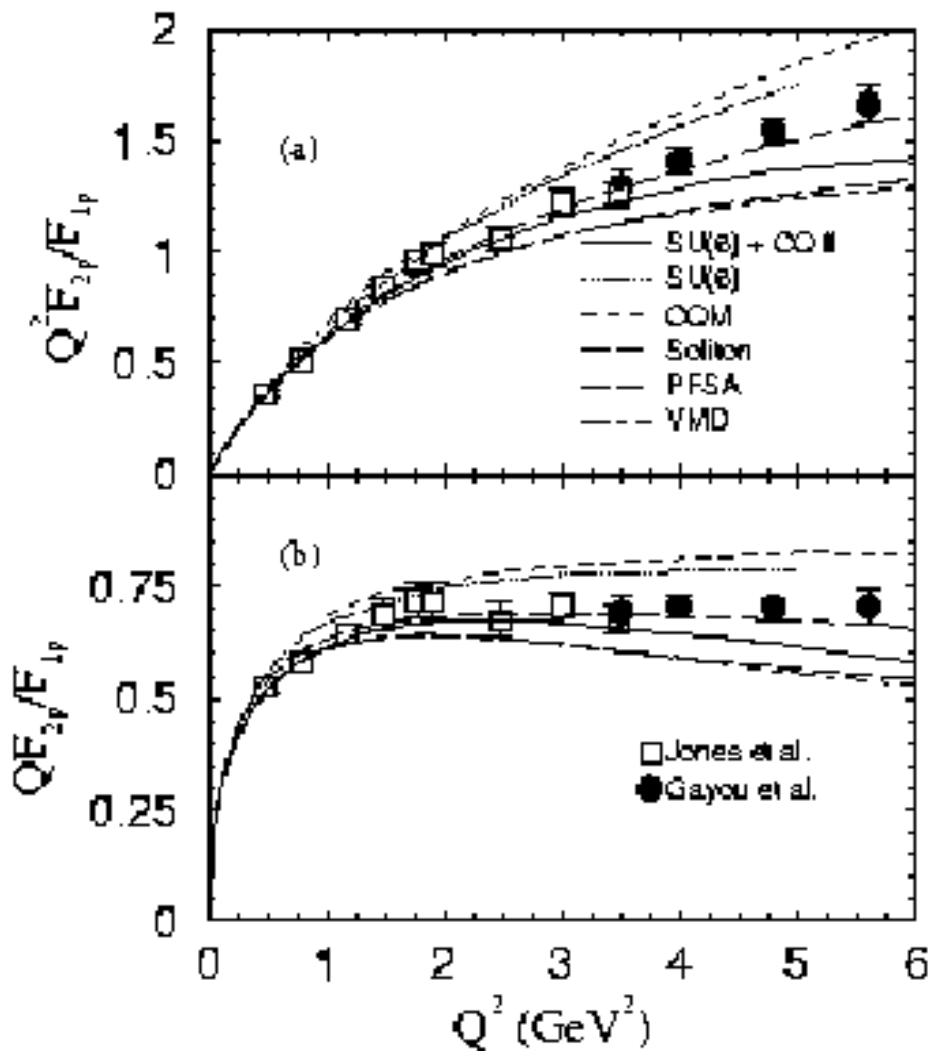
JLab – Hall A Qattan *et al.*

P.R.L. 94 (05) 142301



$$\sigma_R = \frac{\epsilon}{\tau} G_E^2 + G_M^2 + \epsilon \sigma_{2\gamma}(\epsilon, Q^2)$$

# space-like form factor puzzle: where is onset of QCD scaling?



pQCD  $\frac{F_2}{F_1} \rightarrow \frac{1}{Q^2}$



JLab polar. transfer data

$$\frac{F_2}{F_1} \sim \frac{1}{Q}$$

$Q^2 \leq 10$  no scaling!

# exclusive Drell-Yan cross section

parametrizzazione dati sperimentali (analisi di Fourier)

$$\int d\phi \frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left[ \underbrace{1 + \lambda \cos^2 \theta}_{\text{annichilazione collineare} \sim \text{QPM}} + \underbrace{\mu \sin^2 \theta \cos \phi}_{\text{1 spin-flip } R_{LT}} + \underbrace{\frac{\nu}{2} \sin^2 \theta \cos 2\phi}_{\text{2 spin-flip } R_{TT}} + \alpha \cos \theta + \beta \sin \theta \cos \phi + \gamma \sin \theta \sin \phi \right]$$

twist 2

$\lambda \sim 1 \gg \mu; \nu \sim 0.3$   
 $1 - \lambda \neq 2\nu$  violazione regola di Lam-Tung

$h_1^\perp \otimes \bar{h}_1^\perp$

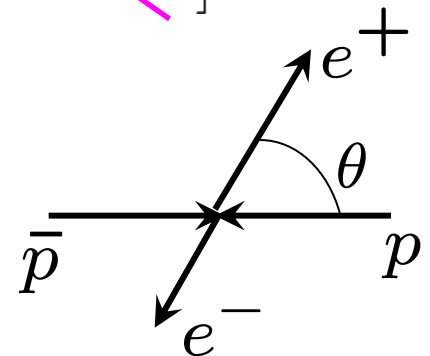
$$\frac{d\sigma}{d\cos\theta} = a(q^2) (1 + R \cos^2 \theta) - b(q^2) \text{Re}[G_M A^*] \cos \theta$$

$$a(q^2) = \frac{\alpha^2 \pi}{2q^2} \frac{1}{\tau} \sqrt{1 - \frac{1}{\tau}} (\tau |G_M|^2 + |G_E|^2)$$

$$b(q^2) = \frac{2\pi\alpha^2}{q^2} \frac{\tau - 1}{\tau}, \quad \tau = \frac{q^2}{4M^2}, \quad R = \frac{\tau |G_M|^2 - |G_E|^2}{\tau |G_M|^2 + |G_E|^2} = \frac{\tau - r}{\tau + r}$$

$$\sigma_{tot} = a(q^2) \left( 2 + \frac{2}{3} R \right)$$

$R$  angular asymmetry  $r = \frac{|G_E|^2}{|G_M|^2}$





(continua)

$$\left. \begin{aligned} R &= \frac{\tau |G_M|^2 - |G_E|^2}{\tau |G_M|^2 + |G_E|^2} = \frac{\tau - r}{\tau + r} & r &= \frac{|G_E|^2}{|G_M|^2} \\ \sigma_{tot} &= \frac{4\pi\alpha^2}{3q^2} \sqrt{1 - \frac{1}{\tau}} \left( |G_M|^2 + \frac{1}{2\tau} |G_E|^2 \right) \end{aligned} \right\} |G_M|, \quad |G_E|$$

$$\frac{d\sigma}{d\cos\theta} = a(q^2) \left( 1 + R \cos^2\theta \right) - b(q^2) \operatorname{Re}[G_M A^*] \cos\theta$$

$q^2 \leq 0$  Rosenbluth plot  
change  $E$ ,  $\theta_e$  at fixed  $q^2$   
→ linear plot in  $\varepsilon$

$q^2 \geq 0$  measure  $\theta$  at fixed  $q^2$   
→ get  $R \rightarrow r$   
 $\cos^2\theta$  typical of Born diagram

non  $\cos^2\theta$   
→ explore  $2\gamma$   
mechanisms?

need good coverage over whole range in  $\theta$

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) + \tau F_2(Q^2) & \text{with } \tau &= \frac{q^2}{4M^2} & \text{threshold } \tau = 1 \Rightarrow |G_E| = |G_M| \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned}$$



poor exp. angular coverage; data mainly from  $\sigma_{tot}$  with  $|G_E| = |G_M|$   
true only at threshold →  $|G_E|$  is unknown!

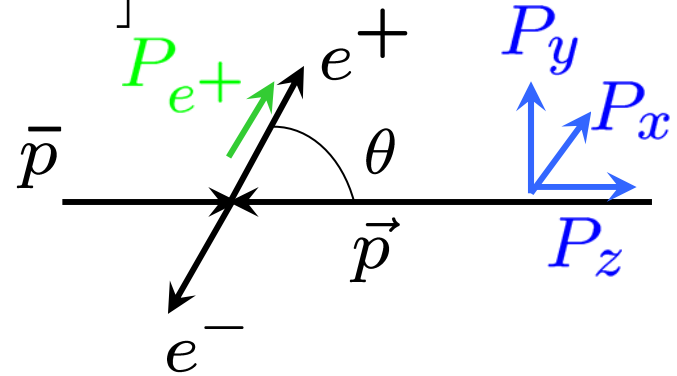
# polarized exclusive Drell-Yan cross section

$$A_y = \frac{1}{P_y} \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{b(q^2)}{2\sqrt{\tau-1} d\sigma^0} \times \sin\theta \left[ \cos\theta \operatorname{Im}[G_M G_E^*] - \sqrt{\frac{\tau-1}{\tau}} \operatorname{Im}[G_E A^*] \right] \quad \bar{p} \vec{p} \rightarrow e^+ e^-$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^0}{d\Omega} (1 + \vec{A} \cdot \vec{P})$$

$$A_x = -P_{e+} \frac{b(q^2)}{2\sqrt{\tau-1} d\sigma^0} 2 \sin\theta \operatorname{Re}[G_M G_E^*]$$

$$A_z = P_{e+} \frac{b(q^2) \sqrt{\tau}}{2\sqrt{\tau-1} d\sigma^0} 2 \cos\theta |G_M|^2$$



$A_x, A_z$  require polarization of the electron:  $P_{e+} \neq 0$

FSI  $\rightarrow$  T-odd mechanisms are allowed :  $\vec{p}_{e+} \times \vec{p} \cdot \vec{P}$  generates  $A_y \neq 0$

phases of FF from residual interactions (FSI) of baryon system :

interference of channels with different phases (  $\operatorname{Im}[G_E^* G_M]$  )

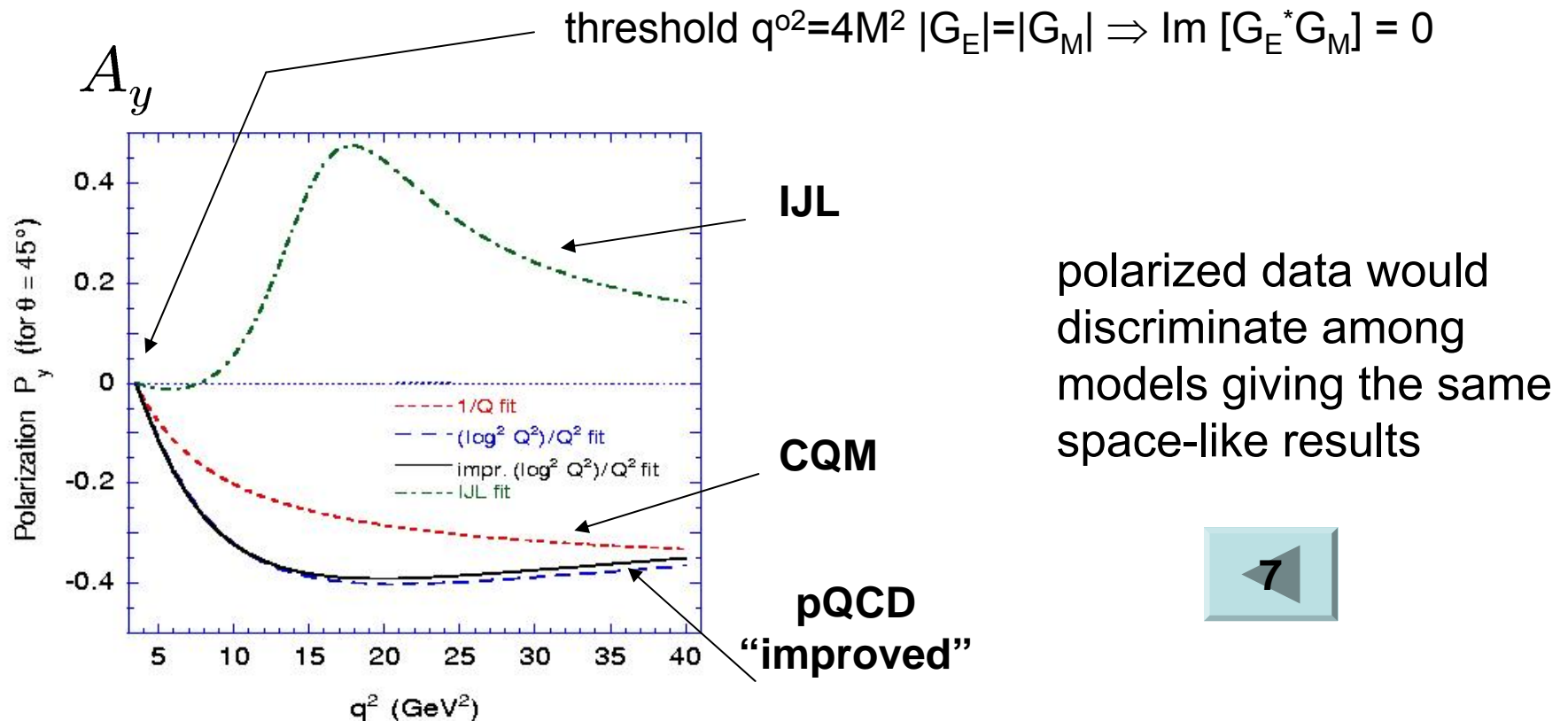


$$A_y \propto \operatorname{Im}(G_E^* G_M) \propto \sin(\delta_M - \delta_E)$$

ambiguity  $\delta \leftrightarrow \pi - \delta$  solved by

$$\frac{A_y}{A_x} \propto -\frac{\operatorname{Im}(G_E^* G_M)}{\operatorname{Re}(G_E^* G_M)} \propto \tan(\delta_E - \delta_M)$$

# Interesting model selectivity



Brodsky *et al.*  
 P.R. D69 (04) 054022

but no polarization data available  $\rightarrow$  phases of  $G_{E/M}$  unknown !

# puzzles in the time-like domain

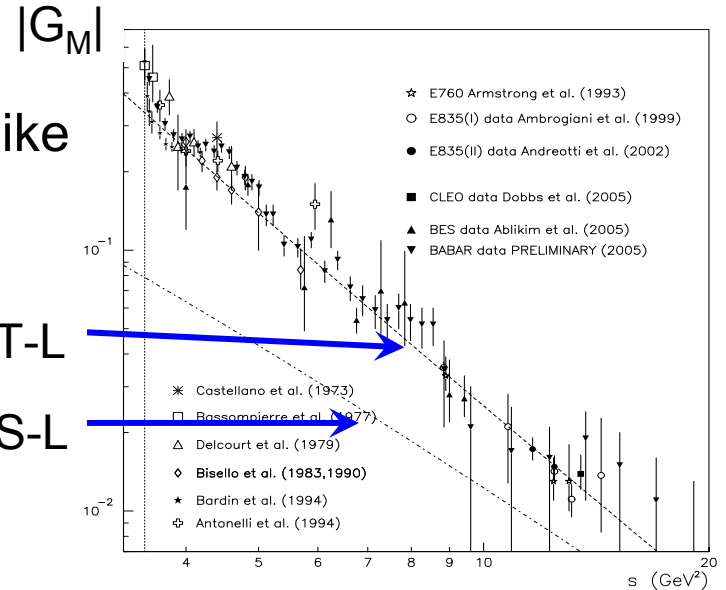
- space like  $\leftarrow$  analytic continuation  $\rightarrow$  time-like

$$\lim_{q^2 \rightarrow \infty} |G_M(q^2 \geq 0)| = G_M(-q^2 \geq 0)$$

$$\text{but } |G_M^p(q^2)| \approx 2 G_M^p(-q^2)$$

fit to pQCD T-L

fit to pQCD S-L

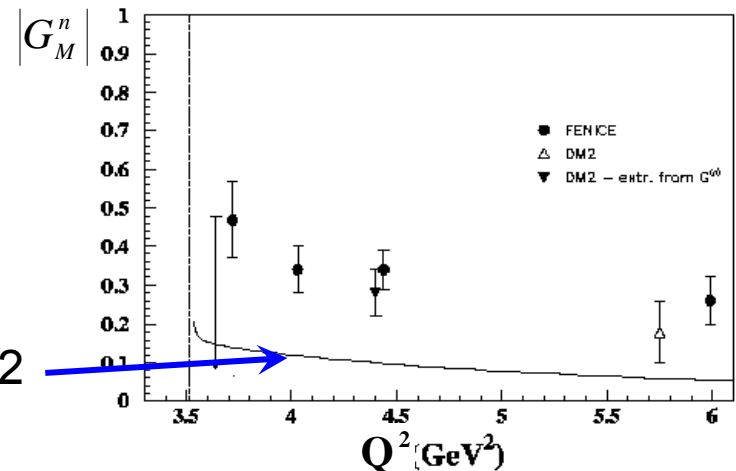


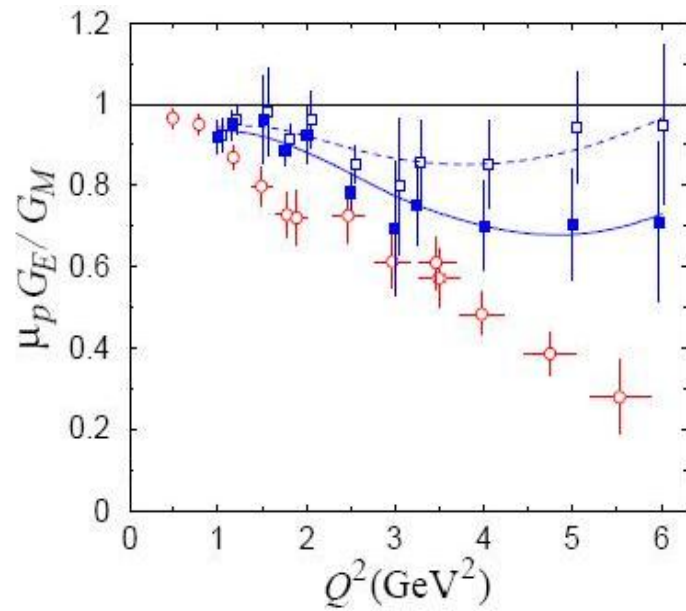
only 1 measurement for neutron (ADONE-1998)

$$\text{again with } |G_E^n| = |G_M^n|$$

$$\bullet \text{ pQCD + analyticity } \rightarrow \left| \frac{G_M^n}{G_M^p} \right|^2 \approx \left( \frac{e_d}{e_u} \right)^2 = \frac{1}{4}$$

$$\text{but } |G_M^p| \leq |G_M^n| \quad \text{fit to } |G_M^p| / 2$$

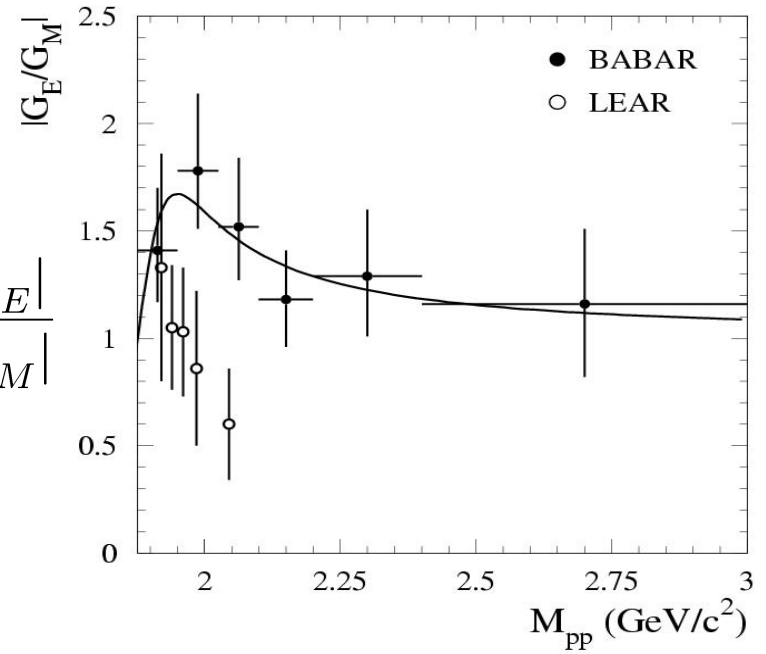




space-like

continued

$$\frac{G_E}{G_M} < 1 < \frac{|G_E|}{|G_M|}$$

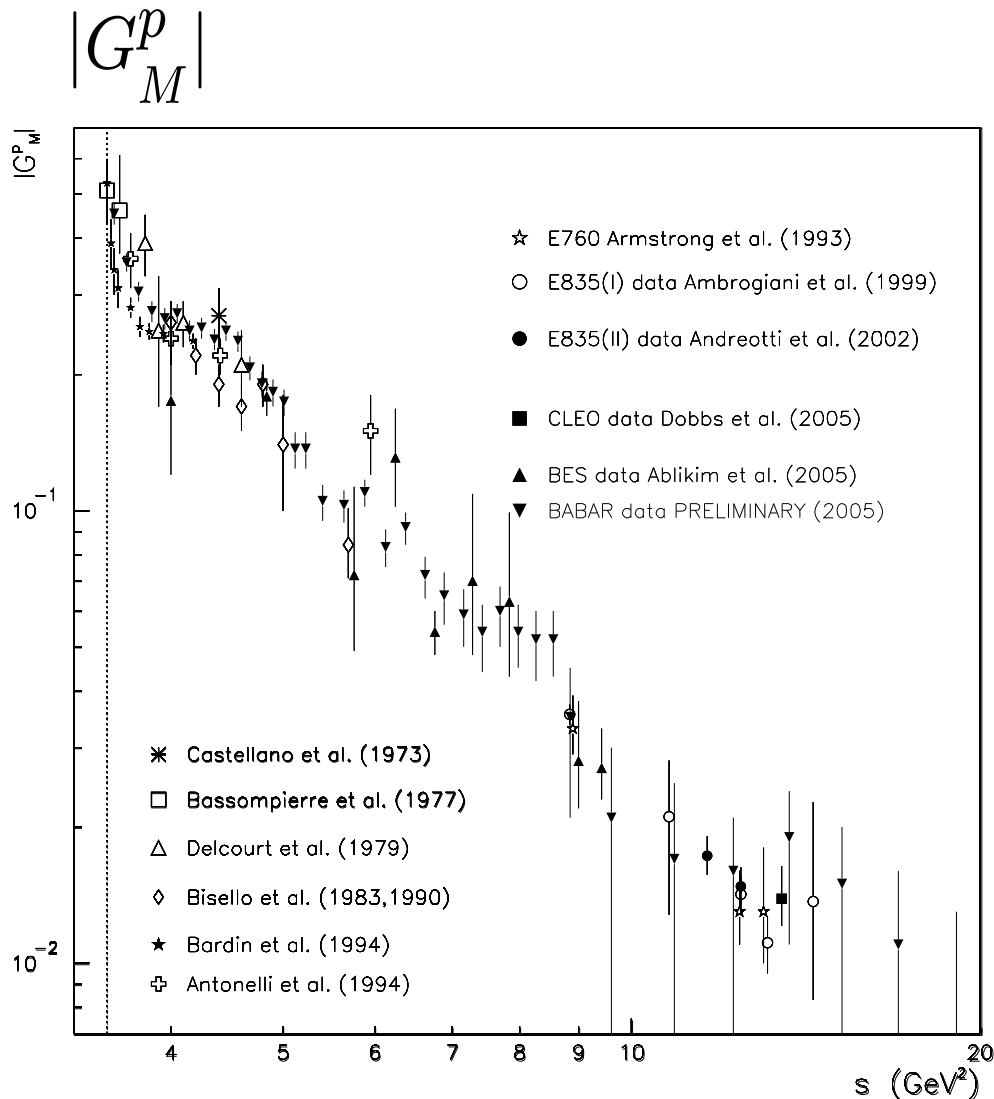


time-like

# World data

$$e^+e^- \rightarrow p\bar{p}$$

$$p\bar{p} \rightarrow e^+e^-$$



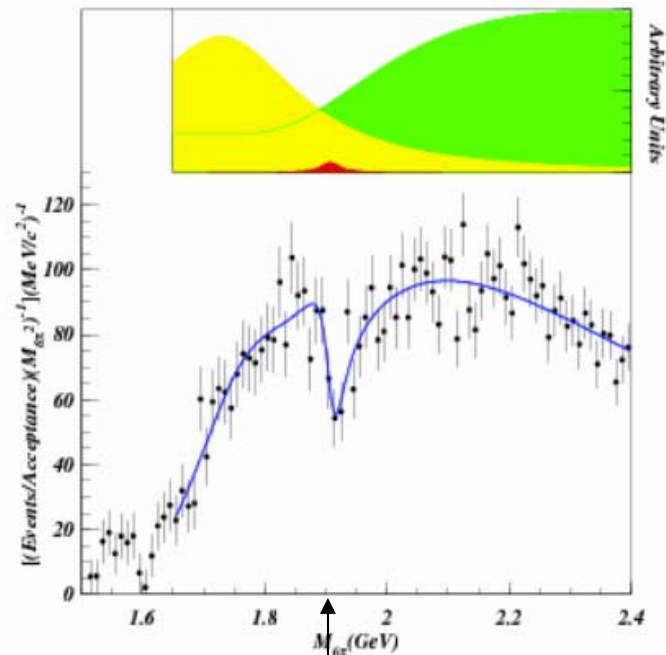
ADONE	Q <sup>2</sup> = 4.4 GeV	(1973)
<b>CERN</b>	<b>Q<sup>2</sup> ~ 3.6</b>	<b>(1977)</b>
Orsay-DM1	Q <sup>2</sup> ~ 3.75-4.56	(1979)
Orsay-DM2	Q <sup>2</sup> = 4-5	(1983)
<b>LEAR</b>	<b>Q<sup>2</sup> ~ 3.5-4.2</b>	<b>(1994)</b>
<b>E760</b>	<b>Q<sup>2</sup> ~ 8.9-13</b>	<b>(1993)</b>
FENICE	Q <sup>2</sup> ~ 3.7-6	(1994)
<b>E835</b>	<b>Q<sup>2</sup> ~ 8.8-18.4</b>	<b>(1999)</b>
	<b>11.6-18.2</b>	<b>(2003)</b>
CLEO	Q <sup>2</sup> ~ 11-12	(2005)
BES	Q <sup>2</sup> ~ 4-9	(2005)
BaBar	Q <sup>2</sup> ~ 2-20	(2005)

## below threshold ?

world data for  $|G_M|$  show  
steep rise at threshold

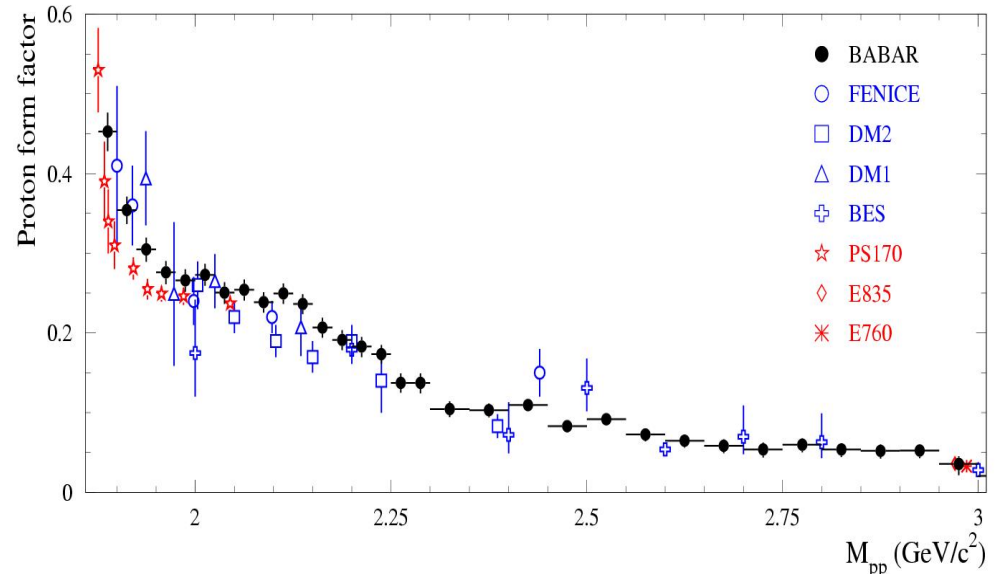


tail of resonance at  $q^2 \leq 4M^2$  :  
 $\rho(1900)$  or baryonium or..?



21-Gen-10

$W \sim 1.9 \text{ GeV}$



should show up as a dip in some  
hadronic cross section

E687 diffractive photoproduction of  $6\pi$