

Riassunto della lezione precedente

- QCD = teoria di gauge non abeliana \rightarrow antiscreening ;
comportamento asintotico antiintuitivo: alti $Q^2 \rightarrow$ liberta` asintotica (pQCD)
bassi $Q^2 \rightarrow$ confinamento
- Correzioni perturbative (pQCD) hanno come punto di partenza all'ordine "0"
il modello a partoni (QPM), dedotto da risultati di Deep Inelastic Scattering
(DIS)
- Linee generali della teoria dello scattering con sonde elettromagnetiche:
 - sezione d'urto inclusiva
 - sezione d'urto inclusiva elastica: caso della particella scalare

Scattering inclusivo elastico su particella scalare libera

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2}{Q^4} E'^2 \cos^2 \frac{\theta_e}{2} \frac{E'}{E} |F_1(Q^2)|^2 \equiv \sigma_{\text{Mott}} \frac{E'}{E} |F_1(Q^2)|^2$$

Coulomb scattering elastico da
particella puntiforme



The diagram consists of three arrows pointing upwards from text labels to specific parts of the equation above. The first arrow points from 'Coulomb scattering elastico da particella puntiforme' to the $\frac{4\alpha^2}{Q^4}$ term. The second arrow points from 'rinculo bersaglio' to the $\frac{E'}{E}$ term. The third arrow points from 'struttura bersaglio' to the $|F_1(Q^2)|^2$ term.

rinculo
bersaglio

struttura
bersaglio

Breit frame \rightarrow fattore di forma

$$\mathbf{P} = -\mathbf{q}/2$$



$$\mathbf{P}' = +\mathbf{q}/2$$

$$v = 0$$

$$R^\mu = (2E, \mathbf{0})$$

$$q^\mu = (0, \mathbf{q})$$

$$J^\mu = (J^0, \mathbf{0}) \rightarrow 2E F_1(Q^2)$$



$$F_1(Q^2) \equiv F_1(|\mathbf{q}|^2) = \int d\mathbf{r} \, \rho(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}}$$

↑
fattore di forma di
carica
materia
.....

↑
distribuzione di
carica
materia
.....

Bersaglio = particella di Dirac libera puntiforme

Esempio: $e^- + \mu^- \rightarrow e'^- + \mu^-$

$$J^\mu = \bar{u}(P') \gamma^\mu u(P)$$

$$H^{\mu\nu} \equiv L^{\mu\nu} \quad \text{with } k^{(\prime)} \leftrightarrow P^{(\prime)}$$

$$\begin{aligned} \frac{1}{2} \sum_{\text{spin}} H^{\mu\nu} &= \frac{1}{2} \text{Tr} \left[(\not{P}' + M) \gamma^\mu (\not{P} + M) \gamma^\nu \right] \\ &= 2 \left[P'^\mu P^\nu + P'^\nu P^\mu - (P \cdot P' - M^2) g^{\mu\nu} \right] \end{aligned}$$

$$L_{\mu\nu} H^{\mu\nu} \stackrel{\text{TRF}}{=} 16EE'M^2 \cos^2 \frac{\theta_e}{2} \left(1 + \frac{Q^2}{2M^2} \tan^2 \frac{\theta_e}{2} \right)$$

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left(1 + \frac{Q^2}{2M^2} \tan^2 \frac{\theta_e}{2} \right)$$

interazione
magnetica di
spin con γ^*

Bersaglio = particella di Dirac libera con struttura

3 vettori indipendenti $P^\mu, P'^\mu, \gamma^\mu$ (+ invarianza per time-reversal, parità)

$$J^\mu = \bar{u}(P') \Gamma^\mu u(P) = \bar{u}(P') \left[\Gamma_1 P^\mu + \Gamma_2 P'^\mu + \Gamma_3 \gamma^\mu \right] u(P)$$

$$\Gamma_i(Q^2, P^2, P'^2) \equiv \Gamma_i(Q^2)$$

conservazione della corrente $q_\mu J^\mu = 0$

$$\begin{aligned} q_\mu \bar{u}(P') \Gamma^\mu u(P) &= \\ &= \bar{u}(P') \left[\Gamma_1 P \cdot (P' - P) + \Gamma_2 P' \cdot (P' - P) + \Gamma_3 \gamma \cdot (P' - P) \right] u(P) \\ &= \bar{u}(P') \left[(P \cdot P' - M^2) (\Gamma_1 - \Gamma_2) + \Gamma_3 (\not{P}' - \not{P}) \right] u(P) = 0 \\ &\Rightarrow \Gamma_1 = \Gamma_2 \end{aligned}$$

$$J^\mu = \bar{u}(P') \left[\Gamma_1(Q^2) R^\mu + \Gamma_3(Q^2) \gamma^\mu \right] u(P)$$

eq. di Dirac
 $\not{P} u = M u$
 $\bar{u} \not{P} = \bar{u} M$

Decomposizione di Gordon (on-shell)

$$J^\mu = \bar{u}(P') \left[\Gamma_1(Q^2) R^\mu + \Gamma_3(Q^2) \gamma^\mu \right] u(P)$$

$$\bar{u} \gamma^\mu u = \bar{u} \left[\frac{R^\mu}{2M} + \frac{i}{2M} \sigma^{\mu\nu} q_\nu \right] u \quad \text{with } \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

cioe' $R^\mu \leftrightarrow 2M \gamma^\mu - i \sigma^{\mu\nu} q_\nu$



proof flow-chart

- da destra, inserire def. di $\sigma^{\mu\nu}$
- usare eq. di Dirac
- usare $\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu}$
- usare eq. Dirac \rightarrow sinistra

$$J^\mu = \bar{u}(P') \left[F_1(Q^2) \gamma^\mu + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2(Q^2) \right] u(P)$$

$F_1 = 2M\Gamma_1 + \Gamma_3$	Dirac form factor	$F_1(0) = 1$
$F_2 = -2M\Gamma_1$	Pauli form factor	$F_2(0) = \kappa$

Bersaglio = particella di Dirac libera e composita

$$\frac{1}{2} \sum_{\text{spin}} H^{\mu\nu} = \frac{1}{2} \text{Tr} \left[(\not{p}' + M) \Gamma^\mu (\not{p} + M) \Gamma^\nu \right]$$

$$\Gamma^\mu = F_1 \gamma^\mu + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2$$

.....

Sezione d'urto

$$\frac{d\sigma}{dE' d\Omega} = \sigma_{\text{Mott}} \left[\left(F_1^2 + \frac{Q^2}{4M^2} F_2^2 \right) + \frac{Q^2}{2M^2} (F_1 + F_2)^2 \tan^2 \frac{\theta_e}{2} \right] \delta \left(\nu - \frac{Q^2}{2M} \right)$$



$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[\left(F_1^2 + \frac{Q^2}{4M^2} F_2^2 \right) + \frac{Q^2}{2M^2} (F_1 + F_2)^2 \tan^2 \frac{\theta_e}{2} \right]$$

struttura interna
(difficilmente separabile)

Formula di Rosenbluth

Definizione fattori
di forma di Sachs

(Yennie, 1957)

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$\text{with } \tau = \frac{Q^2}{4M^2}$$

N.B.: infatti, in Breit frame + riduzione nonrel. \rightarrow

$$\begin{aligned} J^0 &\sim G_E \\ \mathbf{J} &\sim \frac{G_M}{2M} \boldsymbol{\sigma} \times \mathbf{q} \end{aligned}$$

distribuzione di carica/magnetica
del bersaglio



$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \sigma_{\text{Mott}} \frac{E'}{E} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta_e}{2} \right] \\ &\equiv \sigma_{\text{Mott}} \frac{E'}{E} \left[A(Q^2) + B(Q^2) \tan^2 \frac{\theta_e}{2} \right] \end{aligned}$$

separazione piu` facile

Separazione di Rosenbluth

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta_e}{2} \right]$$

- larghi θ_e (larghi Q^2) \rightarrow estrarre G_M
- piccoli θ_e (piccoli Q^2) \rightarrow estrarre G_E per differenza
- Rosenbluth plot



$$\epsilon (1 + \tau) \frac{E}{E'} \frac{1}{\sigma_{\text{Mott}}} \frac{d\sigma}{d\Omega} = \epsilon G_E^2 + \tau G_M^2$$

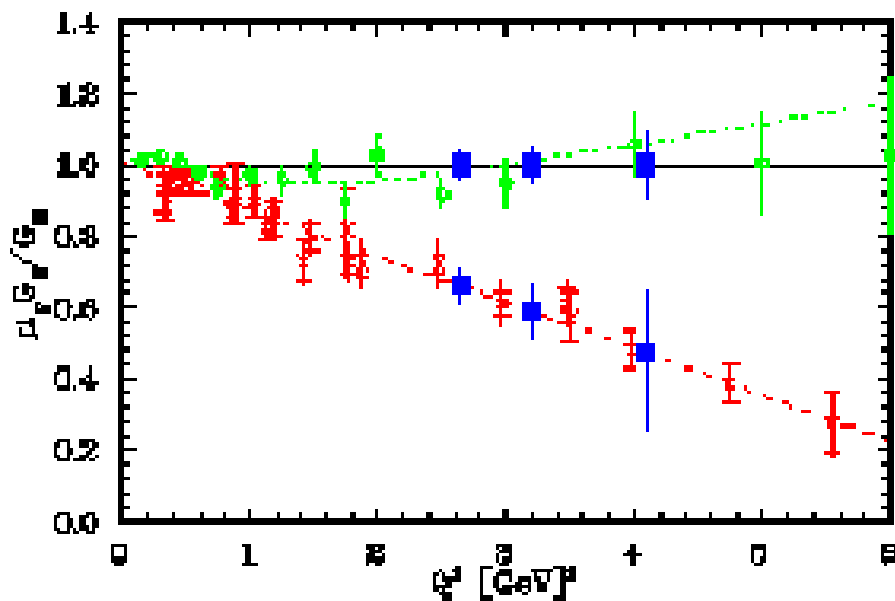
$$\epsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right]^{-1}$$

polarizz. trasversa lineare di γ^*

misure con diverse $(E, \theta_e) \rightarrow$ plot in ϵ a fisso Q^2

intercetta a $\epsilon = 0 \rightarrow G_M$

pendenza in $\epsilon \rightarrow G_E$



Rosenbluth plot

$$\mu_p \frac{G_E^p}{G_M^p} \rightarrow \text{cost}$$



$$F_2 \sim \frac{F_1}{Q^2}$$

pQCD scaling

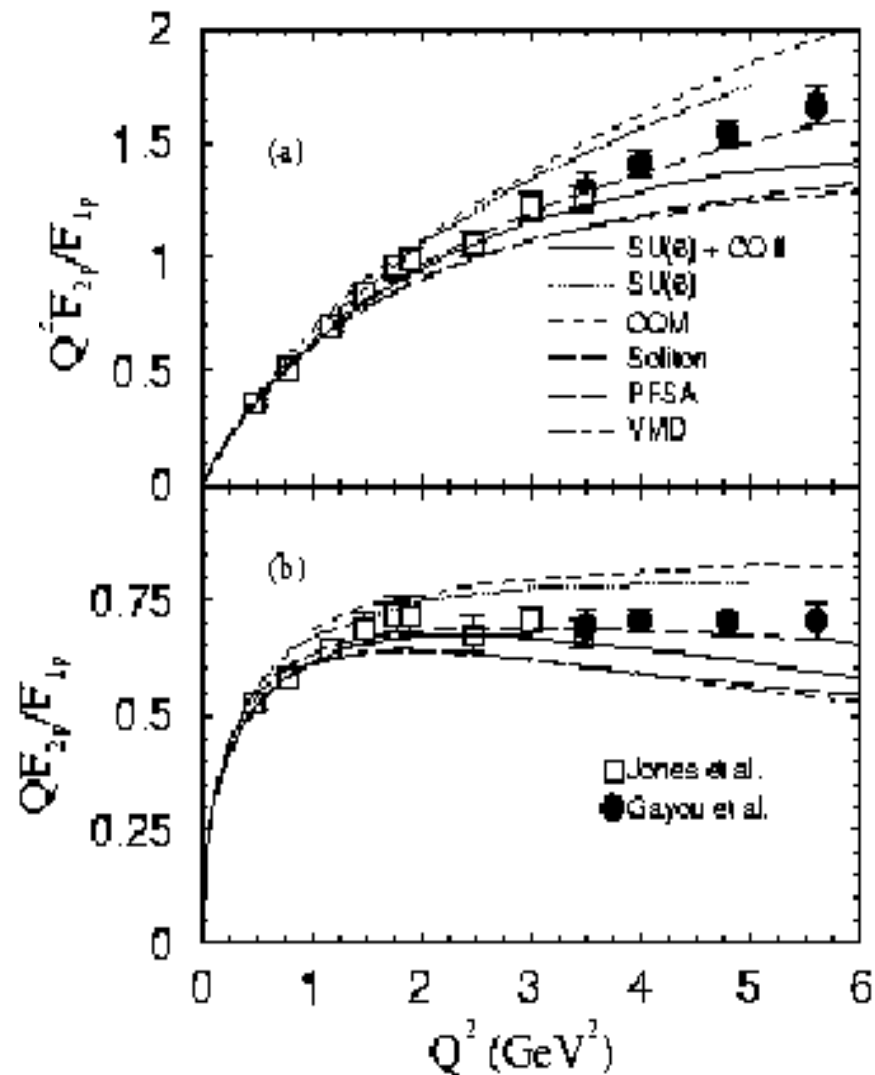
JLAB data

(ottenuti con e^- scattering
e doppia polarizzazione
→ più precisi)



$$F_2 \sim \frac{F_1}{Q}$$

$Q^2 \lesssim 10 \text{ (GeV/c)}^2$ ancora regime
non perturbativo



Sezione d'urto (an)elastica inclusiva per particella di Dirac composita

Risultato generale :
$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[A(\nu, Q^2) + B(\nu, Q^2) \tan^2 \frac{\theta_e}{2} \right]$$

Procedura :

- 2 vettori indipendenti P, q
- base tensoriale: $b_1=g^{\mu\nu}, b_2=q^\mu q^\nu, b_3=P^\mu P^\nu,$
 $b_4=(P^\mu q^\nu + P^\nu q^\mu), b_5=(P^\mu q^\nu - P^\nu q^\mu),$
 $b_6=\varepsilon_{\mu\nu\rho\sigma} q^\rho P^\sigma$
- tensore adronico $W^{\mu\nu} = \sum_i c_i (q^2, P \cdot q) b_i$
- invarianza per parità e time-reversal,
 conservazione della corrente $q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu = 0$
- sistema lineare con c_6 indeterminato ($=0$), $c_5=0$,
 c_1 e c_3 dipendenti da c_2 e c_4
- Risultato finale :

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) q^2 c_2(q^2, P \cdot q) + \frac{\tilde{P}^\mu \tilde{P}^\nu}{M^2} \left(-\frac{M^2 q^2}{P \cdot q} \right) c_4(q^2, P \cdot q)$$

$$\tilde{P}^\mu = P^\mu - \frac{P \cdot q}{q^2} q^\mu$$



(continua)

- struttura $\varepsilon_{\mu\nu\rho\sigma} q^\rho P^\sigma$ proibita da invarianza per parita`
- struttura $(P^\mu q^\nu - P^\nu q^\mu)$ proibita da invarianza per time-reversal
- strutture $(P^\mu q^\nu + P^\nu q^\mu)$, $q^\mu q^\nu$ trascurabili perche` $\sim m_e^2$, ma non proibite (violazione della conservazione della corrente)
- hermiticity $W^{\mu\nu} = (W^{\nu\mu})^* \rightarrow c_{2,4}$ funzioni reali

$$L_{\mu\nu} W^{\mu\nu} = 4EE' \cos^2 \frac{\theta_e}{2} \left(W_2 + 2W_1 \tan^2 \frac{\theta_e}{2} \right)$$



$$\begin{aligned} \frac{d\sigma}{dE' d\Omega} &= \frac{\alpha^2}{Q^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu} \\ &= \sigma_{\text{Mott}} \left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2) \tan^2 \frac{\theta_e}{2} \right] \end{aligned}$$

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[W_2 + 2W_1 \tan^2 \frac{\theta_e}{2} \right]$$

Riepilogo

Scattering inclusivo su particella di Dirac libera e composita

anelastico

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2) \tan^2 \frac{\theta_e}{2} \right]$$

elastico

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left[(F_1^2 + \tau F_2^2) + 2\tau (F_1 + F_2)^2 \tan^2 \frac{\theta_e}{2} \right] \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$W_2^{\text{el}} \leftrightarrow (F_1^2 + \tau F_2^2) \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$2W_1^{\text{el}} \leftrightarrow 2\tau (F_1 + F_2)^2 \delta \left(\nu - \frac{Q^2}{2M} \right)$$

elastico puntiforme

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left(1 + 2\tau \tan^2 \frac{\theta_e}{2} \right) \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$W_2^{\text{el}} \leftrightarrow \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$$W_1^{\text{el}} \leftrightarrow \tau \delta \left(\nu - \frac{Q^2}{2M} \right)$$

$F_1 \rightarrow 1$
 $F_2 \rightarrow 0$

DIS regime

$$Q^2 \longrightarrow \infty$$
$$x_B = \frac{Q^2}{2P \cdot q} \quad \text{fissato}$$

TRF : $v \rightarrow \infty$ velocemente come Q^2

poiche' $Q^2 = -(q^0)^2 + \mathbf{q}^2 = -v^2 + \mathbf{q}^2$

dunque

$$|\mathbf{q}| \rightarrow \infty \quad \text{velocemente come } Q^2$$

dipendente dal frame

indipendente dal frame

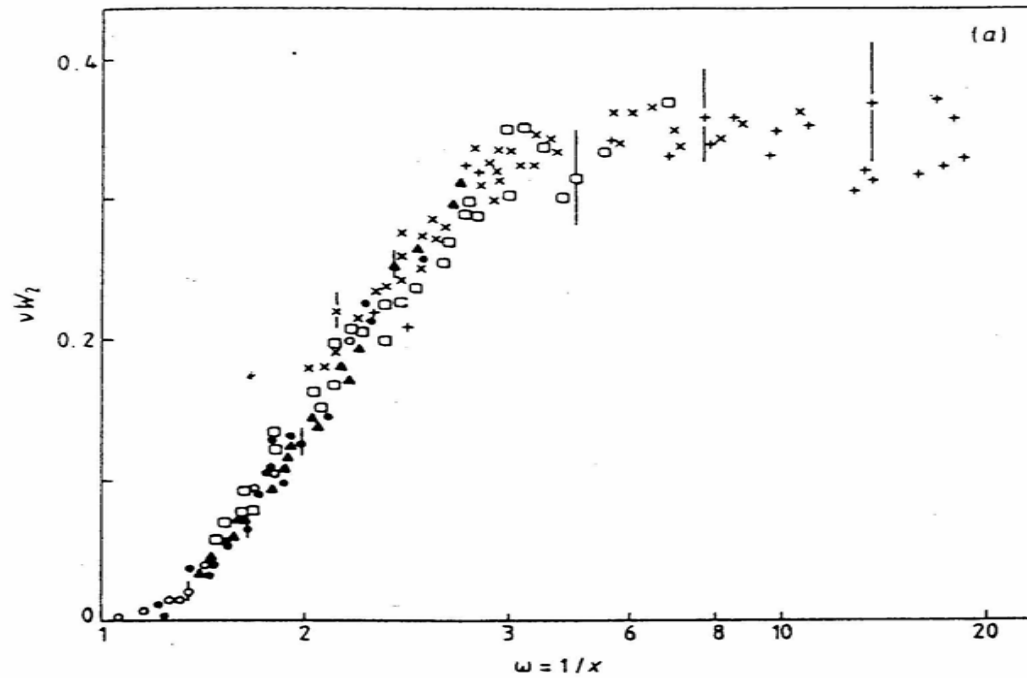
Scaling

$$\begin{aligned} W_2^{\text{el}} &\leftrightarrow \delta\left(\nu - \frac{Q^2}{2M}\right) & \nu W_2^{\text{el}} &\leftrightarrow \delta\left(1 - \frac{Q^2}{2M\nu}\right) \equiv \delta(1 - x_B) \equiv F_2(x_B) \\ W_1^{\text{el}} &\leftrightarrow \tau \delta\left(\nu - \frac{Q^2}{2M}\right) & 2MW_1^{\text{el}} &\leftrightarrow \frac{Q^2}{2M\nu} \delta\left(1 - \frac{Q^2}{2M\nu}\right) \equiv x_B \delta(1 - x_B) \equiv 2F_1(x_B) \end{aligned}$$

Osservazione sperimentale dello scaling = segnale che nella cinematica DIS (cioe' $Q^2, \nu \rightarrow \infty$, x_B fissato) lo scattering si puo' rappresentare come la somma incoerente di scattering elastici da costituenti puntiformi del bersaglio con statistica di Dirac \rightarrow origine del concetto di partone

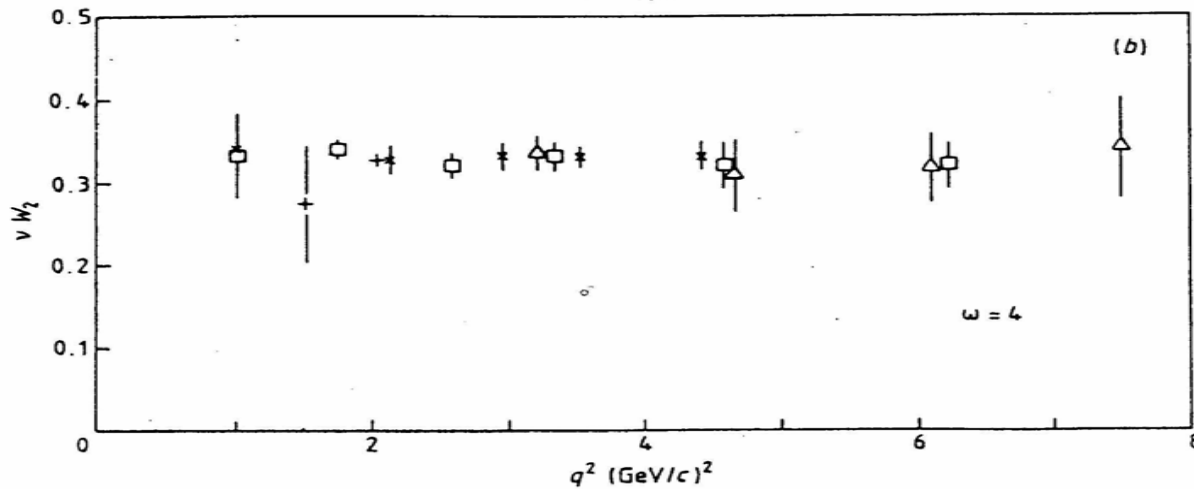
N.B. Analogo dell'esperimento di Rutherford sullo scattering di particelle α da atomi

νW_2



$1/x$

Aitchison
& Hey



Q^2

Figure 4.2 Bjorken scaling: the structure function νW_2 (a) plotted against $\omega = 1/x$ for different q^2 values (Miller *et al* 1972) (b) plotted against q^2 for a single value of $x = 0.25$ ($\omega = 4$) (Friedman and Kendall 1972).

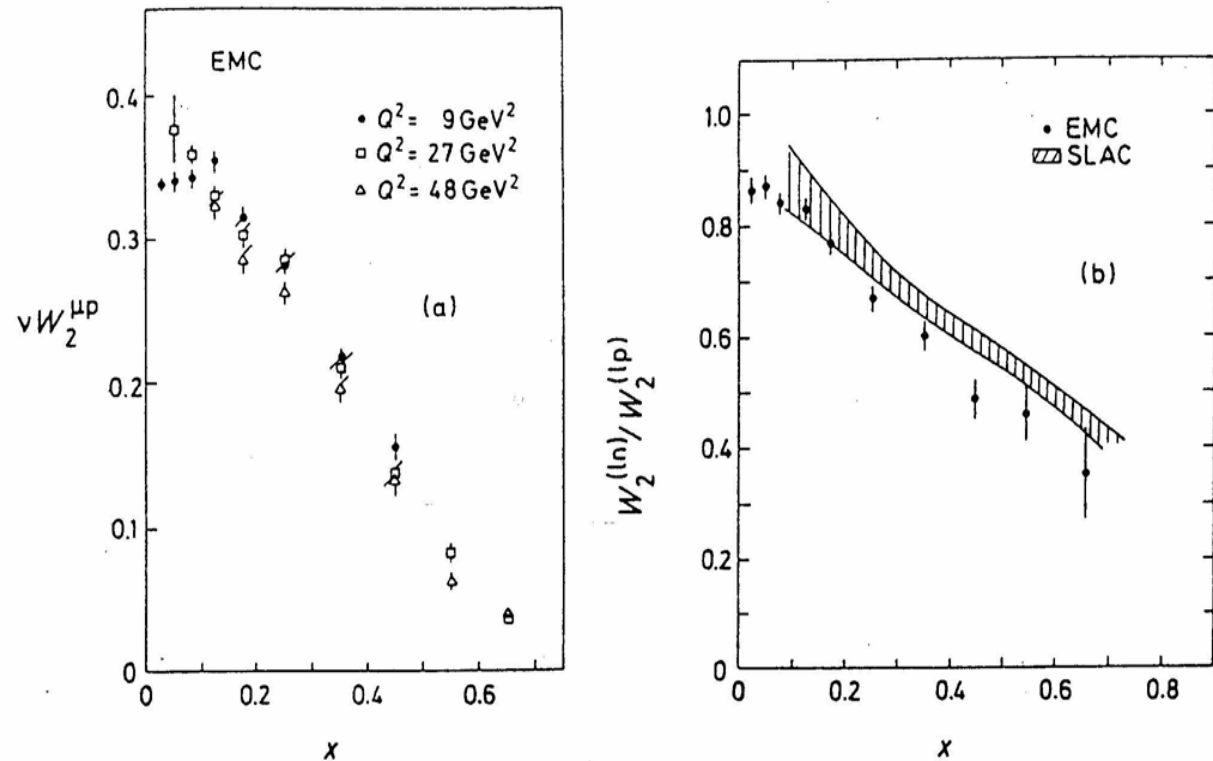


Figure 18.9 Data from the European Muon Collaboration (EMC) for the structure function $\nu W_2^{\mu p}(\nu, Q^2)$ of the proton as a function of $x = Q^2/(2M\nu)$ for various Q^2 values. Exact Bjorken scale invariance would demand that the data points for the same x but different Q^2 should lie on top of one another (a). Part (b) shows the ratio of the neutron and proton structure functions $W_2^{(ln)}(\nu, Q^2)$ and $W_2^{(lp)}(\nu, Q^2)$ ($l = e, \mu$) as a function of x . The shaded band represents the SLAC data obtained from electron scattering in the interval $2 \leq Q^2 \leq 20 \text{ GeV}^2$. The points correspond to preliminary EMC data from muon scattering in the interval $10 \leq Q^2 \leq 80 \text{ GeV}^2$ (after Drees 1983 and Dydak 1983).

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Taylor

osservazione
sperimentale
(DIS con e^- beam
di 7-17 GeV e
 $6^\circ < \theta_e < 10^\circ$)

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Nobel laureate