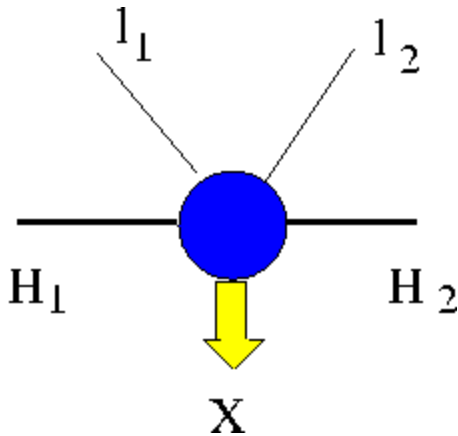


Riassunto della lezione precedente

- verifica sperimentale di QPM in reazioni elettrodeboli :
 1. DIS con (anti)neutrini su nuclei isoscalari
 - sez. d'urto di neutrino domina quella per antineutrino
 - antipartoni soppressi in mezzo nucleare
 2. DIS con (anti)neutrini su protone
 - dominanza di quark valenza / mare per $x_B \rightarrow 1/0$
 3. rapporto di carica per nuclei isoscalari
- regole di somma :
 1. Gottfried sum rule → $SU_f(3)$ rotta per i quark del mare
 2. Momentum sum rule → gluoni portano metà del momento dell'adrone
- verifica fattorizzazione → esplorare nuovi processi fondamentali

Drell - Yan

[Drell & Yan, P.R.L. **25** (70) 316]



adroni in annichilazione $H_{1/2}$ con momento $P_{1/2}$

leptoni prodotti $l_{1/2}$ con momento $k_{1/2}$

energia disponibile nel c.m. degli adroni

$$s = (P_1 + P_2)^2$$

massa invariante della coppia di leptoni

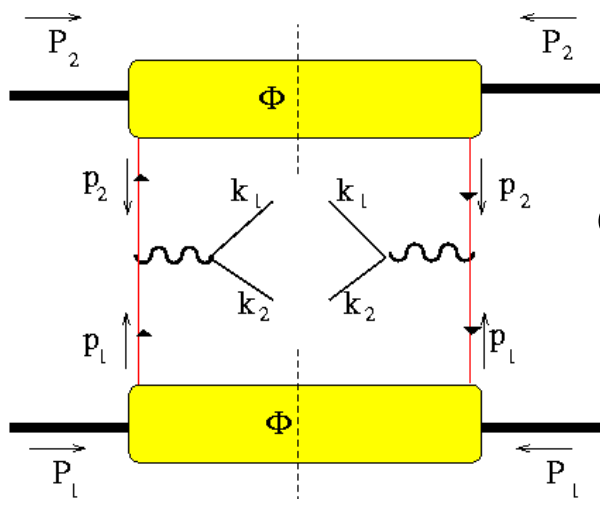
$$M^2 \equiv q^2 = (k_1 + k_2)^2$$

$$q^2 = Q^2 \geq 0 \quad \text{time-like}$$

regime DIS : $q^2, s \rightarrow \infty$ con $\tau = q^2 / s$ fissato $1 \geq \tau \geq 0$

la coppia di leptoni non interagisce con la coppia di adroni iniziali
→ è manifestazione del decadimento dei bosoni di gauge intermedi
prodotti dalla annichilazione adronica

bosoni di gauge a spin 1 con $Q^2 \geq 0 \rightarrow$ risonanze mesoniche vettoriali
→ decadimento → produzione di coppie leptoniche con p_T



$$p_1 = x_1 P_1 \quad \text{con} \quad x_1 = \frac{Q^2}{2P_1 \cdot q} \quad 1 \geq x_{1/2} \geq 0$$

$$p_2 = x_2 P_2 \quad x_2 = \frac{Q^2}{2P_2 \cdot q}$$

energia disponibile della reazione elementare :

$$(p_1 + p_2)^2 \sim 2 p_1 \cdot p_2 \sim x_1 x_2 (P_1 + P_2)^2 = x_1 x_2 s$$

$$\tau = \frac{q^2}{s} = \frac{M^2}{s} = x_1 x_2 \quad 1 \geq \tau \geq 0$$

$$x_F = x_1 - x_2 \quad 1 \geq x_F \geq -1$$

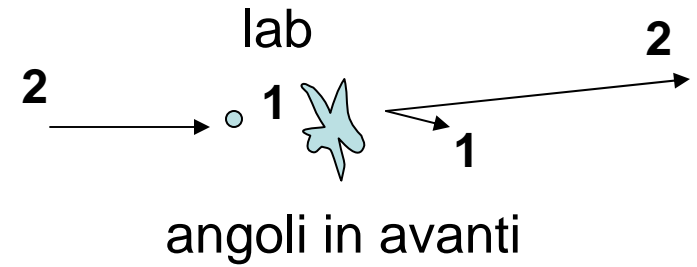
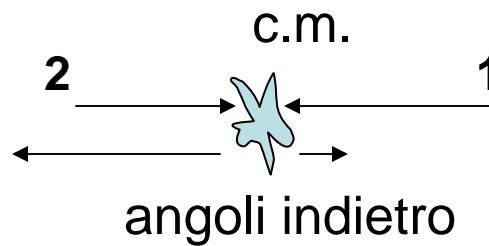
$$x_{1/2} = \frac{1}{2} \left(\pm x_F + \sqrt{x_F^2 + \frac{4M^2}{s}} \right)$$

$x_{1/2}$ = frazione del momento longitudinale \rightarrow

x_F = momento longitudinale della coppia nel c.m. rispetto al momento longitudinale massimo possibile

supponiamo H_2 = fascio e H_1 = target

$$x_F \rightarrow -1 \Leftrightarrow \begin{aligned} x_2 &\rightarrow 1 \\ x_1 &\rightarrow 0 \end{aligned}$$



$$x_F \rightarrow 1 \Leftrightarrow \begin{aligned} x_2 &\rightarrow 0 \\ x_1 &\rightarrow 1 \end{aligned}$$

situazione rovesciata

“solite” formule, applicate al processo DY

$$d\sigma = \frac{1}{F} |\mathcal{M}|^2 dR$$



$$F = 4\sqrt{(P_1 \cdot P_2)^2 - M_1^2 M_2^2} \sim 2s$$

$$dR = (2\pi)^4 \delta(P_1 + P_2 - P_X - k_1 - k_2) \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \frac{d\mathbf{k}_1}{(2\pi)^3 2E_1} \frac{d\mathbf{k}_2}{(2\pi)^3 2E_2}$$

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu} \quad H^{\mu\nu} = \sum_X \langle P_1 S_1, P_2 S_2 | J^\mu | P_X \rangle \langle P_X | J^\nu | P_1 S_1, P_2 S_2 \rangle$$

$$(2\pi)^3 2E_1 (2\pi)^3 2E_2 \frac{d\sigma}{d\mathbf{k}_1 d\mathbf{k}_2} = 2 \frac{1}{2s} (2\pi)^4 \frac{e^4}{Q^4} L_{\mu\nu} \times \frac{1}{(2\pi)^4} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(P_1 + P_2 - P_X - q) H^{\mu\nu}$$

somma sugli stati
di polarizzazione dei
leptoni finali

$$= (2\pi)^4 32\pi^2 \frac{1}{2s} \frac{\alpha^2}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

$$\begin{aligned} \frac{d\mathbf{k}_1}{(2\pi)^3 2E_1} \frac{d\mathbf{k}_2}{(2\pi)^3 2E_2} &= d^4 k_1 d^4 k_2 \\ &= d^4 q d^4(k_1 - k_2) \stackrel{c.m.}{=} \frac{d^4 q d\Omega}{(2\pi)^4 32\pi^2} \end{aligned}$$

$$\frac{d\sigma}{d^4 q d\Omega} = \frac{\alpha^2}{2s Q^4} L_{\mu\nu} W^{\mu\nu}$$

QPM picture

approssimazione: Q^2 non elevato \rightarrow bosone di gauge γ

energia disponibile nel c.m. della reazione elementare :

$$(p_1 + p_2)^2 \sim 2 p_1 \cdot p_2 = x_1 x_2 2 P_1 \cdot P_2 \sim x_1 x_2 (P_1 + P_2)^2 = x_1 x_2 s$$

processo elementare: $(q\bar{q} \rightarrow l\bar{l}) \equiv (e^+e^- \rightarrow l\bar{l})$ elastico

esempio: produzione di $\mu^+\mu^-$ $\frac{d\sigma^{el}}{dQ^2} = \frac{4\pi\alpha^2}{3Q^2} e_f^2 \delta(x_1 x_2 s - Q^2)$

$$\frac{d\sigma}{dQ^2} = \left(\frac{1}{N_c} \sum_f \right) \int_0^1 dx_1 dx_2 \phi_f(x_1) \frac{d\sigma^{el}}{dQ^2} \phi_{\bar{f}}(x_2)$$

N_c modi di creare
la coppia
conservando il colore
nel vertice;

ciascuna ϕ_f porta N_c
 $\rightarrow (N_c \times N_c) / N_c = N_c$
 \rightarrow test di SU_c (3)

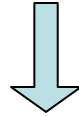
$$= \frac{4\pi\alpha^2}{9Q^4} \sum_f e_f^2 \int_0^1 dx_1 dx_2 \phi_f(x_1) \phi_{\bar{f}}(x_2) \delta \left(x_1 x_2 \frac{s}{Q^2} - 1 \right)$$

$$\begin{aligned} & \sum_f \dots \phi_f(x_1) \phi_{\bar{f}}(x_2) \\ & \equiv \sum_f \dots \left[\phi_f(x_1) \phi_{\bar{f}}(x_2) + \phi_{\bar{f}}(x_1) \phi_f(x_2) \right] \end{aligned}$$

QPM: test sperimentali

a) scaling della sez. d'urto

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9} \sum_f \bar{e}_f^2 \int_0^1 dx_1 dx_2 \phi_f(x_1) \phi_{\bar{f}}(x_2) \delta\left(\frac{x_1 x_2}{\tau} - 1\right)$$



$$Q^3 \frac{d\sigma}{dQ dx_1 dx_2} = \frac{8\pi\alpha^2}{9} \sum_f \bar{e}_f^2 \phi_f(x_1) \phi_{\bar{f}}(x_2) \delta\left(\frac{x_1 x_2}{\tau} - 1\right)$$

oppure, con $\begin{cases} x_F = x_1 - x_2 \\ \tau = x_1 x_2 \end{cases} \quad J = \begin{vmatrix} \frac{\partial x_F}{\partial x_1} & \frac{\partial x_F}{\partial x_2} \\ \frac{\partial \tau}{\partial x_1} & \frac{\partial \tau}{\partial x_2} \end{vmatrix} = x_1 + x_2$



$$M^3 \frac{d\sigma}{dM dx_F} = \int d\tau \frac{d\sigma}{dM dx_F d\tau} = \frac{8\pi\alpha^2}{9} \frac{x_1 x_2}{\sqrt{x_F^2 + 4x_1 x_2}} \sum_f \bar{e}_f^2 \phi_f(x_1) \phi_{\bar{f}}(x_2)$$
$$x_{1/2} = \frac{1}{2} \left(\pm x_F + \sqrt{x_F^2 + \frac{4M^2}{s}} \right)$$

scaling $\forall s \rightarrow$ interazione elementare puntiforme !

$$M^3 \frac{d\sigma}{dM dx_F} \sim 1 - \sqrt{\tau}$$



piccole deviazioni perche`
pQCD $\rightarrow \phi_f(x, \log Q^2)$

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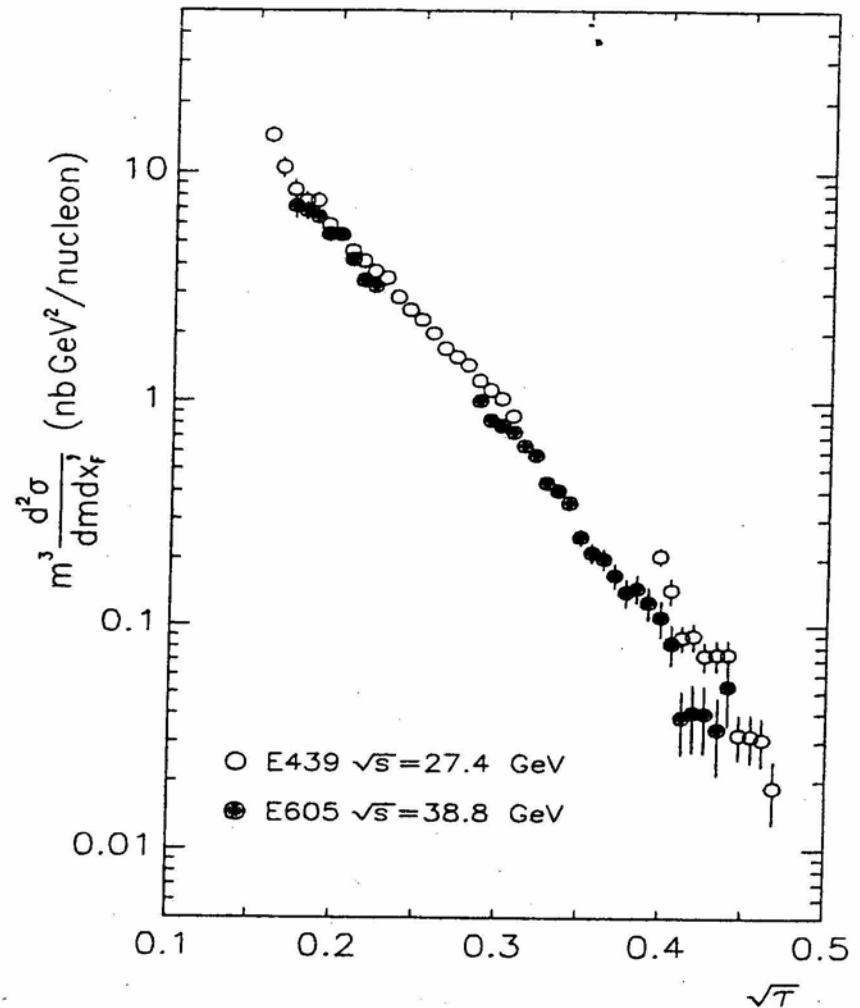


FIG. 12. Scaling form of the dimuon yield versus $\sqrt{\tau}$ comparing this experiment with experiment E439 (Smith *et al.*, Ref. 31, $\sqrt{s} = 27.4$ GeV) for the interval $0 < x'_F < .2$.

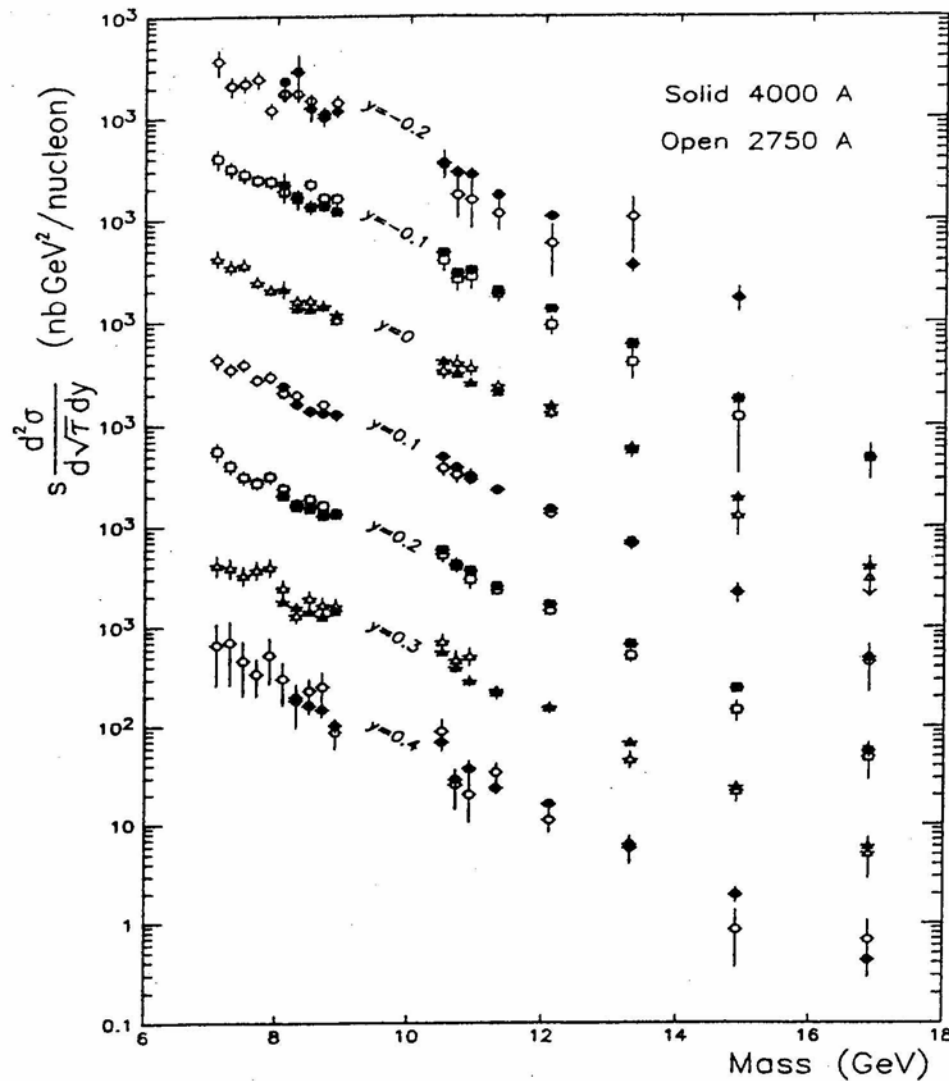


FIG. 10. Scaling form of the dimuon yield, excluding the Υ 's, showing the agreement of the two sets in the region of overlap. The average of both sets is given in Table VIII.

$$\tau = \frac{M^2}{s} \rightarrow \frac{d}{d\sqrt{\tau}} = \sqrt{s} \frac{d}{dM}$$

$$\text{rapidity } y = \frac{1}{2} \log \frac{x_1}{x_2} \rightarrow \frac{d}{dy} = 2x_1 \frac{d}{dx_F}$$

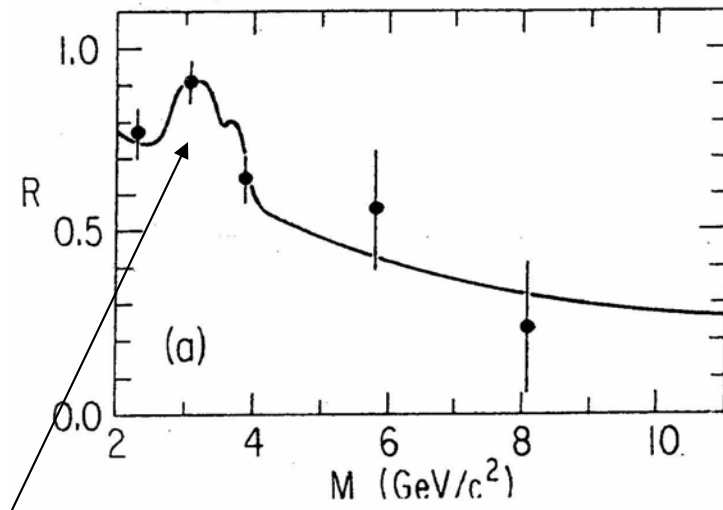
$$M^3 \frac{d\sigma}{dM dx_F} = \sqrt{x_1 x_2} \frac{x_2}{2} s \frac{d\sigma}{d\sqrt{\tau} dy}$$



$$s \frac{d\sigma}{d\sqrt{\tau} dy} \sim \text{const} - \sqrt{\tau} \propto \text{const} - \frac{M}{\sqrt{s}}$$

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b) rapporto di carica di DY su nuclei isoscalari



$M \sim J/\psi \rightarrow$ meccanismo differente

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nuclei isoscalari $\rightarrow n_u = n_d$ (ex. ^{12}C)

$$1 \quad \xleftarrow{\tau \rightarrow 0} \frac{\pi^+ {}^{12}\text{C} \rightarrow \mu^+ \mu^- X}{\pi^- {}^{12}\text{C} \rightarrow \mu^+ \mu^- X} \xrightarrow{\tau \rightarrow 1} \frac{1}{4}$$

perche'?

$\tau = x_1 x_2 \rightarrow 1$ valence area

$$\frac{\pi^+ (u\bar{d}) C(u_1 \dots u_n d_1 \dots d_m)}{\pi^- (d\bar{u}) C(u_1 \dots u_n d_1 \dots d_m)}$$

$$\sim \frac{e_d^2}{e_u^2} = \frac{1}{4}$$

$\tau = x_1 x_2 \rightarrow 0$ sea area

$$\frac{\pi^+ C(\text{..sea quarks..})}{\pi^- C(\text{..sea quarks..})}$$

~ 1

meccanismo elementare

$$q\bar{q} \rightarrow l\bar{l}$$

c) rapporto DY tra π e N

N non ha antiquark di valenza
per annichilazione

$$\rightarrow \frac{\pi^- N \rightarrow \mu^+ \mu^- X}{NN \rightarrow \mu^+ \mu^- X}$$

cresce con $M = Q$ e $\sqrt{\tau}$

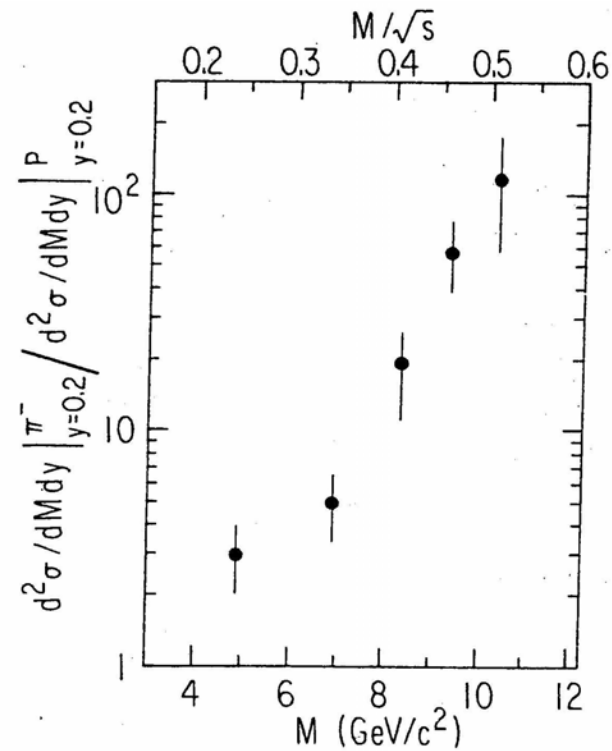


FIG. 1. The ratio of π^- -induced to proton-induced μ -pair cross section at $y_{c.m.} = 0.2$ as a function of mass. Proton data at 225 GeV/c has been calculated from the scaling observed in 200-, 300-, and 400-GeV/c data of Ref. 4.

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QPM picture : discrepanze

$$\gamma^* \rightarrow \mu^+ \mu^- X$$

al crescere di $Q^2 \equiv M^2$ si eccitano
altre risonanze mesoniche vettoriali :

$$M \sim 3 \text{ GeV} \quad J/\psi \rightarrow \mu^+ \mu^- X$$

$$M \sim 9 \div 10.5 \text{ GeV} \quad Y \rightarrow \mu^+ \mu^- X$$

....

$$M \gtrsim 70 \text{ GeV} \quad Z^0, W^\pm$$



meccanismo elementare
diverso da QPM picture (QED)

$$\text{Inoltre} \quad \frac{d\sigma(\text{exp.})}{d\sigma(\text{th.})} \sim K \sim 2 \div 3 !$$

K factor = misura delle
correzioni pQCD
oltre il QPM

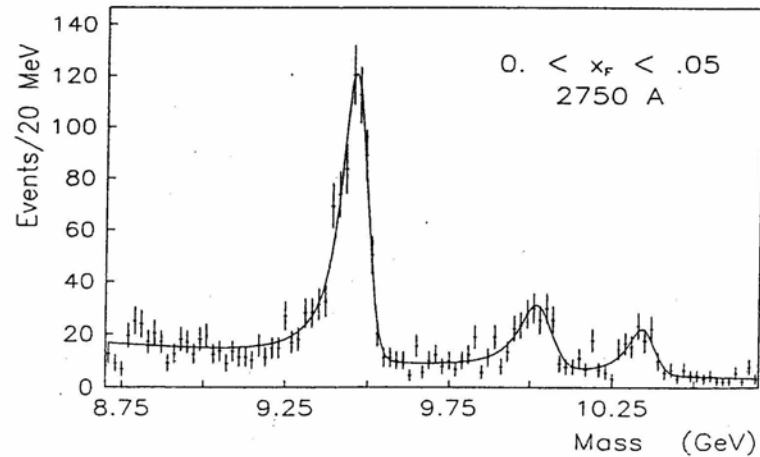
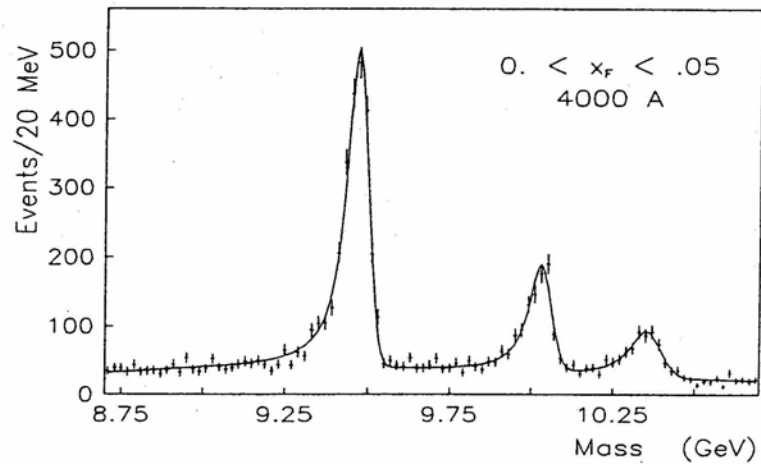


FIG. 16. The fit function used to extract Υ cross sections superimposed on the raw mass spectrum of the two data sets.

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11-Nov-09

spettro della famiglia Υ ($b\bar{b}$)

differente distribuzione in \mathbf{q}_T della coppia di leptoni \rightarrow nuovo meccanismo ?

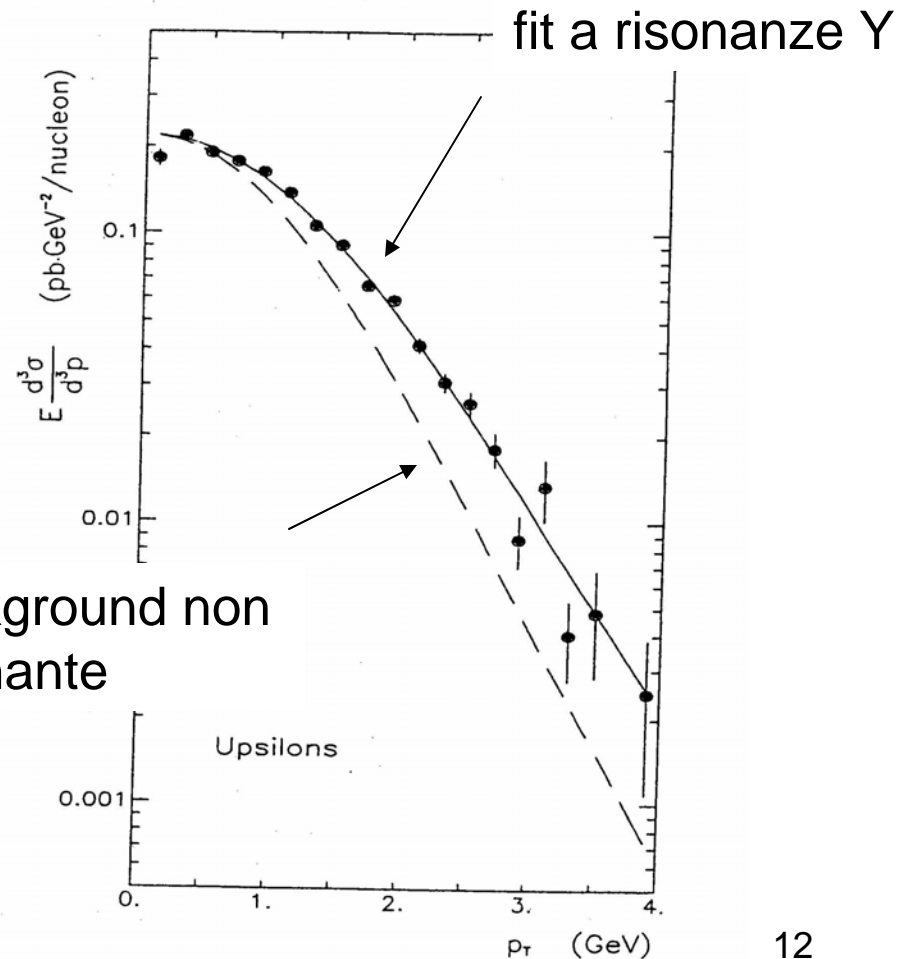
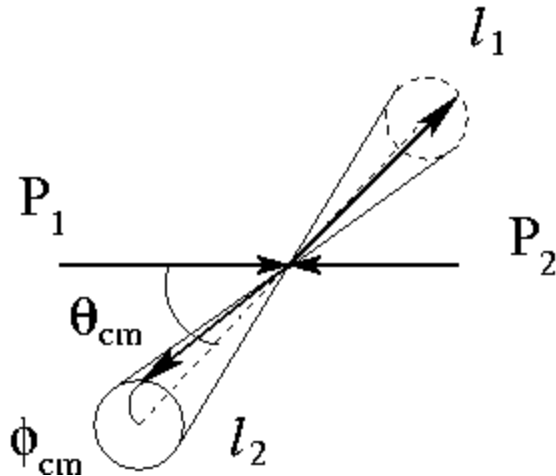


FIG. 19. p_T distribution for the sum of the three Υ 's. The solid curve is a fit to the data, the dashed curve gives the

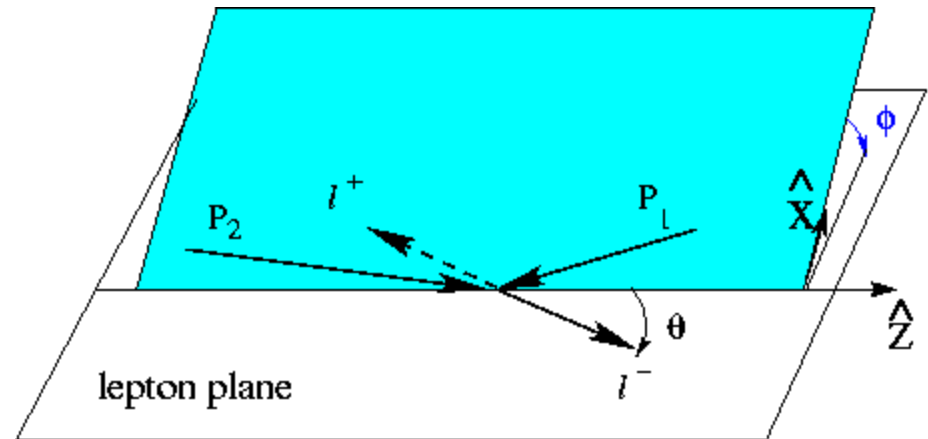
12

Distribuzione angolare della coppia leptonica

c.m. degli adroni



c.m. dei leptoni (Collins-Soper frame)



se $q_T (l_1 l_2) \neq 0 \rightarrow$ direzione di annichilazione non nota
Collins-Soper frame: asse z = direzione "media"

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{1}{N_c} \bar{\sum}_f \int_0^1 dx_1 dx_2 \phi_f(x_1) \frac{d\sigma^{el}(e^+e^- \rightarrow \mu^+\mu^-)}{d\Omega} e_f^2 \phi_{\bar{f}}(x_2) \delta(x_1 x_2 s - Q^2) \\ &= \frac{\alpha^2}{12Q^4} \underline{(1 + \cos^2 \theta)} \bar{\sum}_f e_f^2 \int_0^1 dx_1 dx_2 \phi_f(x_1) \phi_{\bar{f}}(x_2) \delta\left(x_1 x_2 \frac{s}{Q^2} - 1\right) \end{aligned}$$

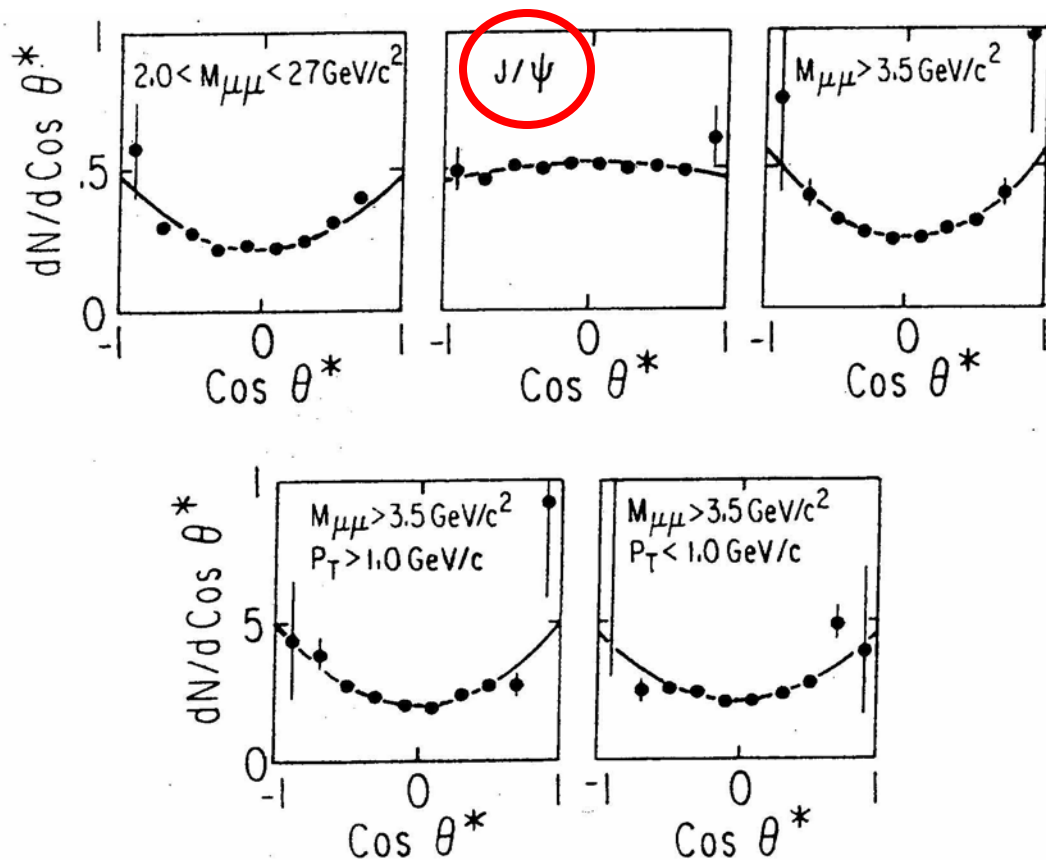


FIG. 3. Helicity angular distributions in three different mass intervals. The $M > 3.5 \text{ GeV}/c^2$ interval is also shown divided in two p_T intervals. The Collins-Soper angle (θ^*) is defined in the text.

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distribuzione angolare
 $\sim (1 + \cos^2 \theta)$
 data dal processo elementare
 $e^+e^- \rightarrow \mu^+\mu^-$

però sulla **risonanza J/ψ**
 distribuzione piatta
 \rightarrow meccanismo diverso

da $\gamma^* \rightarrow \mu^+\mu^-$

$J/\psi \rightarrow \mu^+\mu^-$?

Distribuzione angolare e discrepanze da QPM

$$\begin{aligned} \frac{d\sigma}{d^4q d\Omega} &= \frac{\alpha^2}{2sQ^4} L_{\mu\nu} W^{\mu\nu} \rightarrow \frac{d\sigma}{dq^0 d\mathbf{q} d\Omega} = \dots \rightarrow \frac{d\sigma}{dx_1 dx_2 d\mathbf{q}_T d\Omega} = \dots \\ &\rightarrow \frac{d\sigma}{dx_F d\tau d\mathbf{q}_T d\Omega} = \dots \rightarrow \frac{d\sigma}{dx_F dM d\mathbf{q}_T d\Omega} = \dots \end{aligned}$$

- $W^{\mu\nu}$:
- 3 vettori indipendenti P_1, P_2, q
 - conservazione parità, no polarizzazione \rightarrow struttura simmetrica
 - base tensoriale: $b_1 = g^{\mu\nu}$, $b_2 = q^\mu q^\nu$, $b_3 = P_1^\mu P_1^\nu$,
 $b_4 = (P_1^\mu q^\nu + P_1^\nu q^\mu)$, $b_5 = (P_2^\mu q^\nu + P_2^\nu q^\mu)$,
 $b_6 = (P_1^\mu P_2^\nu + P_1^\nu P_2^\mu)$, $b_7 = P_2^\mu P_2^\nu$
 - tensore adronico $W^{\mu\nu} = \sum_i c_i (q^2, P_1 \cdot q, P_2 \cdot q, P_1 \cdot P_2) b_i$
 - conservazione della corrente $q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu = 0$
 \rightarrow **3 relazioni**
 - gauge-invarianza QED \rightarrow termini lineari in $q^\mu = 0$

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{P_1^\mu P_1^\nu}{M^2} + \frac{W_3}{2M^2} (P_1^\mu P_2^\nu + P_1^\nu P_2^\mu) + W_4 \frac{P_2^\mu P_2^\nu}{M^2}$$

Distribuzione angolare (continua)

$$\frac{d\sigma}{dx_F d\mathbf{q}_T dM d\Omega} \propto \left[\underset{\substack{\uparrow \\ \text{trasversa}}}{W_T} (1 + \cos^2 \theta_{cm}) + \underset{\substack{\uparrow \\ \text{longitudinale}}}{W_L} \sin^2 \theta_{cm} \right]$$

polarizz. del γ^*

trasversa

longitudinale

elementi non diagonali
della matrice densita`
di elicit` della coppia $H_1 H_2$

$$+ \underset{\substack{\uparrow \\ \text{1 spin flip}}}{W_{\uparrow}} \sin 2\theta_{cm} \cos \phi_{cm} + \underset{\substack{\uparrow\uparrow \\ \text{2 spin flip}}}{W_{\uparrow\uparrow}} \sin^2 \theta_{cm} \cos 2\phi_{cm}]$$

$$\left[\frac{d\sigma}{dx_F dM d\mathbf{q}_T} \right]^{-1} \frac{d\sigma}{dx_F dM d\mathbf{q}_T d \cos \theta_{cm}} \propto 1 + \alpha \cos^2 \theta_{cm} , \quad \alpha = \frac{W_T - W_L}{W_T + W_L}$$



—————
 ✱
on-shell (anti)quark
con spin 1/2

$\alpha = 1 \Rightarrow W_L = 0$

$\alpha \neq 1 \Rightarrow \left\{ \begin{array}{l} \text{frame differente con } q_T \neq 0 \\ \text{meccanismo differente } \rightarrow q_T \neq 0 \end{array} \right.$

(Collis-Soper frame $\rightarrow \alpha = 0.85$)

$$p p \rightarrow \mu^+ \mu^- X$$

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$M > 4 \text{ GeV}$ (no J/ψ)

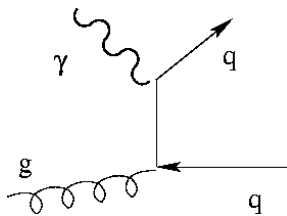
$$d\sigma \sim 1 + \alpha \cos^2 \theta_{cm}$$

$$\alpha = \frac{W_T - W_L}{W_T + W_L}$$

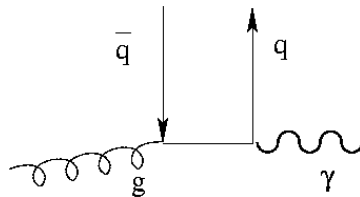
$\alpha \sim 1 \rightarrow W_T$ dominante

Ma sulla **risonanza J/ψ** meccanismo puo` essere diverso.

DY appartiene a classe piu` generale di processi $A+B \rightarrow C+X$ dove meccanismo elementare puo` essere piu` complicato:



QCD Compton



$\gamma - g$ fusion

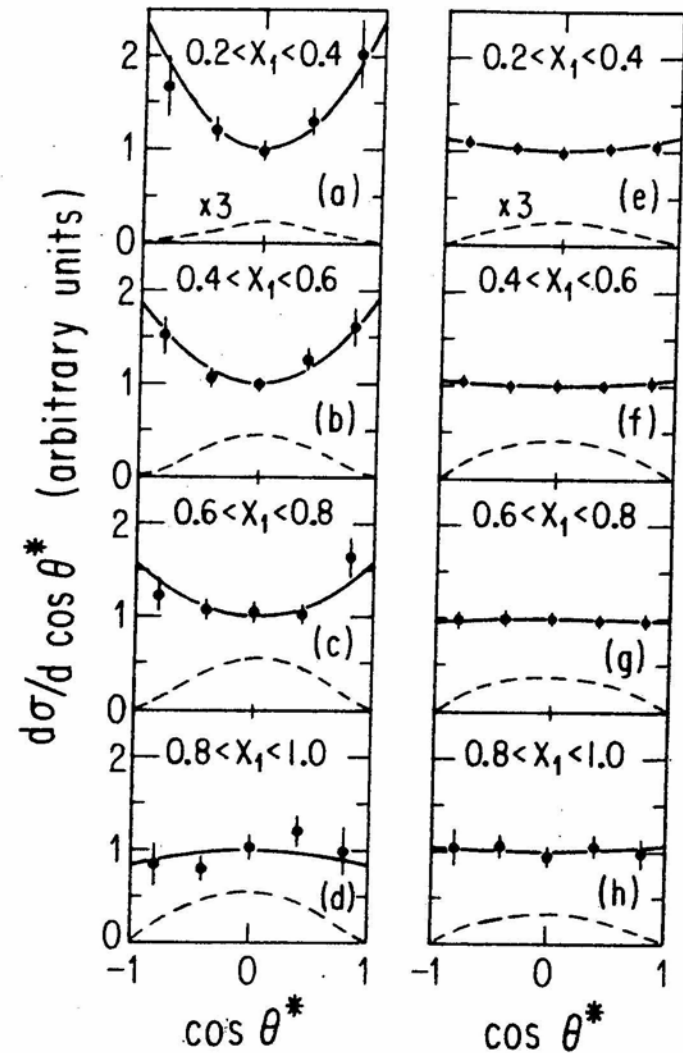


FIG. 1. $d\sigma/d \cos \theta^*$ in the t -channel helicity frame for various x_1 intervals. (a)–(d) Results for the mass continuum with $M > 4 \text{ GeV}$; (e)–(h) results for the J/ψ resonance in the same x_1 intervals. Data are integrated over P_T . The dashed curve shows the variation of detection efficiency with $\cos \theta^*$. The same arbitrary efficiency

correzioni radiative pQCD \rightarrow dipendenza q_T
 q e' off-shell $\rightarrow W_L \neq 0 \rightarrow \alpha(q_T) < 1$

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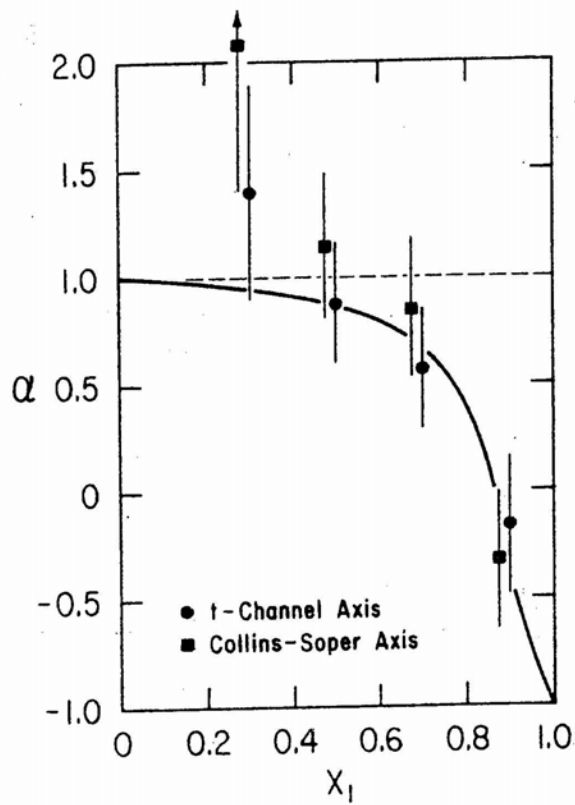


FIG. 2. The dependence of α on x_1 for data with $M > 4$ GeV. The dashed line is the expected result for the naive Drell-Yan model. The solid curve is the QCD prediction of Berger and Brodsky (Ref. 8).

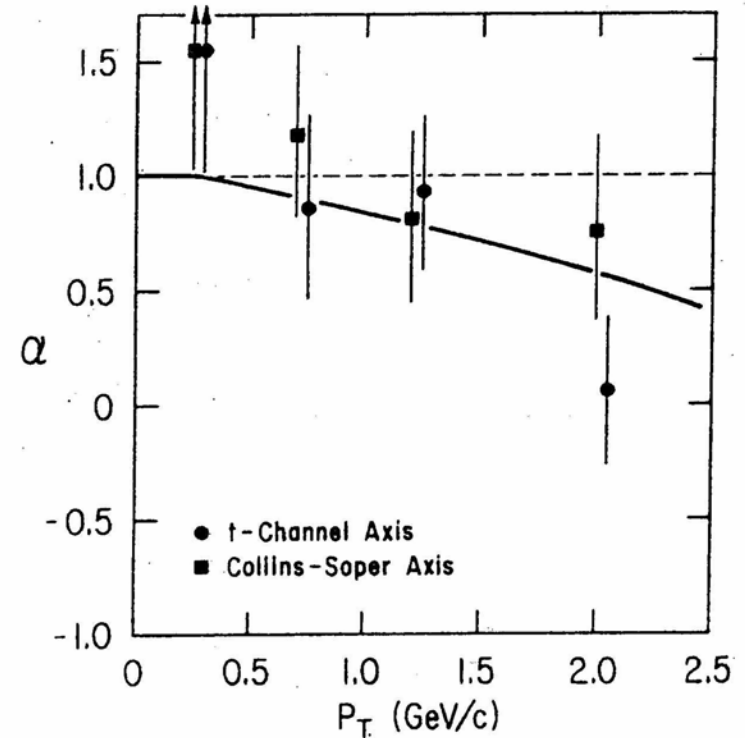


FIG. 3. The dependence of α on P_T for data with $M > 4$ GeV. The smooth curve is the QCD prediction from Kajantie *et al.* (Ref. 7). These authors consider only $x = 0$ while the data are integrated over x_F .