## Slide 4 - 1

$$
\begin{aligned}
S_F(\xi) &= (i\,\partial\!\!\!/ + m)\,\Delta(\xi) \\
&= (i\,\partial\!\!\!/ + m)\left\{\frac{1}{4\pi^2 i}\lim_{\epsilon\to 0}\frac{1}{\xi^2 - i\epsilon} + \dots\right\} \\
&= \lim_{\epsilon\to 0}\frac{1}{4\pi^2 i}\left[i\gamma\cdot\frac{(-2\xi)}{(\xi^2 - i\epsilon)^2} + m\,\frac{1}{\xi^2 - i\epsilon} + \dots\right] \\
&= \frac{1}{4\pi^2}\lim_{\epsilon\to 0}\frac{-2\,\xi\!\!\!/}{(\xi^2 - i\epsilon)^2} + \frac{1}{4\pi^2 i}\lim_{\epsilon\to 0}\frac{m}{\xi^2 - i\epsilon}\dots \qquad (1)
\end{aligned}
$$

## Slide 4 - 2

$$
\begin{aligned}
\mathrm{Tr}\,[S_F(-\xi)\,\gamma^\mu\,S_F(\xi)\,\gamma^\nu] &= \lim_{\epsilon\to 0}\frac{1}{16\pi^4}\,\frac{4}{(\xi^2 - i\epsilon)^4}\,\mathrm{Tr}\,[-\,\xi\!\!\!/\,\gamma^\mu\,\xi\!\!\!/\,\gamma^\nu] \\
&= -\lim_{\epsilon\to 0}\frac{1}{4\pi^4\,(\xi^2 - i\epsilon)^4}\,\xi_\alpha\,\xi_\beta\,\mathrm{Tr}\left[\gamma^\alpha\,\gamma^\mu\,\gamma^\beta\,\gamma^\nu\right] \\
&= -\lim_{\epsilon\to 0}\frac{1}{\pi^4\,(\xi^2 - i\epsilon)^4}\,(2\xi^\mu\xi^\nu - \xi^2\,g^{\mu\nu})\,. \qquad (2)
\end{aligned}
$$

## Slide 5

La definizione di $\gamma_5$ è

$$\gamma_5 = -\frac{i}{4!}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^\mu\,\gamma^\nu\,\gamma^\rho\,\gamma^\sigma\;. \qquad (3)$$

Ne segue che

$$\gamma_5\gamma_\sigma = -\frac{i}{3!}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^\mu\,\gamma^\nu\,\gamma^\rho\;. \qquad (4)$$

La dimostrazione è molto semplice: basta moltiplicare per $\gamma^\sigma$ sia a sinistra che a destra dell'uguaglianza.

Similmente si dimostra che

$$
\begin{aligned}
\gamma_5\gamma_\rho\gamma_\sigma &= \frac{1}{2}\,\epsilon_{\mu\nu\rho\sigma}\,\sigma^{\mu\nu} + \gamma_5\,g_{\rho\sigma}\;, \\
\gamma_5\gamma_\nu\gamma_\rho\gamma_\sigma &= i\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^\mu + \gamma_5\gamma_\nu\,g_{\rho\sigma} - \gamma_5\gamma_\rho\,g_{\nu\sigma} + \gamma_5\gamma_\sigma\,g_{\rho\nu}\;.
\end{aligned}
$$

Basta moltiplicare la prima a destra e sinistra per $\gamma^\rho\gamma^\sigma$, e la seconda a destra e sinistra per $\gamma^\nu\gamma^\rho\gamma^\sigma$.

Dall'ultima equazione si ricava

$$\gamma_\nu\gamma_\rho\gamma_\sigma = \gamma_\nu\,g_{\rho\sigma} - \gamma_\rho\,g_{\nu\sigma} + \gamma_\sigma\,g_{\rho\nu} + i\gamma_5\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^\mu\;, \qquad (5)$$

semplicemente moltiplicando a destra e sinistra per $\gamma_5$.