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$$
\begin{aligned}
&\int d^4x\, e^{iq\cdot x}\,\epsilon(x^0)\,\langle 0|\frac{i}{3\pi^3}\,(2x^\mu x^\nu - x^2 g^{\mu\nu})\,\partial^3(x^2)|0\rangle \\
&= \frac{i}{3\pi^3}\left(g^{\mu\nu}\frac{\partial}{\partial q}\cdot\frac{\partial}{\partial q} - 2\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial q_\nu}\right)\int d^4x\, e^{iq\cdot x}\,\epsilon(x^0)\,\partial^3(x^2) \\
&= \frac{i}{3\pi^3}\left(g^{\mu\nu}\frac{\partial}{\partial q}\cdot\frac{\partial}{\partial q} - 2\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial q_\nu}\right)\frac{i\pi^2}{(2!)\,4}\,(q^2)^2\,\epsilon(q^0)\,\theta(q^2)\;. \qquad (1)
\end{aligned}
$$

Ora abbiamo

$$
\begin{aligned}
\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial q_\nu}\left(q_\alpha g^{\alpha\beta} q_\beta\right)^2 &= \frac{\partial}{\partial q_\mu}\,2\left(q_\alpha g^{\alpha\beta} q_\beta\right)\,2q^\nu \\
&= 2\,2q^\mu\,2q^\nu + 2q^2\,2g^{\mu\nu} \\
&= 8q^\mu q^\nu + 4q^2 g^{\mu\nu} \\
\frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu} &= 12q^2\;, \qquad (2)
\end{aligned}
$$

perché il gradiente covariante spaziale non cambia segno rispetto a quello controvariante, quindi $g^{\mu\nu} \leftrightarrow \delta^{\mu\nu}$.

Inoltre l'operatore derivativo dà risultato nullo su $\epsilon(q^0)$ e, nel senso delle distribuzioni, abbiamo

$$
\begin{aligned}
(q^2)^2\epsilon(q^0)\,\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial q_\nu}\,\theta(q^2) &= (q^2)^2\epsilon(q^0)\,g^{\mu\nu}\,\delta(q^2) \\
&= g^{\mu\nu}\,\epsilon(q^0)\,(q^2)^2|_{q^2=0} = 0\;, \qquad (3)
\end{aligned}
$$

con $\epsilon(q^0)$ finito nel limite $q^2 \to 0$.

Inserendo le Eqs. (2) e (3) nella (1) otteniamo

$$
\begin{aligned}
&\int d^4x\, e^{iq\cdot x}\,\epsilon(x^0)\,\langle 0|\frac{i}{3\pi^3}\,(2x^\mu x^\nu - x^2 g^{\mu\nu})\,\partial^3(x^2)|0\rangle \\
&= \frac{i}{3\pi^3}\,(12q^2 g^{\mu\nu} - 8q^2 g^{\mu\nu} - 16q^\mu q^\nu)\,\frac{i\pi^2}{8}\,\epsilon(q^0)\,\theta(q^2) \\
&= \frac{\epsilon(q^0)\,\theta(q^2)}{6\pi}\,(4q^\mu q^\nu - q^2 g^{\mu\nu})\;.
\end{aligned}
$$

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$$
\begin{aligned}
&\frac{1}{6\pi}\,L_{\mu\nu}\,(4q^\mu q^\nu - q^2 g^{\mu\nu}) \\
&= \frac{1}{6\pi}\,2(k_\mu k'_\nu + k'_\mu k_\nu - k\cdot k' g_{\mu\nu})\,(4q^\mu q^\nu - q^2 g^{\mu\nu})
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{3\pi}\left(8k\cdot q\, k'\cdot q - 2q^2 k\cdot k' - 4q^2 k\cdot k' + 4q^2 k\cdot k'\right)\\
&= (q = k + k')\\
&\simeq \frac{1}{3\pi}\left[8(k\cdot k')^2 - 4(k\cdot k')^2\right] = \frac{4}{3\pi}\,(k\cdot k')^2 = \frac{(2k\cdot k')^2}{3\pi} \simeq \frac{[(k+k')^2]^2}{3\pi} = \frac{(q^2)^2}{3\pi} = \frac{s^2}{3\pi}\ .
\end{aligned}
$$