Lez. 17 15-12-2014

Slide 5

① $2M W^{\mu\nu} = \frac{1}{2\pi}\int d^4\xi \sum_f e_f^2\, e^{iq\cdot\xi} \langle P|[J_f^\mu(\xi), J_f^\nu(0)]|P\rangle$

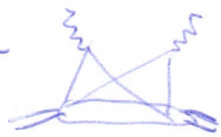
$J_f^\mu(\xi) = \bar\psi_f(\xi)\gamma^\mu\psi_f(\xi)$

corrente di quark libero con flavor f

OPE

$$[J^\mu(\xi), J^\nu(0)] = \mathrm{Tr}[\gamma^\mu, \gamma^\nu]\, \psi(\xi)\bar\psi(0)\,\psi(0)\bar\psi(\xi)$$
$$+ :\bar\psi(\xi)\gamma^\mu\psi(\xi)\bar\psi(0)\gamma^\nu\psi(0):$$
$$+ :\bar\psi(0)\gamma^\nu\psi(0)\bar\psi(\xi)\gamma^\mu\psi(\xi):$$
$$+ :\bar\psi(\xi)\gamma^\mu\psi(\xi)\,\bar\psi(0)\gamma^\nu\psi(0):$$

1° termine escluso perché corrisponde a diagramma  sconnesso

4° termine escluso perché corrisponde a 

Rimangono 2° e 3° termine dove

$\psi(\xi)\bar\psi(0) = S_F(\xi)$ propagatore di particella (con $\xi^0>0$)

$\psi(0)\bar\psi(\xi) = S_F(-\xi)$ propagatore di particella a $\xi^0<0$

$\Longleftrightarrow$ propagatore di antiparticella a $\xi^0>0$

cioè

$$:\bar\psi(0)\gamma^\nu\psi(0)\bar\psi(\xi)\gamma^\mu\psi(\xi): \Longleftrightarrow :\psi(\xi)\gamma^\nu\bar\psi(\xi)\psi(0)\gamma^\mu\bar\psi(0):$$

(1) continua

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \, e^{iq\cdot\xi} \Big\{ \langle P| \bar\psi_f(\xi)\gamma^\mu \overbrace{\psi_f(\xi)\bar\psi_f(0)}\gamma^\nu\psi_f(0)|P\rangle \; +$$

$$\langle P|\psi_f(\xi)\gamma^\nu \overbrace{\bar\psi_f(\xi)\,\psi_f(0)}\gamma^\mu\bar\psi_f(0)|P\rangle \Big\}$$

inseriamo completezza $|0\rangle\langle 0|$

$$\int \frac{d\vec{P}_x}{(2\pi)^3 2P_x^0}\,|P_x\rangle\langle P_x| \int\frac{d\vec{k}}{(2\pi)^3 2k^0}\,|k\rangle\langle k|$$

cioè stato finale visto come prodotto tensoriale di stato di 1 quark libero con vettore d'onda k e del resto $(\equiv X)$

E' giustificato da operatore di corrente J^μ per quark libero

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k \; e^{iq\cdot\xi}$$

$$\times\Big\{ \langle P|\bar\psi_f(\xi)|0\rangle\langle 0|\gamma^\mu \overbrace{\psi_f(\xi)|P_x\rangle|k\rangle\langle k|\langle P_x|\bar\psi_f(0)}\gamma^\nu\psi_f(0)|P\rangle \; +$$

$$\langle P|\psi_f(\xi)|0\rangle\langle 0|\gamma^\nu \overbrace{\bar\psi_f(\xi)|P_x\rangle|k\rangle\langle k|\langle P_x|\psi_f(0)}\gamma^\mu\bar\psi_f(0)|P\rangle \Big\}$$

trasliamo i campi in ξ al punto 0 in modo da avere solo elementi di matrice nello stesso punto spazio-temporale

N.B. Non c'è più bisogno della contrazione, perché così i campi sono già ordinati temporalmente

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k \; e^{iq\cdot\xi}$$

$$\times\Big\{ \langle P|e^{i\hat{P}\cdot\xi}\bar\psi_f(0)e^{-i\hat{P}\cdot\xi}|0\rangle\langle 0|\gamma^\mu e^{i\hat{P}\cdot\xi}\psi_f(0)e^{-i\hat{P}\cdot\xi}|P_x\rangle|k\rangle\langle k|\langle P_x|\bar\psi_f(0)\gamma^\nu\psi_f(0)|P\rangle$$

$$+ \langle P|e^{i\hat{P}\cdot\xi}\psi_f(0)e^{-i\hat{P}\cdot\xi}|0\rangle\langle 0|\gamma^\nu e^{i\hat{P}\cdot\xi}\bar\psi_f(0)e^{-i\hat{P}\cdot\xi}|P_x\rangle|k\rangle\langle k|\langle P_x|\psi_f(0)\gamma^\mu\bar\psi_f(0)|P\rangle \Big\}$$

$$= \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k \; e^{i(P+q-P_x-k)\cdot\xi}$$

$$\times\Big\{ \langle P|\bar\psi_f(0)|0\rangle\langle 0|\gamma^\mu\psi_f(0)|P_x\rangle|k\rangle\langle k|\langle P_x|\bar\psi_f(0)\gamma^\nu\psi_f(0)|P\rangle \; +$$

$$\langle P|\psi_f(0)|0\rangle\langle 0|\gamma^\nu\bar\psi_f(0)|P_x\rangle|k\rangle\langle k|\langle P_x|\psi_f(0)\gamma^\mu\bar\psi_f(0)|P\rangle \Big\}$$

① continua

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k \; e^{i(q+P-P_x-k)\cdot\xi}$$

$$\times\Big\{ \langle P|\bar\psi_f(0)\,\gamma^\mu\,\psi_f(0)|k\rangle\; |P_x\rangle\langle P_x|\; \langle k|\bar\psi_f(0)\,\gamma^\nu\,\psi_f(0)|P\rangle \; +$$

$$\langle P|\psi_f(0)\,\gamma^\nu\,\bar\psi_f(0)|k\rangle\; |P_x\rangle\langle P_x|\; \langle k|\psi_f(0)\,\gamma^\mu\,\bar\psi_f(0)|P\rangle \Big\}$$

$\psi_f(0)|k\rangle \equiv u_k$ campo di particella libera con flavor f e vettore d'onda k

quindi $\psi_f(0)|k\rangle\langle k|\bar\psi_f(0) \; = \; u_k\,\bar u_k \; = \; (\not k + m)\,\delta(k^2-m^2)\,\theta(k^0-m)$

cioè propagatore di particella libera con vettore d'onda k (se ha massa m)

particella $\to \theta(k^0-m)$ cioè propagazione ad energie positive

libera $\to$ on shell $\to \delta(k^2-m^2)$

idem per $\bar\psi_f(0)|k\rangle\langle k|\psi_f(0) \equiv v_k\,\bar v_k$ antiparticella

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k \; e^{i(q+P-P_x-k)\cdot\xi}\; \delta(k^2-m^2)\,\theta(k^0-m)$$

$$\times\Big\{ \langle P|\bar\psi_f(0)\,\gamma^\mu(\not k+m)\,|P_x\rangle\langle P_x|\,\gamma^\nu\psi_f(0)\,|P\rangle \; +$$

$$\langle P|\psi_f(0)\,\gamma^\nu(\not k-m)\,|P_x\rangle\langle P_x|\,\gamma^\mu\bar\psi_f(0)\,|P\rangle \Big\}$$

$$= \sum_f e_f^2 \int d^4P_x \int d^4k\; \delta(k^2-m^2)\,\theta(k^0-m)\;\; \delta(q+P-P_x-k) \;\times\Big\{ \;-\;-\;-\;-\;-\; \Big\}$$

$$= \sum_f e_f^2 \int d^4P_x \int d^4k\; \delta(k^2-m^2)\,\theta(k^0-m) \int d^4p \int \frac{d^4x}{(2\pi)^4}\; e^{i(P-P_x-p)\cdot x}\; \delta(p+q-k)\times\{\ldots\}$$

$$= \sum_f e_f^2 \int d^4P_x \int d^4p\; \delta((p+q)^2-m^2)\,\theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4}\; e^{-ip\cdot x}\;\; e^{i(P-P_x)\cdot x}$$

$$\times\Big\{ \langle P|\bar\psi_f(0)\,\gamma^\mu(\not p+\not q+m)\,|P_x\rangle\langle P_x|\,\gamma^\nu\psi_f(0)\,|P\rangle \; +$$

$$\langle P|\psi_f(0)\,\gamma^\nu(\not p+\not q-m)\,|P_x\rangle\langle P_x|\,\gamma^\mu\bar\psi_f(0)\,|P\rangle \Big\}$$

$$= \sum_f e_f^2 \int d^4P_x \int d^4p\; \delta((p+q)^2-m^2)\;\theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4}\; e^{-ip\cdot x}\Big\{ \langle P|\bar\psi_f(x)\,\gamma^\mu(\not p+\not q+m)|P_x\rangle\langle P_x|\gamma^\nu\psi_f(0)|P\rangle$$

$$+ \langle P|\psi_f(x)\,\gamma^\nu(\not p+\not q-m)|P_x\rangle\langle P_x|\gamma^\mu\bar\psi_f(0)|P\rangle\Big\}$$

$$= \sum_f e_f^2 \int d^4p\; \delta((p+q)^2-m^2)\,\theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4}\, e^{+ip\cdot x}\Big\{ \langle P|\bar\psi_f(x)\gamma^\mu(\not p+\not q+m)\,\gamma^\nu\psi_f(0)\,|P\rangle \; +$$

$$\langle P|\psi_f(x)\,\gamma^\nu(\not p+\not q-m)\,\gamma^\mu\bar\psi_f(0)\,|P\rangle \Big\}$$

① continua

$$2\pi W^{\mu\nu} = \sum_f e_f^2 \int d^4p\; \delta\left((p+q)^2-m^2\right)\theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4}\, e^{-ip\cdot x}$$

$$\times \Big\{ \langle P|\bar\psi_f(x)\,\gamma^\mu\,(\not p+\not q+m)\,\gamma^\nu\,\psi_f(0)|P\rangle +$$

$$\langle P|\psi_f(x)\,\gamma^\nu\,(\not p+\not q-m)\,\gamma^\mu\,\bar\psi_f(0)|P\rangle \Big\}$$

si mettano in evidenza gli indici di Dirac per ciascun operatore nell'elemento di matrice adronico

$$2\pi W^{\mu\nu} = \sum_f e_f^2 \int d^4p\; \delta\left((p+q)^2-m^2\right)\theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4}\, e^{-ip\cdot x}$$

$$\times \Big\{ \langle P|\bar\psi_{f_j}(x)\,(\gamma^\mu)_{il}\,(\not p+\not q+m)_{lm}\,(\gamma^\nu)_{mj}\,\psi_{f_i}(0)|P\rangle +$$

$$\langle P|\psi_{f_j}(x)\,(\gamma^\nu)_{il}\,(\not p+\not q-m)_{lm}\,(\gamma^\mu)_{mj}\,\bar\psi_{f_i}(0)|P\rangle \Big\}$$

$$= \sum_f e_f^2 \int d^4p\; \delta\left((p+q)^2-m^2\right)\theta(p^0+\nu-m)\, \mathrm{Tr}\Big[(\Phi_f)_{ij}(p,P)\,(\gamma^\mu)_{il}\,(\not p+\not q+m)_{lm}\,(\gamma^\nu)_{mj} +$$

$$(\bar\Phi_f)_{ij}(p,P)\,(\gamma^\nu)_{il}\,(\not p+\not q-m)_{lm}\,(\gamma^\mu)_{mj}\Big]$$

con $$(\Phi_f)_{ij}(p,P) = \int \frac{d^4x}{(2\pi)^4}\, e^{-ip\cdot x}\, \langle P|\bar\psi_{f_j}(x)\,\psi_{f_i}(0)|P\rangle$$

$$(\bar\Phi_f)_{ij}(p,P) = \int \frac{d^4x}{(2\pi)^4}\, e^{-ip\cdot x}\, \langle P|\psi_{f_j}(x)\,\bar\psi_{f_i}(0)|P\rangle$$

in termini diagrammatici