“Strong Interactions”

Underlying field theory is a renormalizable non-abelian gauge theory named

Quantum Chromo-Dynamics (QCD)

QCD Lagrangian:

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}(x)\,[i\gamma^\mu D_\mu - m]\,\psi(x) - \frac{1}{4}\left(F^i_{\mu\nu}\right)^2$$

with ψ Dirac field and A^μ vector field and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$$

a,b,c color indices, others understood

$$D_\mu \equiv \partial_\mu - ig\, A^a_\mu t^a$$

covariant derivative (makes Dirac-vector field interaction locally gauge-invariant)

$$[t^a, t^b] = i\, f^{abc}\, t^c$$

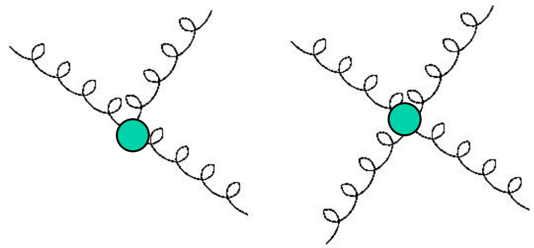
non-abelian theory
t = generators of gauge transformations
f = fine structure constants

QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}(x)\,[i\gamma^{\mu}\,D_{\mu} - m]\,\psi(x) - \frac{1}{4}\left(F^{i}_{\mu\nu}\right)^{2}$$

$$F^{a}_{\mu\nu} = \partial_{\mu}\,A^{a}_{\nu} - \partial_{\nu}\,A^{a}_{\mu} + g\,f^{abc}\,A^{b}_{\mu}\,A^{c}_{\nu}$$

⇒ trilinear and quadrilinear couplings



Consequence? "Maxwell" eqs. for vector fields

$$\frac{\partial\mathcal{L}_{\mathrm{QCD}}}{\partial A^{a}_{\mu}} = \partial_{\nu}\,\frac{\partial\mathcal{L}_{\mathrm{QCD}}}{\partial\,\partial_{\nu}\,A^{a}_{\mu}} \;\Rightarrow\; \partial^{\mu}\,F^{a}_{\mu\nu} + g\,f^{abc}\,A^{\mu\,b}\,F^{c}_{\mu\nu} = -g\,\bar{\psi}\,\gamma_{\nu}\,t^{a}\,\psi$$

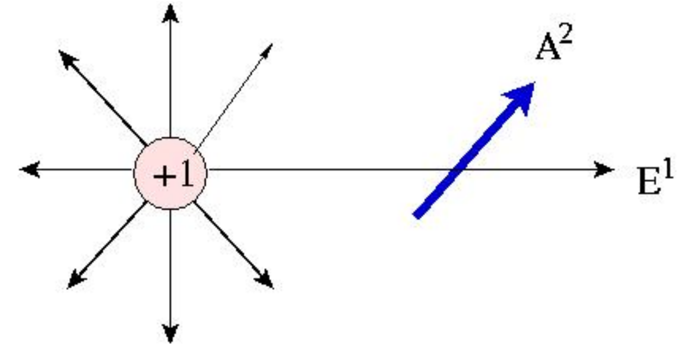
for $\nu = 0$ "Gauss" law for color charge *a* distributed with density ρ^{a} and generating color electric field $E^{a}_{i} = F^{a}_{0i}$

$$\partial_{i}\,E^{a}_{i} = g\,\rho^{a} + g\,f^{abc}\,A^{b}_{i}\,E^{c}_{i}$$

density from pointlike color charge a=1

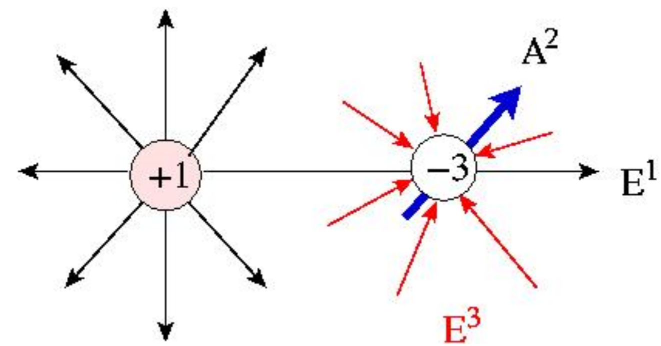
$$\partial_i E_i^1 = g\,\delta(\vec{x})\delta_{a1} \qquad A_i^2$$

vacuum fluctuation



$$\partial_i E_i^3 = g\, f^{321}\, A_i^2\, E_i^1$$

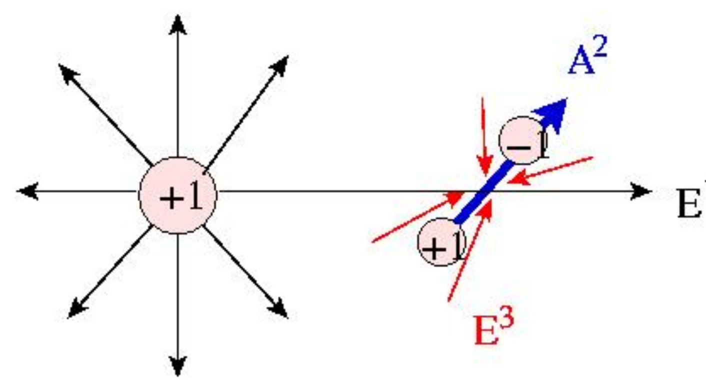
"sink" of field E^3



$$\partial_i E_i^1 = g\,\delta(\vec{x})\,\delta_{a1} + g\, f^{123}\, A_i^2\, E_i^3$$

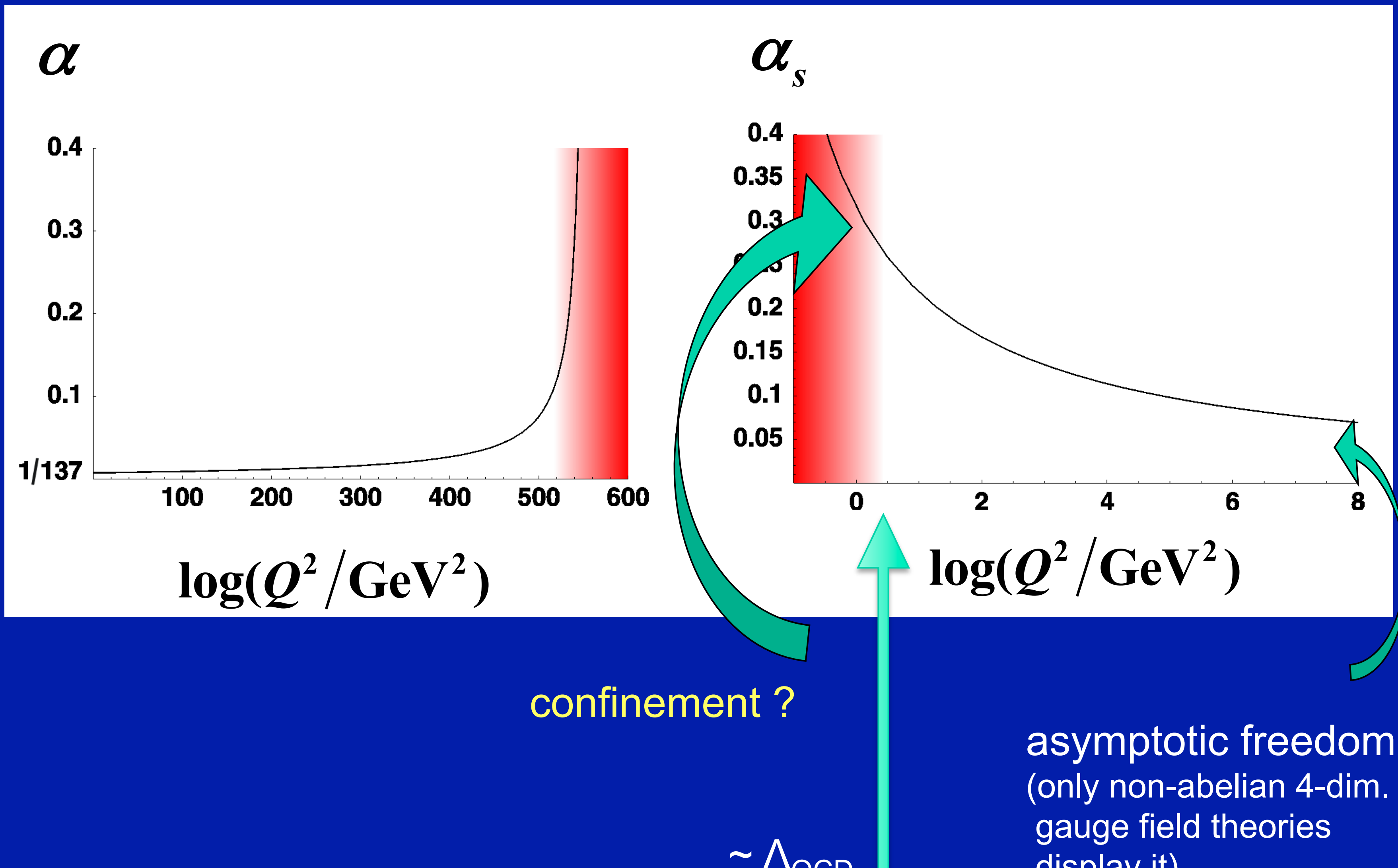
$$\left.\begin{array}{ll} > 0 & \vec{A}^2 \parallel \vec{E}^3 \\ < 0 & \vec{A}^2 \parallel^{-1} \vec{E}^3 \end{array}\right\}$$

dipole charge a=1
pointing to source



➔ getting away from source the color charge a=1 looks stronger! antiscreening

QED : screening QCD : screening + antiscreening (≫)



confinement ?

~ Λ_{QCD}

asymptotic freedom
(only non-abelian 4-dim. gauge field theories display it)

Regimes

$\Lambda_{QCD} \ll$: perturbative regime, calculable with techniques mutuated from QED

$\Lambda_{QCD} \lesssim$: non-perturbative regime, not directly calculable

Structure of hadrons realized at scale ~ Λ_{QCD}

hadrons cannot be deduced directly from the Lagrangian describing the forces that make them !

Alternative: compute QCD on lattice
statistical approach, no direct access to dynamics

Hadronic Physics : study hadronic systems using effective approaches induced by QCD

- ✦ spectroscopy
- ✦ dynamical structure ☜

Historical origin

End of '60's: famous SLAC experiment of Deep Inelastic Scattering (DIS) on proton targets at $7 \lesssim Q^2 \lesssim 10$ $(GeV/c)^2$ and $6^o < \theta_e < 10^o$

- ☛ scaling = the target response does no longer depend on momentum transferred
- ☛ isolated events of diffusion at very large angles
- ☛ the proton behaves like an ensemble of pointlike scattering centers, each one moving independently from the others
- ☛ birth of the Quark Parton Model (QPM)

Bloom *et al.,* P.R.L. **23** (69) 930
Feynman, P.R.L. **23** (69) 1415
Friedman, Kendall,Taylor 1990 NOBEL laureates

Scattering lepton -- hadron

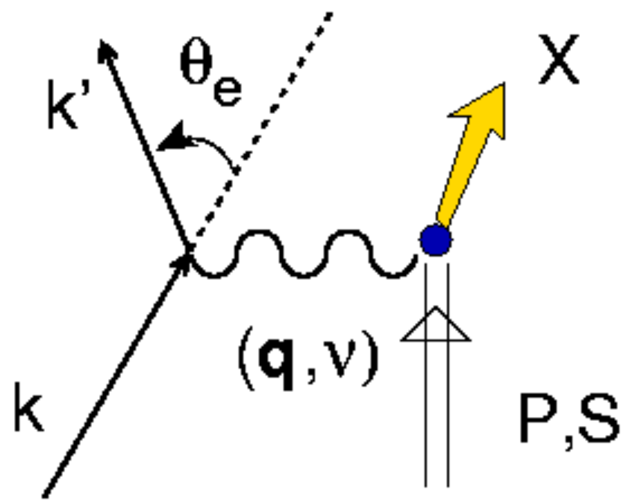
(electron, neutrino, muon) (nucleon, nucleus, photon)

- Quantum ElectroDynamics (QED) known at any order
- leptonic probe explores the whole target volume

$$\alpha = \frac{e^2}{4\pi\hbar c} \sim \frac{1}{137}$$

- α_{em} ~ fine structure constant is small → perturbative expansion
- Born approximation (exchange of one photon only) works well
- virtual photon (γ^*): ($\mathbf{q}$,ν) independent, two different γ^* polarizations (longitudinal and transverse) → two different target responses

prototype reaction
e+p -> e' +X



3 independent 4-vectors
k, *k', P*
+ spin *S*
θ_e scattering angle

definitions and kinematics

e⁻ ultrarelativistic $m_e \ll |\mathbf{k}|, |\mathbf{k}'|$
Target Rest Frame (TRF)

$$
\begin{aligned}
P &= (M, \mathbf{0}) \\
k &= (\sqrt{m_e^2 + |\mathbf{k}|^2}, 0, 0, |\mathbf{k}|) \\
&\sim (E, 0, 0, E) \\
k' &= (\sqrt{m_e^2 + |\mathbf{k}'|^2}, |\mathbf{k}'| \sin\theta_e, 0, |\mathbf{k}'| \cos\theta_e) \\
&\sim (E', E' \sin\theta_e, 0, E' \cos\theta_e) \\
q &= k - k' = (E - E', \mathbf{k} - \mathbf{k}')
\end{aligned}
$$

kinematical invariants

$$
\begin{aligned}
P^2 &= M^2 \quad ; \quad S^2 = 1 \quad ; \quad P \cdot S = 0 \\
k^2 &= k'^2 = m_e^2 \sim 0 \\
q^2 &\sim -2EE'(1 - \cos\theta_e) = -4EE' \sin^2 \frac{\theta_e}{2} \leq 0 \\
&\Longrightarrow Q^2 \equiv -q^2 = 4EE' \sin^2 \frac{\theta_e}{2} \geq 0
\end{aligned}
$$

(cont'ed)

$$\nu = \frac{P \cdot q}{M} \stackrel{\text{TRF}}{=} \frac{M(E-E')}{M} = E - E' \quad \text{energia trasferita}$$

$$y = \frac{P \cdot q}{P \cdot k} \stackrel{\text{TRF}}{=} \frac{M(E-E')}{ME} = \frac{E-E'}{E} \quad \text{frazione di energia trasferita}$$

$$0 \leq y \leq 1$$

$$x_B = \frac{Q^2}{2P \cdot q} \stackrel{\text{TRF}}{=} \frac{Q^2}{2M\nu} \qquad 0 \leq x_B \leq 1$$

elastic limit ($x_B = 1$)

anelastic limit ($x_B = 0$)

final invariant mass

$$W = (P+q)^2 = M^2 + Q^2\left(\frac{1}{x_B} - 1\right) \geq M^2$$

Q is our "lense"

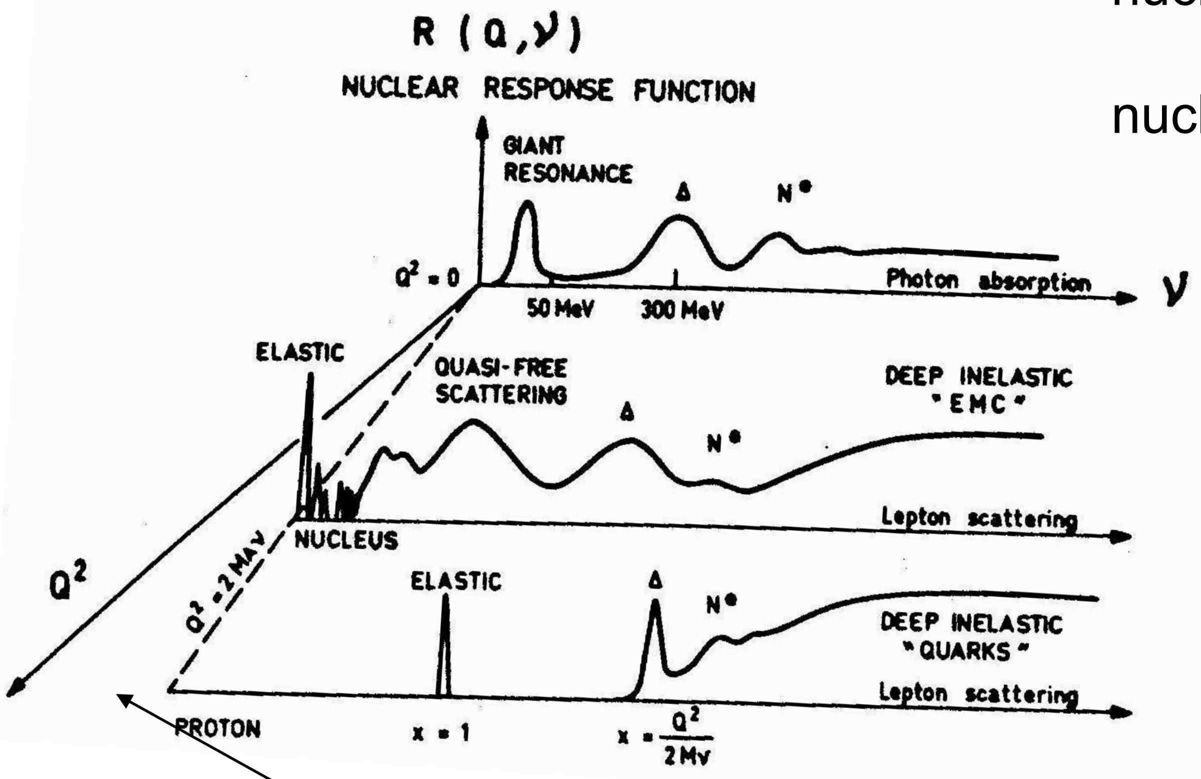
Q [GeV]	$\lambda \sim 1/Q$ [fm]	target
0.02	10	nuclei
0.1	2	
0.2	1	mesons / baryons
1	0.2	partons
……	……	??

N.B. 1 fm = $(200\ \text{MeV})^{-1}$

Frois, Nucl. Phys. **A434** (’ 85) 57c

nucleus M_A $\quad x_A = \dfrac{Q^2}{2M_A\nu}$

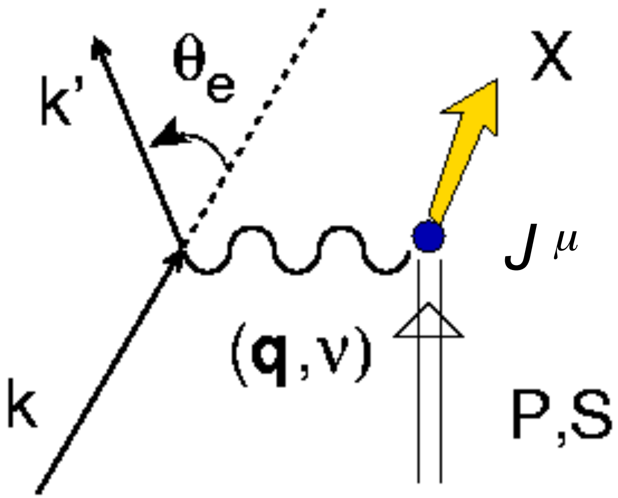
nucleon M $\quad x_B = \dfrac{Q^2}{2M\nu}$



unaccessible domain $\quad \nu \leq \dfrac{Q^2}{2M_A}$

Cross Section

$$\frac{\text{n}^{\text{o}} \text{ events per unit time, scattering center, solid angle}}{\text{n}^{\text{o}} \text{ incident particles per unit time, area}}$$



$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 \, dR$$

flux

$$\mathcal{F} = 4\sqrt{(P \cdot k)^2 - P^2 k^2} \stackrel{\text{TRF}}{=} 4ME$$

phase space

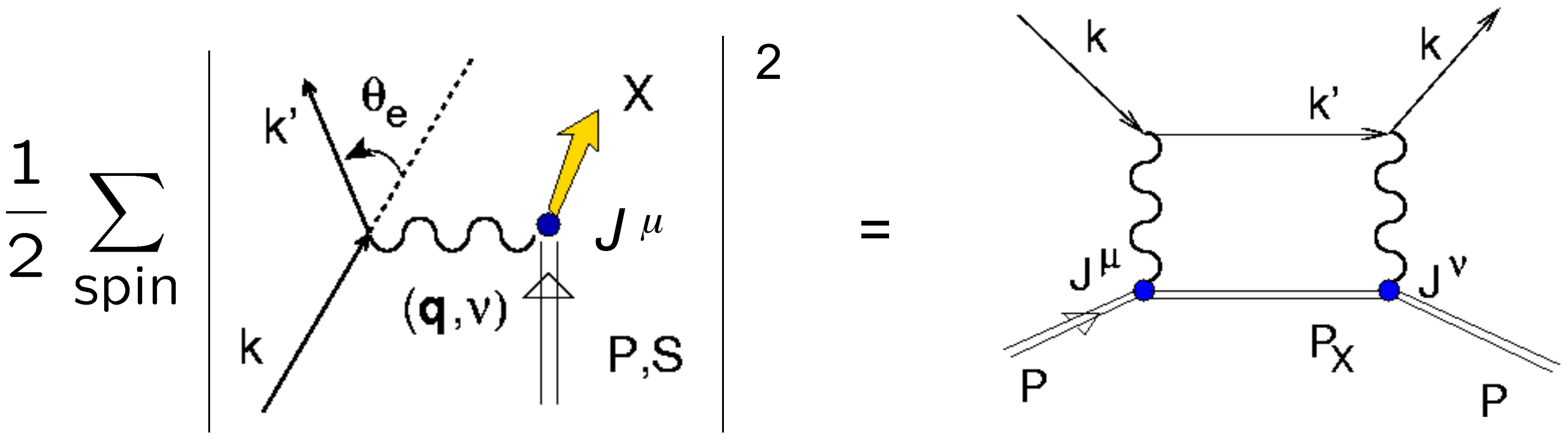
$$dR = (2\pi)^4 \delta(P + q - P_X) \, \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \, \frac{d\mathbf{k}'}{(2\pi)^3 2E'}$$

$$\frac{E' dE' d\Omega}{16\pi^3}$$

scattering amplitude

$$\mathcal{M} = \bar{u}(k')\gamma_\mu u(k) \, \frac{e^2}{Q^2} \, \langle P_X | J^\mu(0) | P, S \rangle$$

Leptonic and Hadronic Tensors

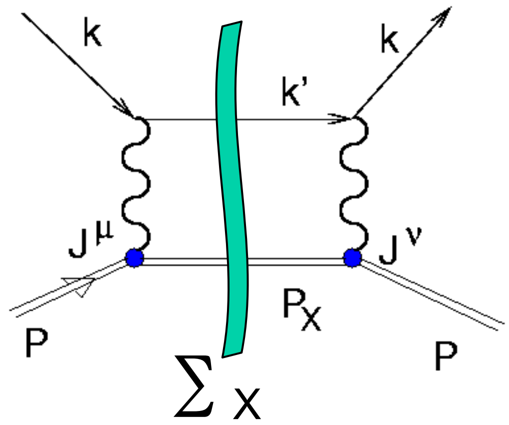


$$\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu}$$

leptonic tensor $L^{\mu\nu} = 2k^{\mu}k'^{\nu} + 2k^{\nu}k'^{\mu} - Q^2 g^{\mu\nu}$

hadronic tensor $H^{\mu\nu} = \langle P|J^{\mu}(0)|P_X\rangle\langle P_X|J^{\nu}(0)|P\rangle$

Inclusive Scattering



hadronic tensor

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_X \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta(P+q-P_X)\,H^{\mu\nu}$$

cross section for inclusive scattering (general formula)

$$\frac{d\sigma}{dE'd\Omega} = \frac{E'}{E}\,\frac{\alpha^2}{Q^4}\,L_{\mu\nu}\,W^{\mu\nu}$$

$$\alpha = \frac{e^2}{4\pi}$$

$$Q^2 = 4EE'\sin^2\frac{\theta_e}{2}$$ large angles are suppressed !

Inclusive Elastic Scattering

$$\sum_X \ldots|P_X\rangle\langle P_X|\ldots = \ldots|P'\rangle\langle P'|\ldots$$

$W'=(P+q)^2=M^2$

hadronic tensor

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_X \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta(P+q-P_X)\,H^{\mu\nu}$$

$$= \delta\left((P+q)^2-M^2\right)\,H^{\mu\nu} = \delta(2P\cdot q-Q^2)\,H^{\mu\nu}$$

$$\overset{\text{TRF}}{=} \delta(2M\nu-Q^2)\,H^{\mu\nu} = \frac{1}{2M}\delta\left(\nu-\frac{Q^2}{2M}\right)\,H^{\mu\nu}$$

ν ↔ Q : scaling

$$\frac{d\sigma}{dE'd\Omega} = \frac{E'}{E}\frac{\alpha^2}{4M^2Q^4}L_{\mu\nu}\,H^{\mu\nu}\,\delta\left(\nu-\frac{Q^2}{2M}\right)$$

$$\int dE'\,\frac{d\sigma}{dE'\,d\Omega} = \frac{d\sigma}{d\Omega} = \left(\frac{E'}{E}\right)^2\frac{\alpha^2}{4M^2Q^4}L_{\mu\nu}\,H^{\mu\nu}$$

various cases

target = free scalar particle

$$H^{\mu\nu} = \langle P|J^\mu|P'\rangle\langle P'|J^\nu|P\rangle$$

2 independent 4-vectors: $R=P+P'$, $q=P-P'$ ⇒ $J^\mu \approx F_1 R^\mu + F_2 q^\mu$

$F_{1,2}(q^2, P^2, P'^2) = F_{1,2}(q^2)$

current conservation $q_\mu J^\mu = 0$ ⇒ $F_2(q^2) = -\frac{R\cdot q}{q^2} F_1(q^2)$

define : $\tilde{P}^\mu = R^\mu - \frac{R\cdot q}{q^2} q^\mu \longrightarrow J^\mu = \tilde{P}^\mu F_1(Q^2)$

N.B. for on-shell particles $q\bullet R = 0$; but in general for off-shell $q\cdot\tilde{P} = 0$

$$L_{\mu\nu}H^{\mu\nu} = \left(2k\cdot\tilde{P}k'\cdot\tilde{P} - \tilde{P}^2 k\cdot k'\right) 2\,|F_1(Q^2)|^2$$

$$\overset{\text{TRF}}{\sim} 16EE'M^2\,|F_1(Q^2)|^2 \cos^2\frac{\theta_e}{2}$$

Inclusive elastic scattering on free scalar target

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2}{Q^4} E'^2 \cos^2\frac{\theta_e}{2} \frac{E'}{E} |F_1(Q^2)|^2 \equiv \sigma_{\mathrm{Mott}} \frac{E'}{E} |F_1(Q^2)|^2$$

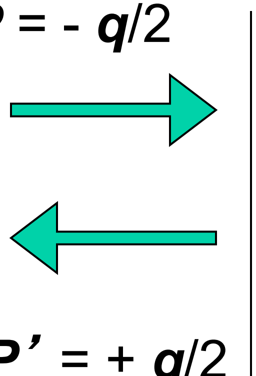
elastic Coulomb scattering on pointlike target

target recoil

target structure

Breit frame → target form factor

$\boldsymbol{P} = -\,\boldsymbol{q}/2$



$\boldsymbol{P}' = +\,\boldsymbol{q}/2$

$\nu = 0$

$R^\mu = (2E, \mathbf{0})$
$q^\mu = (0, \mathbf{q})$

$J^\mu = (J^0, \mathbf{0}) \approx (2E\, F_1(Q^2), \mathbf{0})$

$$F_1(Q^2) \to F_1(|\mathbf{q}|^2) = \int d\mathbf{r}\ \rho(\mathbf{r})\ e^{i\,\mathbf{q}\cdot\mathbf{r}}$$

form factor for
charge
matter
…..

distribution of
charge
matter
…..

works only in the
non-relativistic limit

in fact, ρ is a static density, while Breit frame changes with $Q^2=\mathbf{q}^2$:
boost makes $|\,\mathbf{P}'=+\mathbf{q}/2 > \neq |\,\mathbf{P}=-\mathbf{q}/2 >$ ⇒ density interpretation is lost

target = pointlike free Dirac particle

Example: $e^- + \mu^- \to e^{-\prime} + \mu^-$

$$J^\mu = \bar{u}(P')\,\gamma^\mu\,u(P)$$

$$H^{\mu\nu} \equiv L^{\mu\nu} \quad \text{with } k^{(\prime)} \leftrightarrow P^{(\prime)}$$

$$\frac{1}{2}\sum_{\text{spin}} H^{\mu\nu} = \frac{1}{2}\text{Tr}\left[(\not{P}' + M)\,\gamma^\mu\,(\not{P} + M)\gamma^\nu\right]$$

$$= 2\left[P'^\mu P^\nu + P'^\nu P^\mu - (P\cdot P' - M^2)g^{\mu\nu}\right]$$

$$L_{\mu\nu}H^{\mu\nu} \overset{\text{TRF}}{=} 16EE'M^2\cos^2\frac{\theta_e}{2}\left(1 + \frac{Q^2}{2M^2}\tan^2\frac{\theta_e}{2}\right)$$

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}}\,\frac{E'}{E}\left(1 + \frac{Q^2}{2M^2}\tan^2\frac{\theta_e}{2}\right)$$

spin magnetic interaction with γ^*

target = free Dirac particle with structure

3 independent 4-vectors P^μ, P'^μ, γ^μ (+ parity and time-reversal invariance)

$$J^\mu = \bar{u}(P')\Gamma^\mu u(P) = \bar{u}(P')\left[\Gamma_1 P^\mu + \Gamma_2 P'^\mu + \Gamma_3 \gamma^\mu\right] u(P)$$

$$\Gamma_i(Q^2, P^2, P'^2) \equiv \Gamma_i(Q^2)$$

current conservation $q_\mu J^\mu = 0$

$$\begin{aligned}
& q_\mu \bar{u}(P')\Gamma^\mu u(P) = \\
= \;& \bar{u}(P')\left[\Gamma_1 P\cdot(P'-P) + \Gamma_2 P'\cdot(P'-P) + \Gamma_3 \gamma\cdot(P'-P)\right] u(P) \\
= \;& \bar{u}(P')\left[(P\cdot P' - M^2)(\Gamma_1 - \Gamma_2) + \Gamma_3(\not{P}' - \not{P})\right] u(P) = 0 \\
\Rightarrow \;& \Gamma_1 = \Gamma_2 \\
J^\mu = \;& \bar{u}(P')\left[\Gamma_1(Q^2) R^\mu + \Gamma_3(Q^2)\gamma^\mu\right] u(P)
\end{aligned}$$

Dirac eq.

$$\not{P} u = M u$$

$$\bar{u}\,\not{P} = \bar{u} M$$

Gordon Decomposition (on-shell)

$$J^\mu = \bar{u}(P') \left[\Gamma_1(Q^2) R^\mu + \Gamma_3(Q^2)\gamma^\mu \right] u(P)$$

$$\bar{u}\gamma^\mu u = \bar{u} \left[\frac{R^\mu}{2M} + \frac{i}{2M}\sigma^{\mu\nu} q_\nu \right] u \qquad \text{with } \sigma^{\mu\nu} = \tfrac{i}{2}[\gamma^\mu, \gamma^\nu]$$

namely $R^\mu \Leftrightarrow 2M\,\gamma^\mu - i\,\sigma^{\mu\nu}\,q_\nu$

proof flow-chart

- from right handside, insert def. of $\sigma^{\mu\nu}$
- use Dirac eq.
- use $\{\gamma^\mu,\gamma^\nu\} = 2\,g^{\mu\nu}$
- use Dirac eq. → left handside

$$J^\mu = \bar{u}(P') \left[F_1(Q^2)\gamma^\mu + \frac{i}{2M}\sigma^{\mu\nu} q_\nu\, F_2(Q^2) \right] u(P)$$

F_1	$=$	$2M\Gamma_1 + \Gamma_3$	Dirac form factor	$F_1(0) = 1$
F_2	$=$	$-2M\Gamma_1$	Pauli form factor	$F_2(0) = \kappa$

target = free Dirac particle with structure

$$\frac{1}{2}\sum_{\text{spin}} H^{\mu\nu} = \frac{1}{2}\text{Tr}\left[(\not{p}' + M)\Gamma^{\mu}(\not{p} + M)\Gamma^{\nu}\right]$$

$$\Gamma^{\mu} = F_1\gamma^{\mu} + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}F_2$$

$\ldots\ldots$

cross section

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}}\left[\left(F_1^2 + \frac{Q^2}{4M^2}F_2^2\right) + \frac{Q^2}{2M^2}(F_1 + F_2)^2\tan^2\frac{\theta_e}{2}\right]\delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}}\frac{E'}{E}\left[\left(F_1^2 + \frac{Q^2}{4M^2}F_2^2\right) + \frac{Q^2}{2M^2}(F_1 + F_2)^2\tan^2\frac{\theta_e}{2}\right]$$

internal structure
(not easy to extract)

Rosenbluth formula

Define Sachs form factors

(Yennie, 1957)

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

with $\tau = \frac{Q^2}{4M^2}$

N.B. reason: in Breit frame + non-rel. reduction $\rightarrow$

$$J^0 \sim G_E$$
$$\mathbf{J} \sim \frac{G_M}{2M} \sigma \times \mathbf{q}$$

charge / magnetic target distribution

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau\, G_M^2 \, \tan^2 \frac{\theta_e}{2} \right]$$
$$\equiv \sigma_{\text{Mott}} \frac{E'}{E} \left[A(Q^2) + B(Q^2) \, \tan^2 \frac{\theta_e}{2} \right]$$

easier to handle

Rosenbluth separation

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E} \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau\, G_M^2 \, \tan^2 \frac{\theta_e}{2} \right]$$

- at large θ_e (large Q^2) $\rightarrow$ extract G_M
- small θ_e (small Q^2) $\rightarrow$ extract G_E by difference
- Rosenbluth plot

$$\epsilon\,(1+\tau)\,\frac{E}{E'}\,\frac{1}{\sigma_{\text{Mott}}}\,\frac{d\sigma}{d\Omega} = \epsilon G_E^2 + \tau G_M^2$$

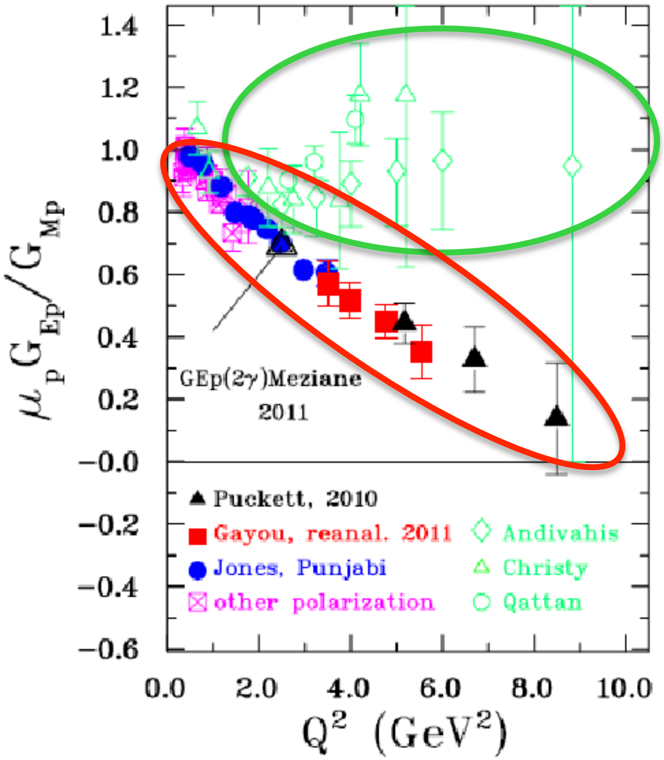
$$\epsilon = \left[1 + 2(1+\tau) \tan^2 \frac{\theta_e}{2} \right]^{-1}$$

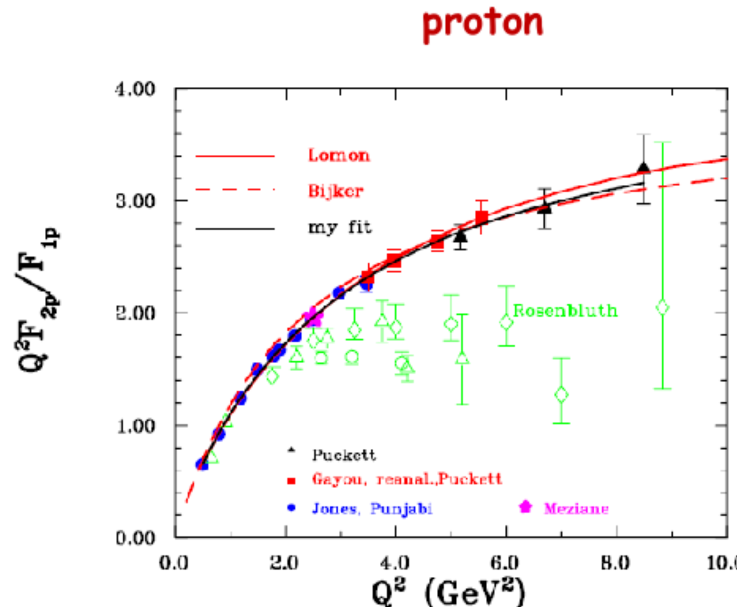
linear transverse polarization of γ^*

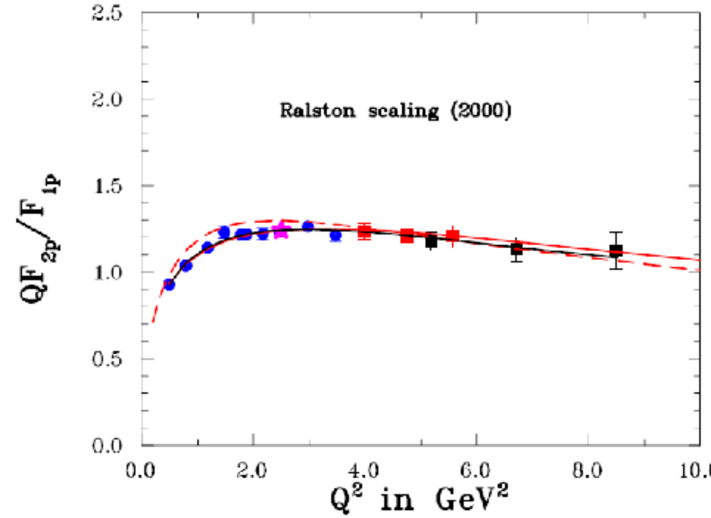
measurements at different (E, θ_e) $\rightarrow$ plot in ε at fixed Q^2

crossing at ε=0 $\rightarrow$ G_M

slope in ε $\rightarrow$ G_E







Rosenbluth plot

$$\mu_p \frac{G_E^p}{G_M^p} \to \text{cost} \quad \Rightarrow \quad F_2 \sim \frac{F_1}{Q^2}$$

pQCD scaling

JLAB data

(obtained with more precise $\vec{e}$- N → e- $\vec{N}$)

$$\Rightarrow \quad F_2 \sim \frac{F_1}{Q}$$

$Q^2 \sim 10\ (\text{GeV}/c)^2$ not yet perturbative regime??