# Spin-dependent Fragmentation Functions

Rainer Jakob, University of Wuppertal

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outline of the talk: (first version)

- systematic scheme for fragmentation functions (10 min)
- model calculations (10 min)
- model-independent bounds (10 min)
- processes involving fragmentation functions (10 min)
- presentation of a database on FF ( 5 min )

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- model calculations (20 min)
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- systematic scheme for fragmentation functions (20 min)
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- model-independent bounds (10 min)
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different aspects learned from works by Jaffe/Ji, Soper/Ralston, Mulders/Levelt/Tangerman/Boer/. . .

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*normal* fragmentation function

$D^{q\to h}(z)$ or $D_1(z)$ or $\hat{f}_1(z)$

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"Collins" functions

$H_1^{\perp(1)}$ or $\Delta\hat{D}_{H/i}$

Collins

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Ralston/Soper, Jaffe/Ji

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Ralston/Soper, Jaffe/Ji

two-hadron interference FF

$H_1^{\triangleleft(1)}(z)$ or $\delta q_I(z)$ and company

Jaffe/Jin/Tang, Bianconi/Boffi/R.J./Radici

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**??**

polarised

fragmentation function

$\Delta D_f^h(z)$ with

$h=\pi, K, \ldots$ unpol.

Glück/Reya

nce FF

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higher twist fragmentation functions
$\hat{e}(z)$ or $E(z)$ and company
$\hat{h}_L(z)$ ,$H_L(z)$, $\hat{g}_T(z)$ ,$G_T(z)$
Jaffe/Ji, Mulders/Tangerman/Boer/R.J.

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Glück/Reya

"transversity" fragmentation function

$H_1(z)$ or $\hat{h}_1(z)$

spin-1 hadron FF

$D_{1LL}, D_{1LT}, D_{1TT}, G_{1LT},$

$G_{1TT}, H_{1LL}^{\perp}, H_{1LT'}, H_{1LT}^{\perp},$

$H'_{1TT}, H_{1TT}^{\perp}$

Bacchetta/Mulders

nce FF

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Jaffe/Ji, Mulders/Tangerman/Boer/

$D^{q\to h}(z$

"Collins" functions

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Collins

??

arised

tion function

$(z)$ with

$, K, \ldots$ unpol.

Glück/Reya

...and some more

"transversity" fragmentation f

$H_1(z)$ or $\hat{h}_1(z)$

spin-1 hadron F

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nce FF

$H_1^{\triangleleft(1)}(z)$ or $\delta q_I(z)$ and company

Jaffe/Jin/Tang, Bianconi/Boffi/R.J./Radici

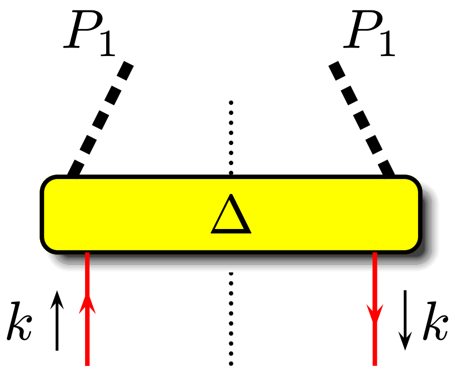
# don’t worry !

**the number of independent fragmentation functions is limited**

**(actually, to just a few at leading twist)**

**and there is a simple systematics behind**

definition of correlation function for fragmentation process $q \to h_1 \quad X$




Soper, Collins

$$\Delta_{ij}(k, P_1, \quad) = \sum_X \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi}\, \langle 0|\mathcal{U}(0,\xi)\psi_i(\xi)|P_1, \quad ; X\rangle\langle P_h, \quad ; X|\overline{\psi}_j(0)|0\rangle$$

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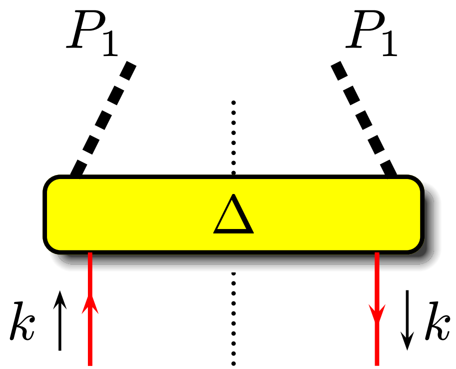


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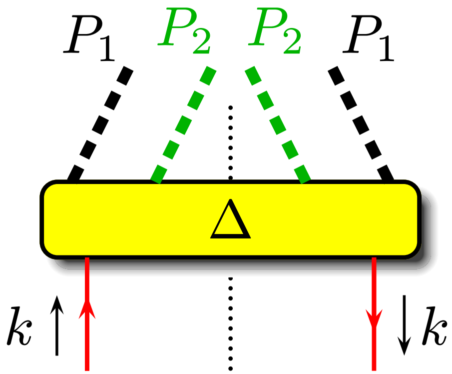
$\Delta$ is a $4 \times 4$ matrix in Dirac space, and depends on the momentum vectors:

$k$, $P_1$,

and possibly spin vectors $S$ (spin-1/2) , or spin vector and tensor $S$, $T$ (spin-1), etc.

definition of correlation function for fragmentation process $q \to h_1 h_2 X$




Soper, Collins

$$\Delta_{ij}(k, P_1, P_2) = \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle 0|\mathcal{U}(0,\xi)\psi_i(\xi)|P_1, P_2; X\rangle\langle P_h, P_2; X|\overline{\psi}_j(0)|0\rangle$$

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$\Longrightarrow$ construct most general ansatz — Soper/Ralston, Mulders/Tangerman

for instance for one spin- 0 hadron in the final state:

$$\Delta(k, P_h) = B_1 M_h + B_2 \not{P}_h + B_3 \not{k} + (B_4/M_h)\, \sigma_{\mu\nu} P_h^\mu k^\nu$$

with $B_i = B_i(P_h \cdot k, k^2)$

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only terms which are in accordance with constraints from:

**1.) hermiticity 2.) behavior under parity transformation**

note: $B_4$ is T-odd, i.e. it would be forbidden by a constraint from the behavior of Dirac fields under time-reversal, if there were no final state interactions

⟹ construct most general ansatz Soper/Ralston, Mulders/Tangerman

for instance for one spin-1/2 hadron in the final state:

$$\Delta(k, P_h, S) = B_1 M_h + B_2 \not{P}_h + B_3 \not{k} + (B_4/M_h)\, \sigma_{\mu\nu} P_h^\mu k^\nu$$
$$+ i\, B_5 (k\cdot S)\gamma_5 + B_6 M_h \not{S}\gamma_5 + (B_7/M_h)(k\cdot S)\, \not{P}_h \gamma 5$$
$$+ (B_8/M_h)(k\cdot S)\, \not{k}\gamma_5 + i\, B_9\, \sigma_{\mu\nu}\gamma_5 S^\mu P_h^\nu$$
$$+ i\, B_{10}\, \sigma_{\mu\nu}\gamma_5 S^\mu k^\nu + i\, (B_{11}/M_h^2)(k\cdot S)\sigma_{\mu\nu}\gamma_5 k^\mu P_h^\nu$$
$$+ (B_{12}/M_h)\, \epsilon_{\mu\nu\rho\sigma}\gamma^\mu P_h^\nu k^\rho S^\sigma$$

with $B_i = B_i(P_h\cdot k, k^2)$

only terms which are in accordance with constraints from:

**1.) hermiticity 2.) behavior under parity transformation**

note: $B_4$,$B_5$, and $B_{12}$ are T-odd, i.e. they would be forbidden by a constraint from the behavior of Dirac fields under time-reversal, if there were no final state interactions

fragmentation functions are obtained from the correlation function $\Delta$
by projection with Dirac matrices $\Gamma$,
and integration over components of the quark momentum

$$\Delta^{[\Gamma]}(z) \equiv \frac{1}{4z} \int dk^+ \int d^2\mathbf{k}_T \, Tr\,[\Delta\Gamma]\Big|_{k^-=P_h^-/z}$$

or

$$\Delta^{[\Gamma]}(z, -z\mathbf{k}_T) \equiv \frac{1}{4z} \int dk^+ \, Tr\,[\Delta\Gamma]\Big|_{k^-=P_h^-/z\ ;\mathbf{k}_T}$$

with the Dirac matrices $\Gamma \in \{\gamma^-, \ \gamma^-\gamma_5, \ i\sigma^{\alpha -}\gamma_5, \ldots,$ higher twist$\}$

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interpretation in the context of LC quantisation: $\Gamma$ determines quark spin states

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$[\gamma^-]$ :

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interpretation in the context of LC quantisation: $\Gamma$ determines quark spin states

$[\gamma^-]:$ 

$[\gamma^-\gamma_5]:$ 

fragmentation functions are obtained from the correlation function $\Delta$

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$$\Delta^{[\Gamma]}(z) \equiv \frac{1}{4z} \int dk^{+} \int d^2\mathbf{k}_T \; Tr\left[\Delta\Gamma\right]\Big|_{k^-=P_h^-/z\;;\mathbf{k}_T}$$

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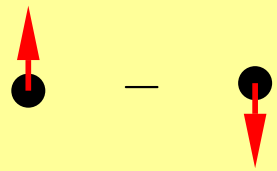
$[\gamma^-]:$ 

$[\gamma^-\gamma_5]:$ 

$[i\sigma^{\alpha -}\gamma_5]:$ 

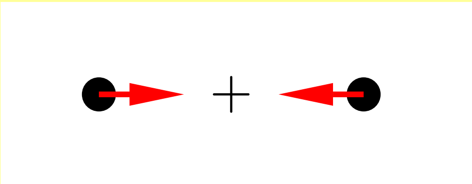
## interpretation

Kogut, Soper, Jaffe

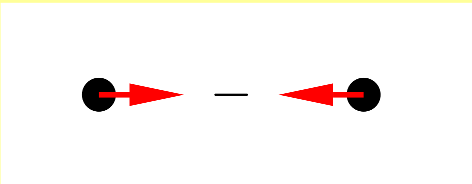
“good” components of Dirac field in LC quantisation

$$\psi_\pm = P_\pm\,\psi = \frac{1}{2}\gamma^\mp\gamma^\pm\,\psi$$

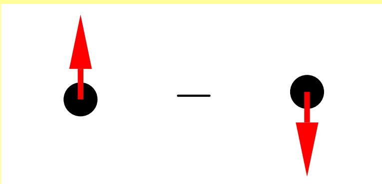
helicity / chiral projectors $P_{R/L} = (1 \pm \gamma_5)/2$



$$\begin{aligned}\overline{\psi}\,\gamma^+\,\psi &= \sqrt{2}\,\psi_+^\dagger\,(P_R P_R + P_L P_L)\;\psi_+ \\ &= \overline{R}R + \overline{L}L\end{aligned}$$



$$\begin{aligned}\overline{\psi}\,\gamma^+\gamma_5\,\psi &= \sqrt{2}\,\psi_+^\dagger\,(P_R P_R - P_L P_L)\,\psi_+ \\ &= \overline{R}R - \overline{L}L\end{aligned}$$



$$\begin{aligned}\overline{\psi}\,\mathrm{i}\sigma^{i+}\gamma_5\,\psi &= \sqrt{2}\,\psi_+^\dagger\;\left(P_L\gamma^i P_R - P_R\gamma^i P_L\right)\psi_+ \\ &= \overline{L}R - \overline{R}L\end{aligned}$$

**independent fragmentation functions at leading twist for**

# one unpolarised hadron

**observed in a quark-jet**

# FF for unpolarised hadron (without $k_T$)

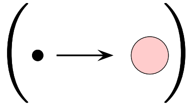
$$\Delta^{[\gamma^-]}(z) = D_1 \qquad \left(\bullet \longrightarrow \circ\right)$$

$$\Delta^{[\gamma^-\gamma_5]}(z) = 0$$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z) = 0$$

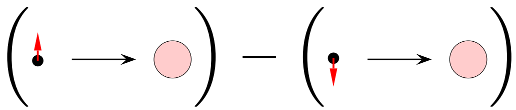
# FF for unpolarised hadron with $k_T$

$$\Delta^{[\gamma^-]}(z, \mathbf{k}_T) = D_1$$



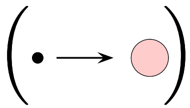
$$\Delta^{[\gamma^-\gamma_5]}(z, \mathbf{k}_T) = 0$$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z, \mathbf{k}_T) = 0 + \frac{\epsilon_T^{ij} k_{Tj}}{M_h} \; H_1^\perp$$



# FF for unpolarised hadron with $k_T$

$$\Delta^{[\gamma^-]}(z, \mathbf{k}_T) \;=\; D_1$$

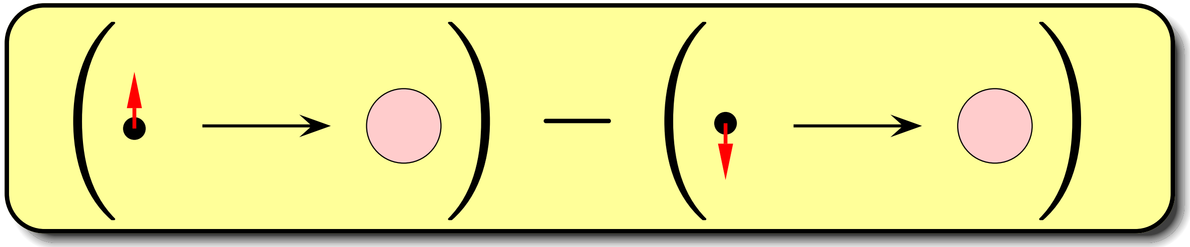


$$\Delta^{[\gamma^-\gamma_5]}(z, \mathbf{k}_T) \;=\; 0$$

T-odd FF: $H_1^\perp$ (Collins)

transv. pol. quark $\longrightarrow$ unpol. hadron

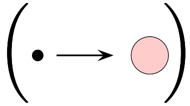
$$\Delta^{[i\sigma^{i-}\gamma_5]}(z, \mathbf{k}_T) \;=\; 0 + \frac{\epsilon_T^{ij} k_{Tj}}{M_h}\, \mathbf{H}_1^\perp$$



# FF for unpolarised hadron with $k_T$

**remark:**

$$\Delta^{[\gamma^-]}(z,\mathbf{k}_T) = D_1$$



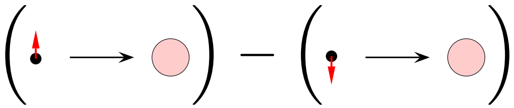
**$\Delta D$ is forbidden by parity constraint**

long. pol. quark $\longrightarrow$ unpol. hadron

$$\Delta^{[\gamma^-\gamma_5]}(z,\mathbf{k}_T) = 0$$

~~$\Delta D_f^h$~~

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z,\mathbf{k}_T) = 0 + \frac{\epsilon_T^{ij}k_{Tj}}{M_h} H_1^\perp$$



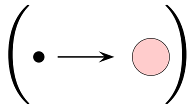
## independent fragmentation functions at leading twist for

# one spin-1/2 hadron

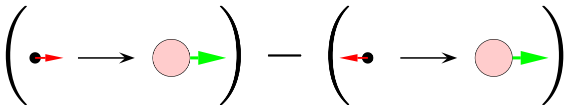
## observed in a quark-jet

# FF for spin-1/2 hadron (without $k_T$)

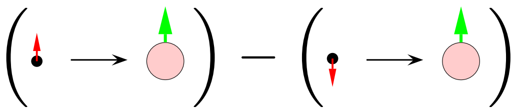
$$\Delta^{[\gamma^-]}(z \quad) = D_1$$



$$\Delta^{[\gamma^- \gamma_5]}(z \quad) = \lambda_h G_1$$

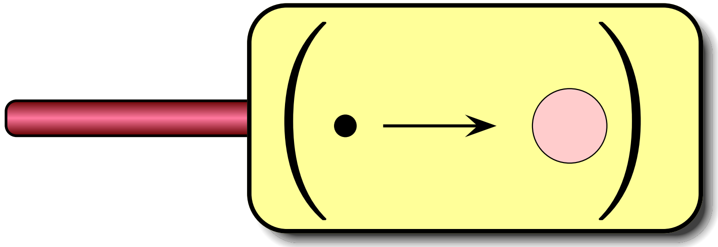


$$\Delta^{[i\sigma^{i-} \gamma_5]}(z \quad) = S_{hT}^i H_1$$



# FF for spin-1/2 hadron (without $k_T$)

$$\Delta^{[\gamma^-]}(z \quad) = D_1$$



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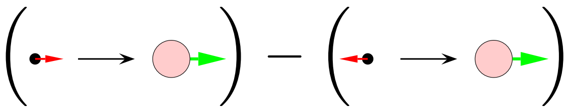
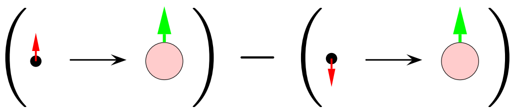


$$\Delta^{[i\sigma^{i-}\gamma_5]}(z \quad) = S^i_{hT} H_1$$



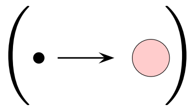
unpolarised FF

probability a quark fragments into

hadron $h$ plus rest $X$

# FF for spin-1/2 hadron (without $k_T$)

$$\Delta^{[\gamma^-]}(z) = D_1$$



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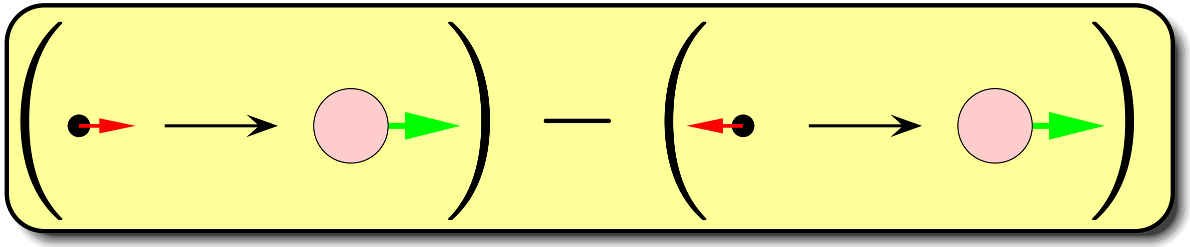
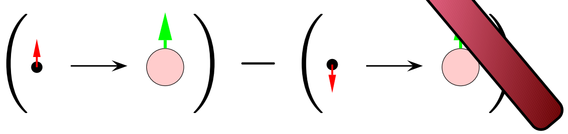


$$\Delta^{[i\sigma^{i-}\gamma_5]}(z) = S_{hT}^i H_1$$



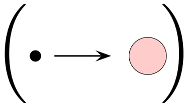
unpolarised FF

probability a quark fragments into hadron $h$ plus rest $X$

preference in **longitudinal** spin transfer

# FF for spin-1/2 hadron (without $k_T$)

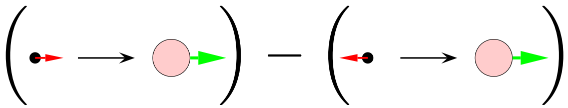
$$\Delta^{[\gamma^-]}(z \quad) = D_1$$



unpolarised FF

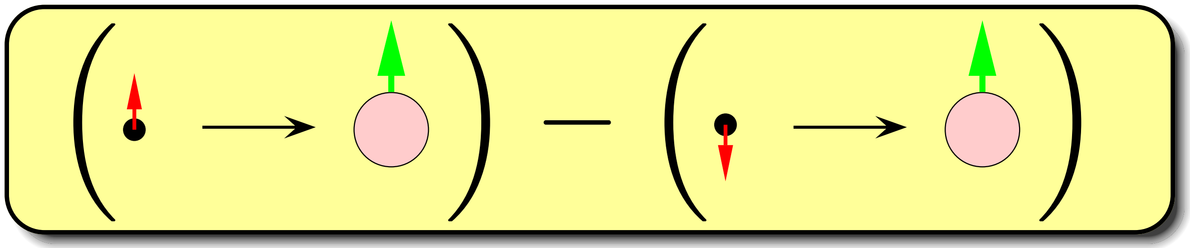
probability a quark fragments into hadron $h$ plus rest $X$

$$\Delta^{[\gamma^-\gamma_5]}(z \quad) = \lambda_h G_1$$



preference in **longitudinal** spin transfer

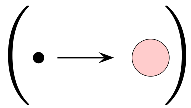
$$\Delta^{[i\sigma^{i-}\gamma_5]}(z \quad) = S_{hT}^i H_1$$



preference in **transverse** spin transfer

# FF for spin-1/2 hadron (without $k_T$)

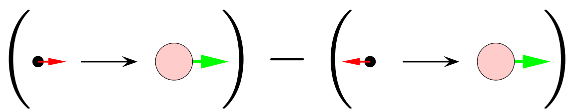
$$\Delta^{[\gamma^-]}(z) = D_1$$



unpolarised FF

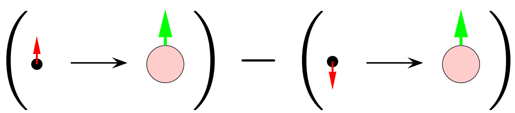
probability a quark fragments into hadron $h$ plus rest $X$

$$\Delta^{[\gamma^-\gamma_5]}(z) = \lambda_h G_1$$



preference in **longitudinal** spin transfer

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z) = S_{hT}^{i} H_1$$



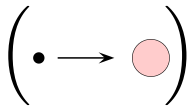
preference in **transverse** spin transfer

the kind of information we are after to understand the process of hadronization:

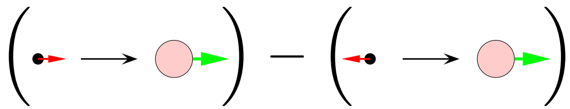
…how much of the quark spin state is *remembered* by a leading hadron after fragmentation ?

## FF for spin-1/2 hadron (without $k_T$)

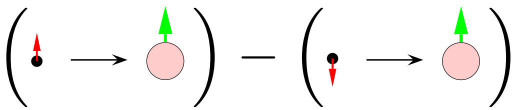
$$\Delta^{[\gamma^-]}(z \quad) = D_1$$



$$\Delta^{[\gamma^-\gamma_5]}(z \quad) = \lambda_h G_1$$



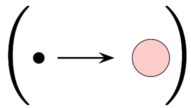
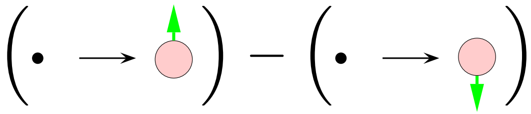
$$\Delta^{[i\sigma^{i-}\gamma_5]}(z \quad) = S_{hT}^i H_1$$



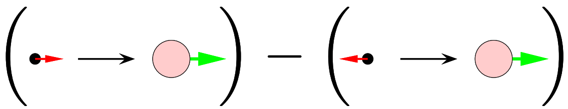
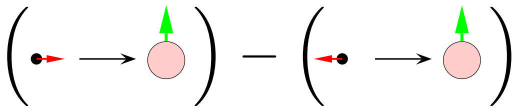
# FF for spin-1/2 hadron with $k_T$

Mulders/Tangerman

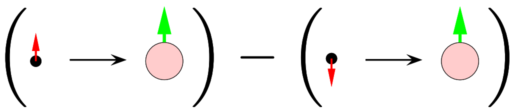
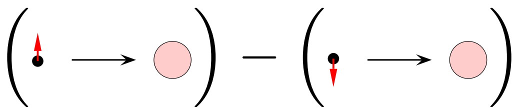
$$\Delta^{[\gamma^-]}(z,\mathbf{k}_T) = D_1 + \frac{\epsilon_{T\,ij} k_T^i S_{hT}^j}{M_h} D_{1T}^\perp$$

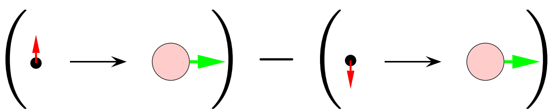
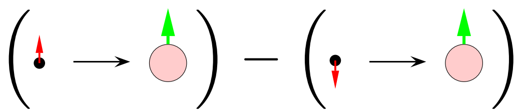
$$\Delta^{[\gamma^-\gamma_5]}(z,\mathbf{k}_T) = \lambda_h G_{1L} + \frac{\mathbf{k}_T \cdot \mathbf{S}_{\mathbf{h}T}}{M_h} G_{1T}$$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z,\mathbf{k}_T) = S_{hT}^i H_{1T} + \frac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^\perp$$

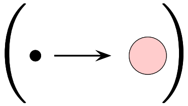
$$+ \frac{k_T^i}{M_h}\left(\lambda_h \quad H_{1L}^\perp \quad + \frac{\mathbf{k}_T \cdot \mathbf{S}_{\mathbf{h}T}}{M_h} \quad H_{1T}^\perp \right)$$

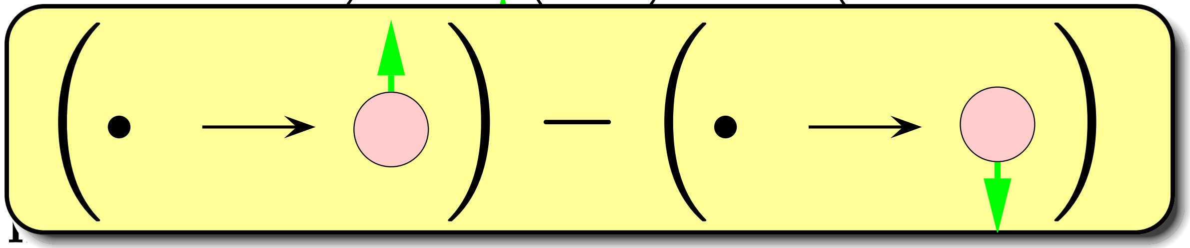



# FF for spin-1/2 hadron with $k_T$

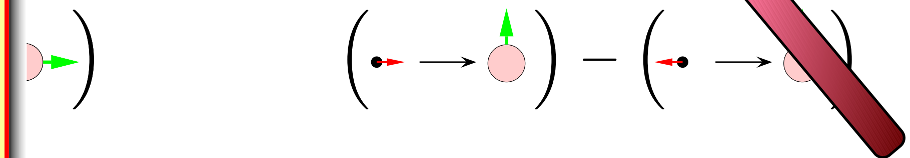
Mulders/Tangerman

$$\Delta^{[\gamma^-]}(z, \mathbf{k}_T) = D_1 + \frac{\epsilon_{Tij} k_T^i S_{hT}^j}{M_h} \mathbf{D}_{1T}^{\perp}$$





$$\Delta^{[\gamma^-\gamma_5]}(z, \mathbf{k}_T) = \lambda_h G_{1L} + \frac{\quad}{M_h} G_{1T}$$



**2 T-odd FF**

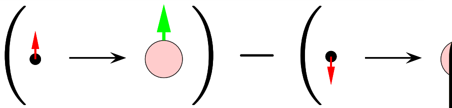
$D_{1T}^{\perp}$ and $H_1^{\perp}$ (Collins)

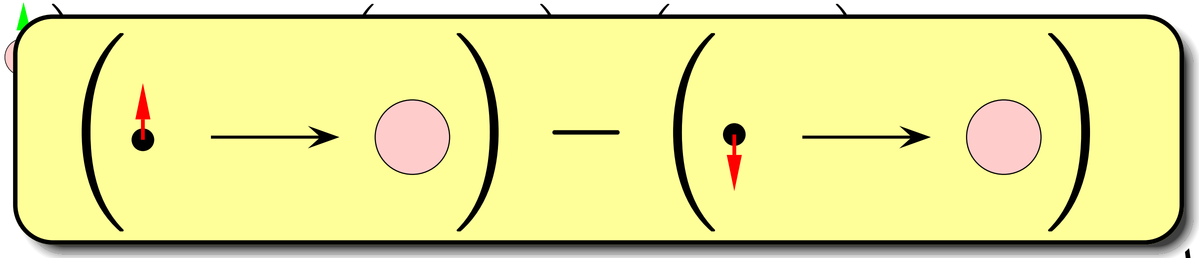
unpol. quark $\longrightarrow$ transv. pol. hadron

and

transv. pol. quark $\longrightarrow$ unpol. hadron

$$+ \frac{\epsilon_T^{ij} k_{Tj}}{M_h} \mathbf{H}_1^{\perp}$$





$$+ \frac{\quad}{M_h} \left( \lambda_h H_{1L} + \frac{\quad}{M_h} H_{1T} \right)$$

Mulders/Tangerman

**3 "unexpected" FF**

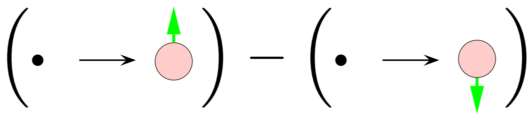
$G_{1T}$, $H_{1L}^{\perp}$ and $H_{1T}^{\perp}$

long. pol. quark $\longrightarrow$ transv. pol. hadron

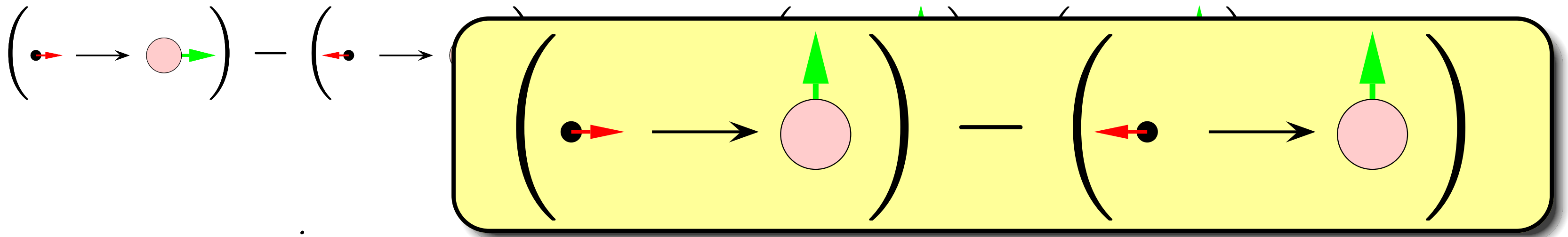
and

transv. pol. quark $\longrightarrow$ long. pol. hadron

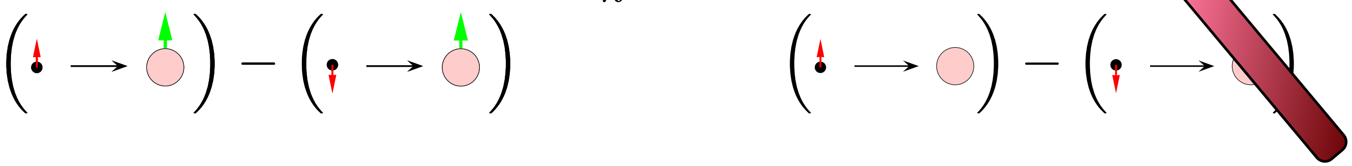
$$\frac{\epsilon_T^{ij} k_T^i S_{hT}^j}{M_h} D_{1T}^{\perp}$$



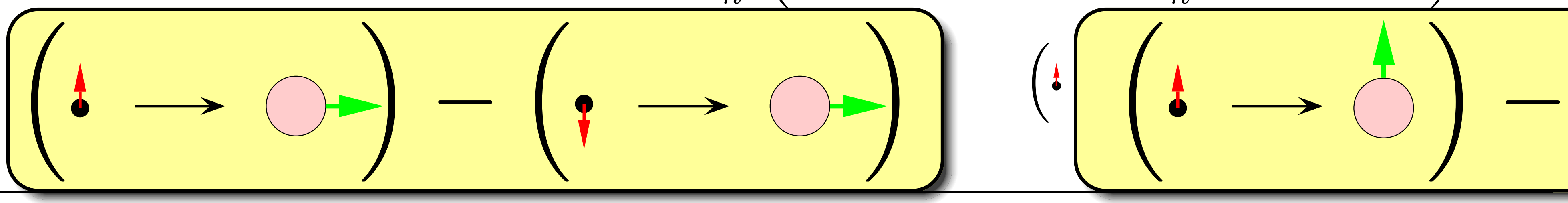
$$+ \frac{\mathbf{k}_T \cdot \mathbf{S}_{hT}}{M_h} \mathbf{G_{1T}}$$



$$\Delta^{[i\sigma^{i-}\gamma_5]}(z, \mathbf{k}_T) = S_{hT}^i H_{1T} + \frac{\ldots}{M_h} H_1^{\perp}$$



$$+ \frac{k_T^i}{M_h} \left( \lambda_h \, \mathbf{H_{1L}^{\perp}} + \frac{\mathbf{k}_T \cdot \mathbf{S}_{hT}}{M_h} \mathbf{H_{1T}^{\perp}} \right)$$



## independent fragmentation functions at leading twist for

# two unpolarised hadrons

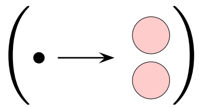
## observed in the same quark-jet

two hadron (interference) fragmentation functions

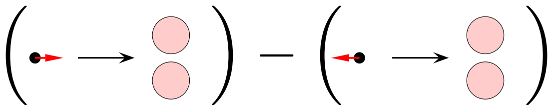
# FF for two unpolarised hadrons (without $k_T$)

$(R \equiv P_1 - P_2)$

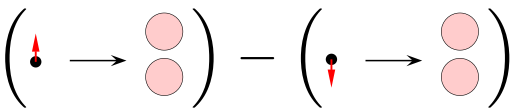
$$\Delta^{[\gamma^-]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = D_1$$



$$\Delta^{[\gamma^- \gamma_5]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} \; G_1^\perp$$



$$\Delta^{[i\sigma^{i-}\gamma_5]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} \; H_1^{\sphericalangle} + \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} \; H_1^\perp$$

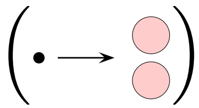


Bianconi/Boffi/R.J./Radici

# FF for two unpolarised hadrons (without $k_T$)

$(R \equiv P_1 - P_2)$

$$\Delta^{[\gamma^-]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = D_1$$

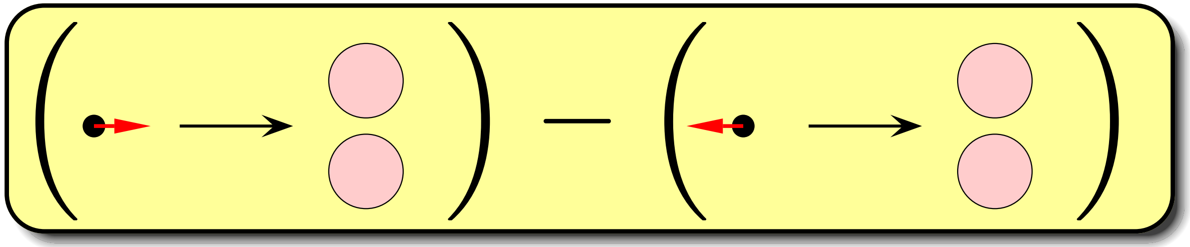


**T-odd FF**

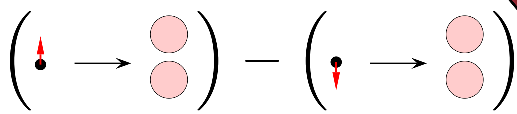
$G_1^\perp$

long. pol. quark $\longrightarrow$ two hadrons

$$\Delta^{[\gamma^-\gamma_5]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} \mathbf{G}_1^\perp$$



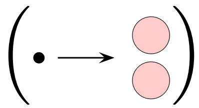
$$\Delta^{[i\sigma^{i-}\gamma_5]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^\sphericalangle + \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^\perp$$



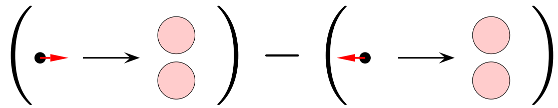
# FF for two unpolarised hadrons (without $k_T$)

$$(R \equiv P_1 - P_2)$$

$$\Delta^{[\gamma^-]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = D_1$$



$$= \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} \, G_1^{\perp}$$



**T-odd FF**

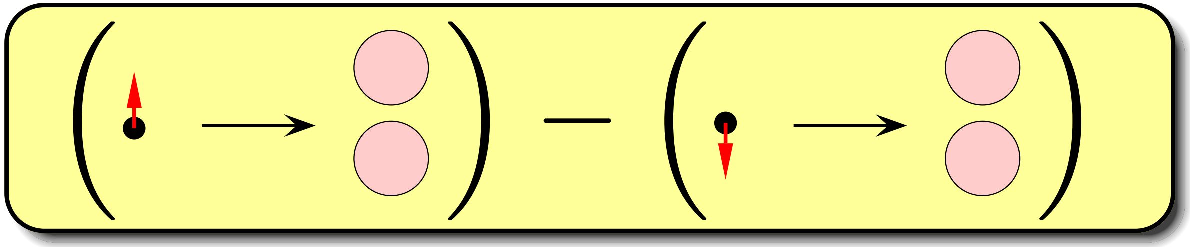
$H_1^{\sphericalangle}$ and $H_1^{\perp}$

transv. pol. quark $\longrightarrow$ two hadrons two

'variants' of the Collins function

come with $\mathbf{R}_T$ and $\mathbf{k}_T$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z_1, z_2, \mathbf{k}_T, \mathbf{R}_T) = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} \mathbf{H}_1^{\sphericalangle} + \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} \mathbf{H}_1^{\perp}$$



## independent fragmentation functions at leading twist for

# one spin-1 hadron

## observed in a quark-jet

$$\Delta^{[\gamma^-]}(z,\mathbf{k}_T) = D_1 + \frac{\epsilon_{T\,ij} k_T^i S_{hT}^j}{M_h} D_{1T}^{\perp}$$

$$\Delta^{[\gamma^-\gamma_5]}(z,\mathbf{k}_T) = \lambda_h G_{1L} + \frac{\mathbf{k}_T \cdot \mathbf{S}_{\mathbf{h}T}}{M_h} G_{1T}$$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z,\mathbf{k}_T) = S_{hT}^i H_{1T} + \frac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^{\perp}$$

$$+ \frac{k_T^i}{M_h}\left(\lambda_h \quad H_{1L}^{\perp} \quad + \frac{\mathbf{k}_T \cdot \mathbf{S}_{\mathbf{h}T}}{M_h} \quad H_{1T}^{\perp} \quad \right)$$

Bacchetta/Mulders

$$\Delta^{[\gamma^-]}(z,\mathbf{k}_T) = D_1 + \frac{\epsilon_{T\,ij} k_T^i S_{hT}^j}{M_h} D_{1T}^\perp + \epsilon_T^{\mu\nu} S_{T\,\nu} \frac{k_{T\,\mu}}{M} D_{1T}^\perp + S_{LL}\, D_{1LL} + \frac{\mathbf{S}_{LT}\cdot\mathbf{k}_T}{M} D_{1LT} + \frac{\mathbf{k}_T\cdot\mathbf{S}_{TT}\cdot\mathbf{k}_T}{M^2} D_{1TT}$$

$$\Delta^{[\gamma^-\gamma_5]}(z,\mathbf{k}_T) = \lambda_h G_{1L} + \frac{\mathbf{k}_T\cdot\mathbf{S_h}_T}{M_h} G_{1T} + \epsilon_T^{\mu\nu} S_{LT\,\nu} \frac{k_{T\,\mu}}{M} G_{1LT} + -\epsilon_T^{\mu\nu} S_{TT\,\nu\rho} \frac{k_T^\rho k_{T\,\mu}}{M^2} G_{1TT}$$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z,\mathbf{k}_T) = S_{hT}^i H_{1T} + \frac{\epsilon_T^{ij} k_{T\,j}}{M_h} H_1^\perp + S_{LL}\frac{\epsilon_T^{ij} k_{T\,j}}{M} H_{1LL}^\perp + \epsilon_T^{ij} S_{LT\,j} H_{1LT}' + \frac{\mathbf{S}_{LT}\cdot\mathbf{k}_T}{M}\frac{\epsilon_T^{ij} k_{T\,j}}{M} H_{1LT}^\perp + \frac{k_T^i}{M_h}\left(\lambda_h \quad H_{1L}^\perp \quad + \frac{\mathbf{k}_T\cdot\mathbf{S_h}_T}{M_h} \quad H_{1T}^\perp \quad\right) + \epsilon_T^{ij} S_{TT\,jl} \frac{k_T^l}{M} H_{1TT}' + \frac{\mathbf{k}_T\cdot\mathbf{S}_{TT}\cdot\mathbf{k}_T}{M^2}\frac{\epsilon_T^{ij} k_{T\,j}}{M} H_{1TT}^\perp$$

# FF for spin-1 hadron with $k_T$

Bacchetta/Mulders

$$\Delta^{[\gamma^-]}(z,\mathbf{k}_T) = D_1 + \frac{\epsilon_{T\,ij} k_T^i S_{hT}^j}{M_h} D_{1T}^\perp + \epsilon_T^{\mu\nu} S_{T\,\nu} \frac{k_{T\,\mu}}{M} D_{1T}^\perp + S_{LL}\, D_{1LL} + \frac{\mathbf{S}_{LT}\cdot\mathbf{k}_T}{M} D_{1LT} + \frac{\mathbf{k}_T\cdot\mathbf{S}_{TT}\cdot\mathbf{k}_T}{M^2} D_{1TT}$$

$$\Delta^{[\gamma^-\gamma_5]}(z,\mathbf{k}_T) = \lambda_h G_{1L} + \frac{\mathbf{k}_T\cdot\mathbf{S_h}_T}{M_h} G_{1T} + \epsilon_T^{\mu\nu} S_{LT\,\nu} \frac{k_{T\,\mu}}{M} G_{1LT} + -\epsilon_T^{\mu\nu} S_{TT\,\nu\rho} \frac{k_T^\rho k_{T\,\mu}}{M^2} G_{1TT}$$

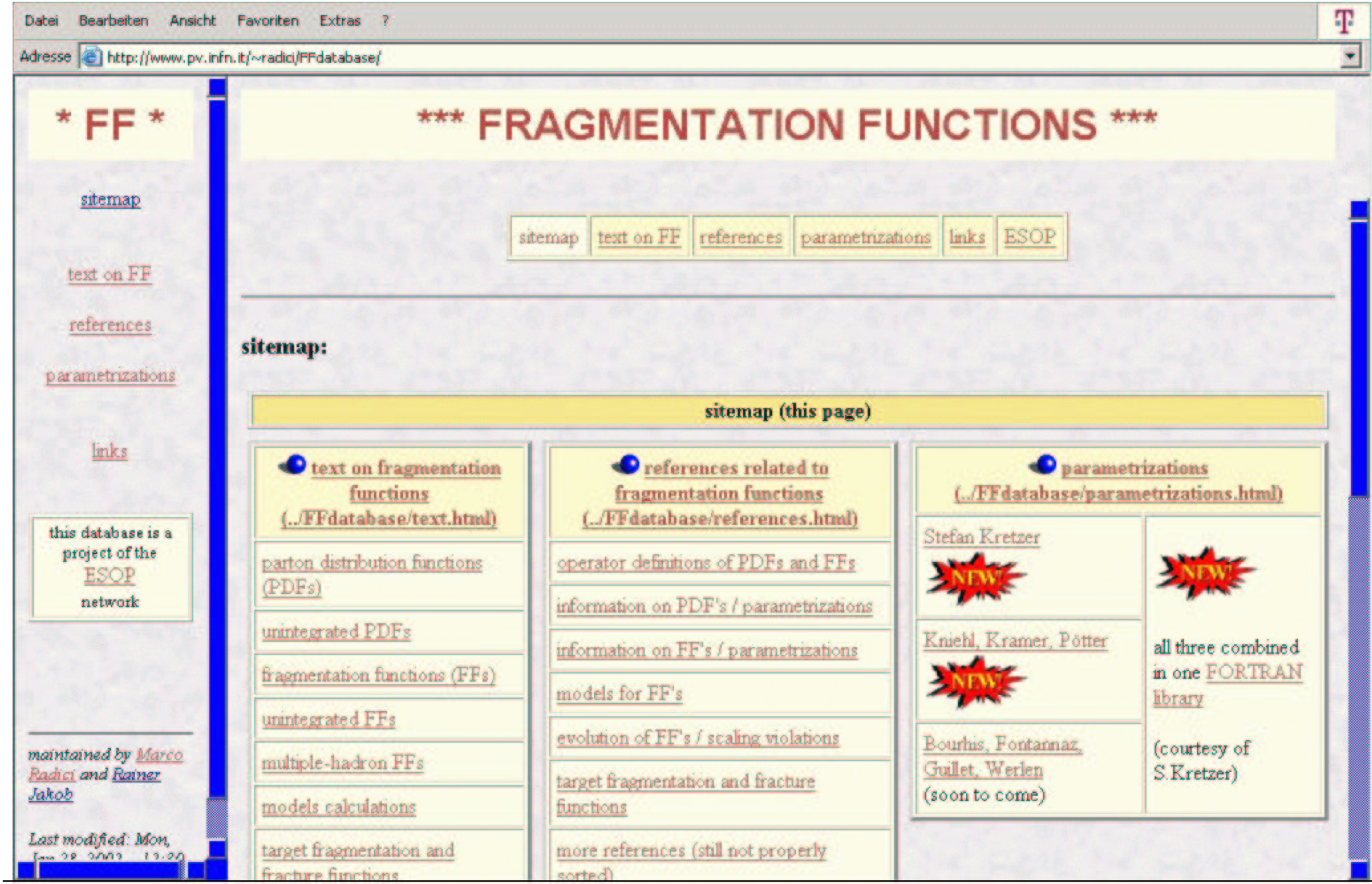
$$\Delta^{[i\sigma^{i-}\gamma_5]}(z,\mathbf{k}_T) = S_{hT}^i H_{1T} + \frac{\epsilon_T^{ij} k_{T\,j}}{M_h} H_1^\perp + S_{LL} \frac{\epsilon_T^{ij} k_{T\,j}}{M} H_{1LL}^\perp + \epsilon_T^{ij} S_{LT\,j}\, H'_{1LT} + \frac{\mathbf{S}_{LT}\cdot\mathbf{k}_T}{M} \frac{\epsilon_T^{ij} k_{T\,j}}{M} H_{1LT}^\perp + \frac{k_T^i}{M_h}\left(\lambda_h \quad H_{1L}^\perp \quad + \frac{\mathbf{k}_T\cdot\mathbf{S_h}_T}{M_h} \quad H_{1T}^\perp \quad\right) + \epsilon_T^{ij} S_{TT\,jl} \frac{k_T^l}{M} H'_{1TT} + \frac{\mathbf{k}_T\cdot\mathbf{S}_{TT}\cdot\mathbf{k}_T}{M^2} \frac{\epsilon_T^{ij} k_{T\,j}}{M} H_{1TT}^\perp$$

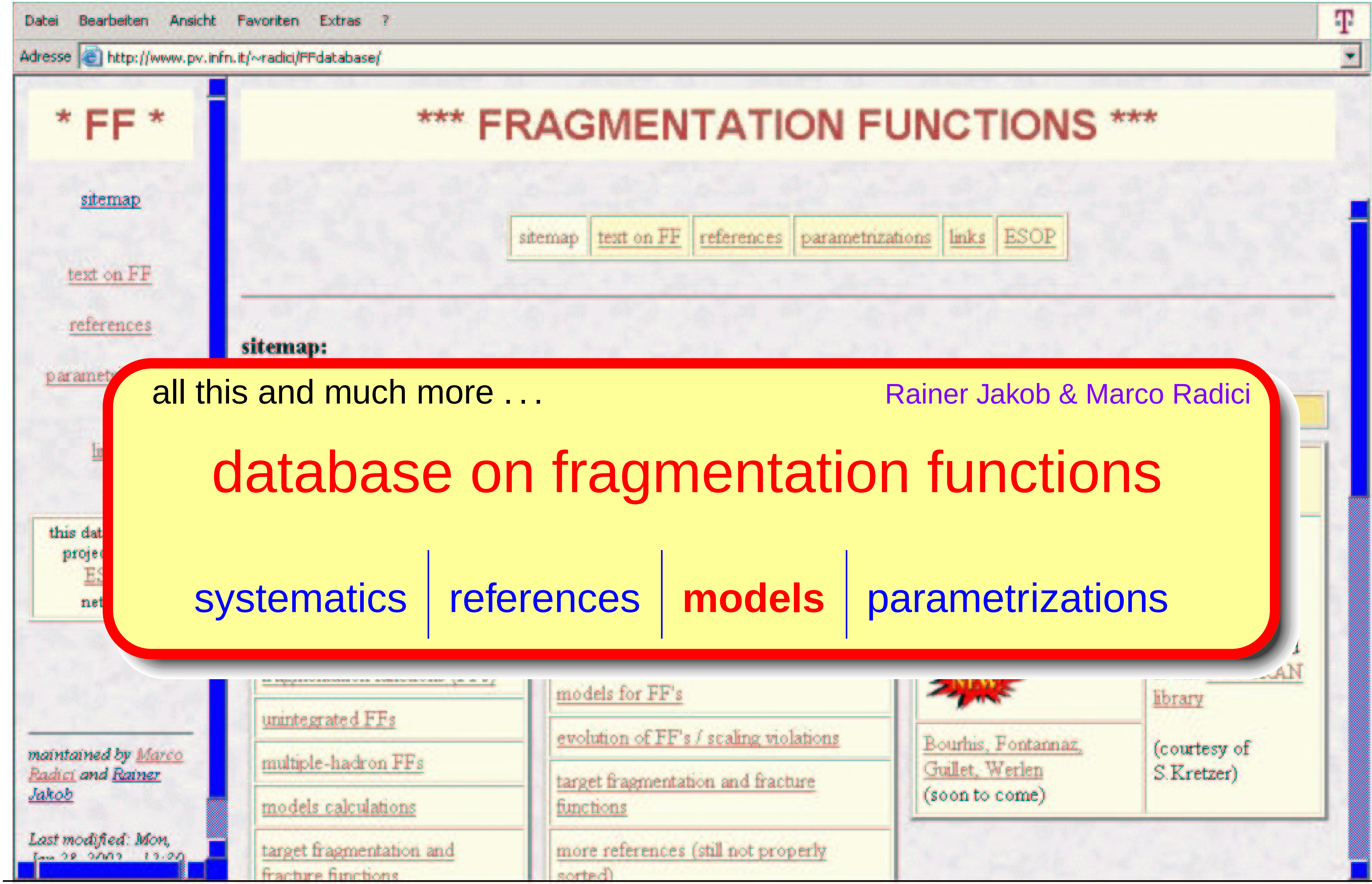
relations between spin-1 FF

and two spin-0 FF

Bacchetta/Radici (in progress)

# FF for spin-1 hadron with $k_T$



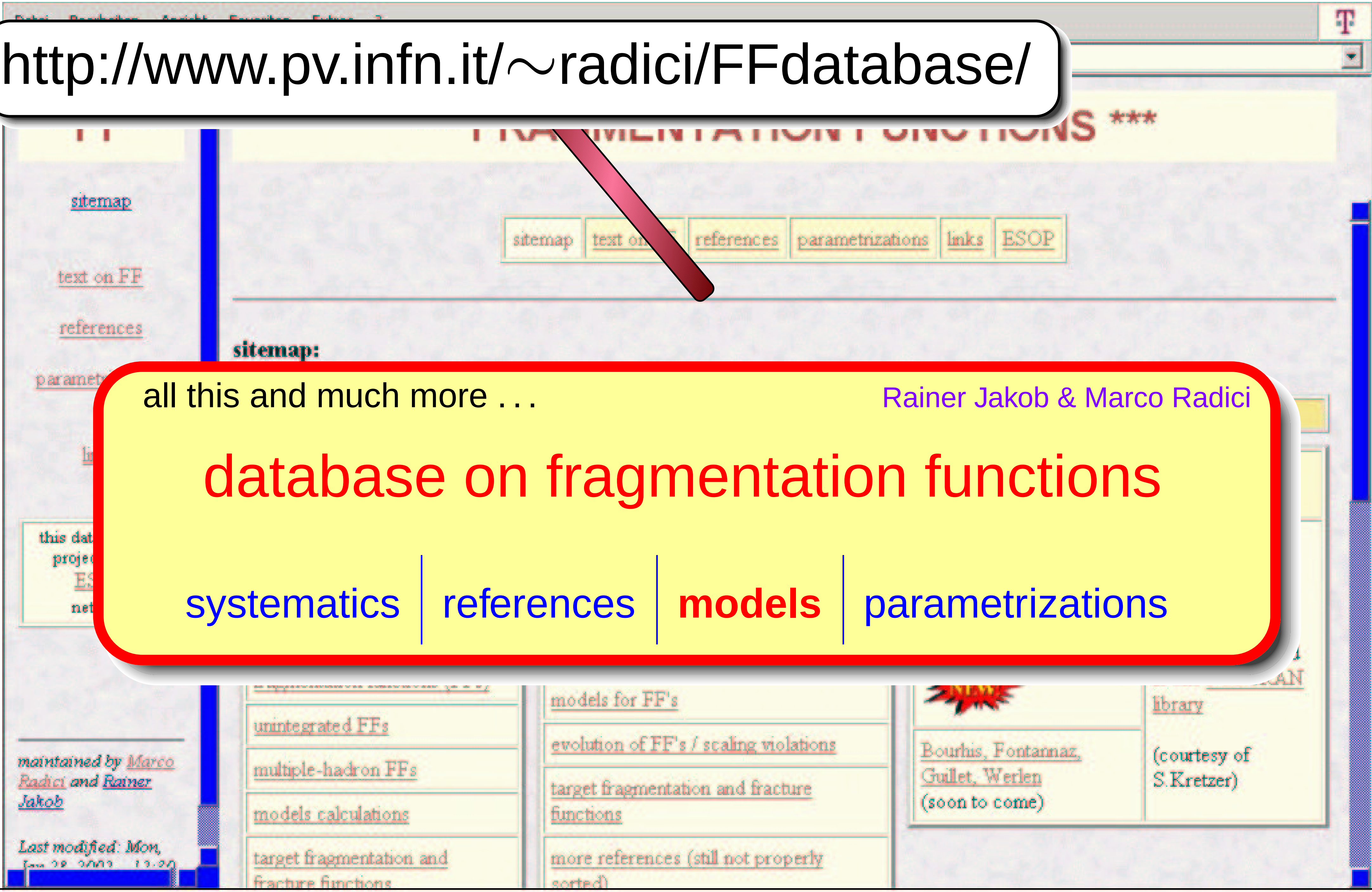



all this and much more . . .

Rainer Jakob & Marco Radici

## database on fragmentation functions

systematics | references | **models** | parametrizations http://www.pv.infn.it/∼radici/FFdatabase/




all this and much more . . .

Rainer Jakob & Marco Radici

## database on fragmentation functions

systematics | references | **models** | parametrizations

# summary

one hadron FF

| | without $\mathbf{k}_T$ | with $\mathbf{k}_T$ |
|---|---|---|
| spin-0 | $D_1$ | $H_1^\perp$ |
| spin-1/2 | $D_1, G_1, H_1$ | $D_{1T}^\perp, H_1^\perp, G_{1T}, H_{1L}^\perp, H_{1T}^\perp$ |
| spin-1 | $D_1, D_{1LL}, G_1, H_1, H_{1LT}$ | $D_{1T}^\perp, H_1^\perp, G_{1T}, H_{1L}^\perp, H_{1T}^\perp,$<br>$D_{1T}^\perp, D_{1LT}, D_{1TT}, G_{1LT}, G_{1TT},$<br>$H_{1LL}^\perp, H'_{1LT}, H_{1LT}^\perp$ |

two hadron FF

| | without $\mathbf{k}_T$ | with $\mathbf{k}_T$ |
|---|---|---|
| spin-0 | $D_1, H_1^\sphericalangle$ | $G_1^\perp, H_1^\perp$ |

higher twist

# conclusions

- the number of independent fragmentation functions is limited and there is a simple systematics behind
- spin-dependent fragmentation functions are not only a tool for the extraction of distribution functions, but provide the key information for understanding hadronic structure