

Riassunto della lezione precedente

- proiezioni a leading twist del correlatore quark-quark Φ :
tre funzioni con interpretazione probabilistica
una (trasversità) è chiral-odd \rightarrow soppressa in DIS inclusivo
- proprietà peculiari della trasversità; evoluzione tipica di PDF di non-singoleto; sensibile soprattutto ai quark di valenza
- formalismo di elicità e diseguaglianza di Soffer
- estrazione di h_1 dai dati :
 - Drell-Yan polarizzato; SSA piccola; possibilità di usare antiprotoni
 - SIDIS \rightarrow funzioni di frammentazione (FF) \rightarrow analisi in twist del correlatore Δ
necessità di considerare adroni finali polarizzati per avere contributo al leading twist
SSA con Λ^\uparrow ; ma meccanismo ignoto

Decomposizione di Δ al leading twist

Base di matrici di Dirac $\{\mathbf{1}, \gamma^\mu, \gamma^\mu \gamma_5, i\gamma_5, i\sigma^{\mu\nu} \gamma_5\}$

$$\Delta(k, P_h, S_h) = \frac{1}{2} [S \mathbf{1} + V_\mu \gamma^\mu + A_\mu \gamma^\mu \gamma_5 + i P \gamma_5 + iT_{\mu\nu} \sigma^{\mu\nu} \gamma_5]$$



$$S = \frac{1}{2} \text{Tr}(\Delta) = C_1(k^2, k \cdot P_h)$$

$$V^\mu = \frac{1}{2} \text{Tr}(\gamma^\mu \Delta) = C_2 P_h^\mu + C_3 k^\mu + C_{10} \epsilon_{\mu\nu\rho\sigma} S_h^\nu P_h^\rho k^\sigma$$

$$A^\mu = \frac{1}{2} \text{Tr}(\gamma^\mu \gamma_5 \Delta) = C_4 S_h^\mu + C_5 p \cdot S_h P_h^\mu + C_6 P_h \cdot S_h k^\mu$$

$$P_5 = \frac{1}{2i} \text{Tr}(\gamma_5 \Delta) = C_{11} k \cdot S_h$$

$$T^\mu = \frac{1}{2i} \text{Tr}(\sigma^{\mu\nu} \Delta) = C_7 P_h^{[\mu} S_h^{\nu]} + C_8 k^{[\mu} S_h^{\nu]} + C_9 k \cdot S_h P_h^{[\mu} k^{\nu]} + C_{12} \epsilon_{\mu\nu\rho\sigma} P_h^\rho k^\sigma$$



Naive T - reversal transformation

$|a\rangle =$  system with some spin and momentum

$|-a\rangle =$  flipping spin and momentum

$|i\rangle, |f\rangle$ initial, final states of the system; T_{if} trans. matrix; T -rev. $\rightarrow |T_{if}|^2 = |T_{f-i}|^2$

naive T - reversal transformation : T_{-i-f}

$$A = |T_{if}|^2 - |T_{-i-f}|^2$$

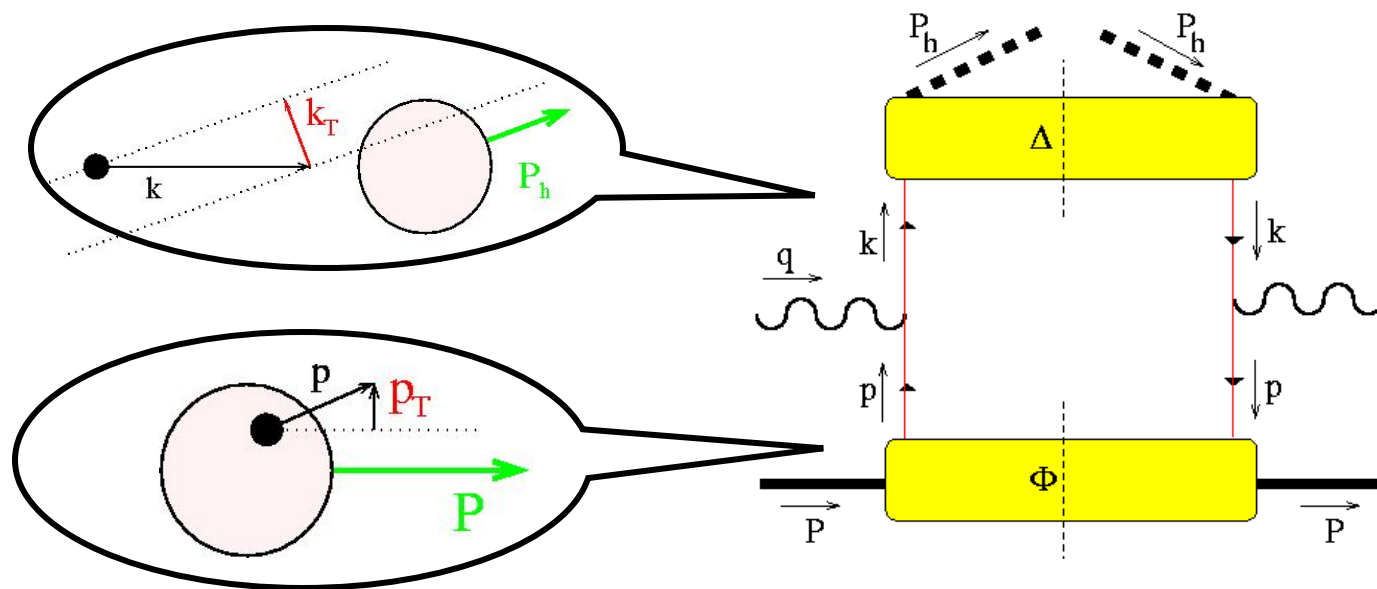
no FSI $\Rightarrow |i\rangle \leftrightarrow |f\rangle$; $A = 0$; T -rev. = naive T -rev.

FSI $\Rightarrow |i\rangle \neq |f\rangle$; T - rev. OK

but $A \neq 0 \propto \Im [\text{Born} \times \text{rescatt.}^*]$



- **ricorda** : in SIDIS $\{P, q, P_h\}$ non sono tutti collineari; nel T-frame, se la sezione d'urto è differenziale anche in $d\mathbf{q}_T$
 → sensibilità ai momenti trasversi dei partoni nel vertice hard
 → struttura più ricca



FF dipendenti da momento trasverso intrinseco

$$\Delta^{[\Gamma]}(z, \mathbf{P}_{hT}, S_h) = \frac{1}{4z} \int dk^+ \text{Tr} [\Delta(k, P_h, S_h) \Gamma] \Big|_{k^- = P_h^- / z}$$



Proiezioni al leading twist

$$D_1 = \bullet \longrightarrow \bigcirc$$

$$D_{1T}^\perp = \left(\bullet \xrightarrow{\nearrow} \bigcirc - \bullet \xrightarrow{\nwarrow} \bigcirc \right)$$

$$\Delta^{[\gamma^-]}(z, \mathbf{P}_{hT}, S_h) = D_1(z, \mathbf{P}_{hT}^2) + \frac{(\mathbf{k}_T \times \mathbf{S}_{hT})_i}{M_h} D_{1T}^\perp(z, \mathbf{P}_{hT}^2)$$

$$G_{1L} = \left(\bullet \xrightarrow{\rightarrow} \bigcirc \xrightarrow{\rightarrow} \bullet \right) - \left(\bullet \xrightarrow{\leftarrow} \bigcirc \xrightarrow{\rightarrow} \bullet \right)$$

$$G_{1T} = \left(\bullet \xrightarrow{\rightarrow} \bigcirc \xrightarrow{\uparrow} \bullet \right) - \left(\bullet \xrightarrow{\leftarrow} \bigcirc \xrightarrow{\uparrow} \bullet \right)$$

$$\Delta^{[\gamma^- \gamma_5]}(z, \mathbf{P}_{hT}, S_h) = \lambda_h G_{1L}(z, \mathbf{P}_{hT}^2) + \frac{\mathbf{k}_T \cdot \mathbf{S}_{hT}}{M_h} G_{1T}^\perp(z, \mathbf{P}_{hT}^2)$$

$$H_{1T} = \left(\bullet \xrightarrow{\uparrow} \bigcirc \right) - \left(\bullet \xrightarrow{\downarrow} \bigcirc \right)$$

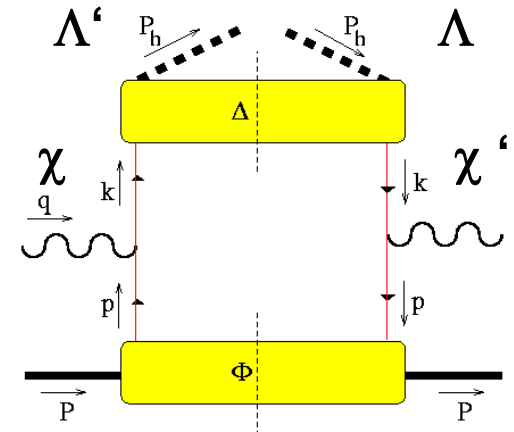
$$H_1^\perp = \left(\bullet \xrightarrow{\uparrow} \bigcirc \right) - \left(\bullet \xrightarrow{\downarrow} \bigcirc \right)$$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z, \mathbf{P}_{hT}, S_h) = S_{hT}^i H_{1T}(z, \mathbf{P}_{hT}^2) + \frac{(\hat{\mathbf{n}}_T \times \mathbf{k}_T)_i}{M_h} H_1^\perp(z, \mathbf{P}_{hT}^2)$$

$$+ \frac{k_T^i}{M_h} \left[\lambda_h H_{1L}^\perp(z, \mathbf{P}_{hT}^2) + \frac{\mathbf{k}_T \cdot \mathbf{S}_{hT}}{M_h} H_{1T}^\perp(z, \mathbf{P}_{hT}^2) \right]$$

$$H_{1L}^\perp = \left(\bullet \xrightarrow{\uparrow} \bigcirc \xrightarrow{\rightarrow} \bullet \right) - \left(\bullet \xrightarrow{\downarrow} \bigcirc \xrightarrow{\rightarrow} \bullet \right) \quad H_{1T}^\perp = \left(\bullet \xrightarrow{\uparrow} \bigcirc \xrightarrow{\nearrow} \bullet \right) - \left(\bullet \xrightarrow{\downarrow} \bigcirc \xrightarrow{\nearrow} \bullet \right)$$

Rappresentazione di elicità di $\Delta(z, \mathbf{P}_{hT}, S_h) \dots$



PFF	chiral even		chiral-odd
	q non pol.	$q \rightarrow$	$q \uparrow$
H non pol.	D_1		H_1^\perp
$H \rightarrow = L$		G_{1L}	H_{1L}^\perp
$H \uparrow = T$	D_{1T}^\perp	G_{1T}	H_1, H_{1T}^\perp

solo per D_1 c'è parametrizzazione da dati sperimentali di $e^+e^- \rightarrow h X$!

naïve T-even

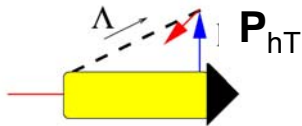
naïve T-odd

(continua)

D_{1T}^\perp e H_{1T}^\perp sono FF su cui il time-reversal non impone condizioni; se le interazioni residue (FSI) tra adrone e jet nello stato finale si annullano
 → entrambe le FF = 0

$$D(h^\uparrow/q) = \frac{1}{2} \left(D_1^q(z, \mathbf{P}_{hT}^2) + D_{1T}^{\perp q}(z, \mathbf{P}_{hT}^2) \frac{\hat{\mathbf{k}} \times \mathbf{P}_{hT} \cdot \mathbf{S}_h}{zM_h} \right)$$

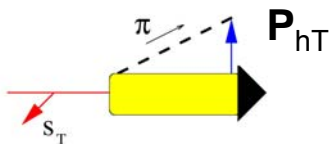
$$D_{1T}^{\perp q}(z, \mathbf{P}_{hT}^2) \frac{\hat{\mathbf{k}} \times \mathbf{P}_{hT} \cdot \mathbf{S}_h}{zM_h} = D(h^\uparrow/q) - D(h^\downarrow/q) \quad D_{1T}^\perp = \left(\bullet \rightarrow \text{circle with green arrow} - \bullet \rightarrow \text{circle with green arrow} \right)$$



$$D(h/q^\uparrow) = D_1^q(z, \mathbf{P}_{hT}^2) + H_{1T}^{\perp q}(z, \mathbf{P}_{hT}^2) \frac{\hat{\mathbf{k}} \times \mathbf{P}_{hT} \cdot \mathbf{S}_q}{zM_h}$$

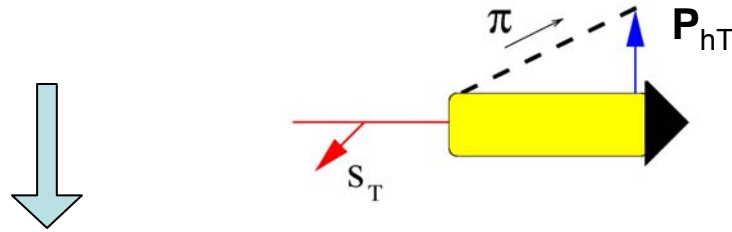
$$H_{1T}^{\perp q}(z, \mathbf{P}_{hT}^2) 2 \frac{\hat{\mathbf{k}} \times \mathbf{P}_{hT} \cdot \mathbf{S}_q}{zM_h} = D(h/q^\uparrow) - D(h/q^\downarrow)$$

$$H_{1T}^\perp = \left(\text{circle with red arrow up} \rightarrow \bullet \right) - \left(\text{circle with red arrow down} \rightarrow \bullet \right)$$



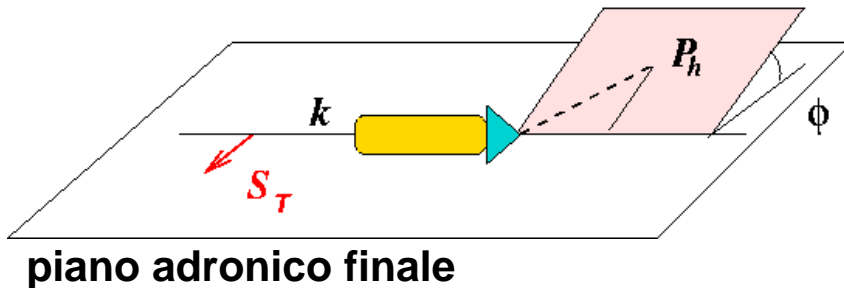
entrambe legano lo spin di un oggetto alla dipendenza da \mathbf{P}_{hT} di un altro oggetto non polarizzato

(continua)



effetto Collins (Collins, Nucl.Phys. **B396** ('93) 161) :

trasferire polarizzazione trasversa del quark di frammentazione non alla polarizz. trasversa dell'adrone, ma al moto orbitale di un adrone non polarizzato \rightarrow SSA dipendente da $\mathbf{P}_{h\perp}$



asimmetria in

$$\sin \phi \propto \mathbf{k} \times \mathbf{P}_h \cdot \mathbf{S}_T$$

tipico effetto non perturbativo
(legato al momento angolare orbitale dei partoni)

Decomposizione di Φ al leading twist

Base di matrici di Dirac $\{\mathbf{1}, \gamma^\mu, \gamma^\mu \gamma_5, i\gamma_5, i\sigma^{\mu\nu} \gamma_5\}$

$$\Phi(p, P, S) = \frac{1}{2} [S \mathbf{1} + V_\mu \gamma^\mu + A_\mu \gamma^\mu \gamma_5 + iP \gamma_5 + iT_{\mu\nu} \sigma^{\mu\nu} \gamma_5]$$

$$S = \frac{1}{2} \text{Tr}(\Phi) = C_1(p^2, p \cdot P)$$

$$V^\mu = \frac{1}{2} \text{Tr}(\gamma^\mu \Phi) = C_2 P^\mu + C_3 p^\mu + C_{10} \epsilon_{\mu\nu\rho\sigma} S^\nu P^\rho p^\sigma$$

$$A^\mu = \frac{1}{2} \text{Tr}(\gamma^\mu \gamma_5 \Phi) = C_4 S^\mu + C_5 p \cdot S P^\mu + C_6 P \cdot S p^\mu$$

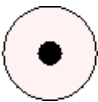
$$P_5 = \frac{1}{2i} \text{Tr}(\gamma_5 \Phi) = C_{11} p \cdot S$$

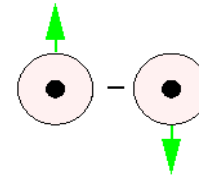
$$T^\mu = \frac{1}{2i} \text{Tr}(\sigma^{\mu\nu} \Phi) = C_7 P^{[\mu} S^{\nu]} + C_8 p^{[\mu} S^{\nu]} + C_9 p \cdot S P^{[\mu} p^{\nu]} + C_{12} \epsilon_{\mu\nu\rho\sigma} P^\rho p^\sigma$$

PDF dipendenti da momento trasverso intrinseco

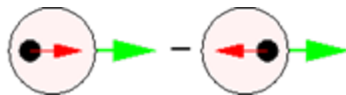
Proiezioni al
leading twist

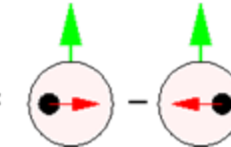
$$\Phi^{[\Gamma]}(x, \mathbf{p}_T, S) = \int dp^- \text{Tr} [\Phi(p, P, S) \Gamma] \Big|_{p^+ = xP^+}$$

$$f_1 = \text{diagram}$$


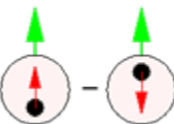
$$f_1^\perp = \text{diagram}$$


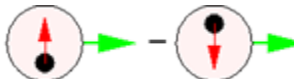
$$\Phi^{[\gamma^+]}(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{\mathbf{p}_T \times \mathbf{S}_T \cdot \hat{\mathbf{P}}}{M} f_{1T}^\perp(x, \mathbf{p}_T^2)$$

$$g_{1L} = \text{diagram}$$


$$g_{1T} = \text{diagram}$$


$$\Phi^{[\gamma^+ \gamma_5]}(x, \mathbf{p}_T, S) = \lambda g_{1L}(x, \mathbf{p}_T^2) + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{p}_T^2)$$

$$h_{1T} = \text{diagram}$$


$$h_{1L}^\perp = \text{diagram}$$


$$h_{1T}^\perp = \text{diagram}$$


$$\Phi^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{p}_T, S) = S_T^i h_1 + \frac{p_T^i}{M} \lambda h_{1L}^\perp + \frac{S_T^i}{M^2} \left(p_T^i p_T^j - \frac{1}{2} \mathbf{p}_T^2 \delta_{ij} \right) h_{1T}^\perp + \frac{(\mathbf{p}_T \times \hat{\mathbf{P}})_i}{M} h_1^\perp$$

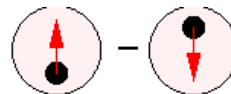
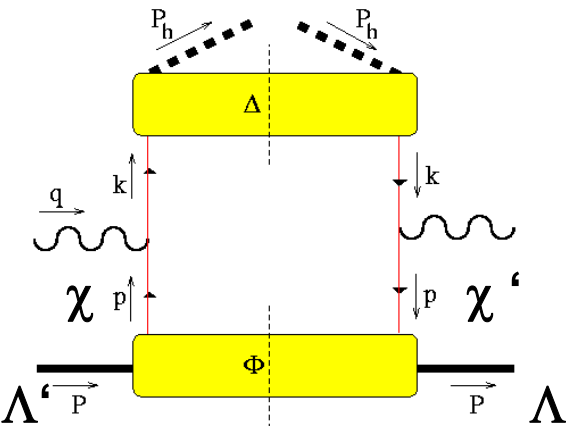
$$h_1^\perp = \text{diagram}$$


Diagram illustrating the twist 2 N[↑] structure function, showing the dependence on the transverse momentum \mathbf{p}_T (pesata con \mathbf{p}_T).

Rappresentazione di elicità di $\Phi(x, \underline{p}_T, S)$

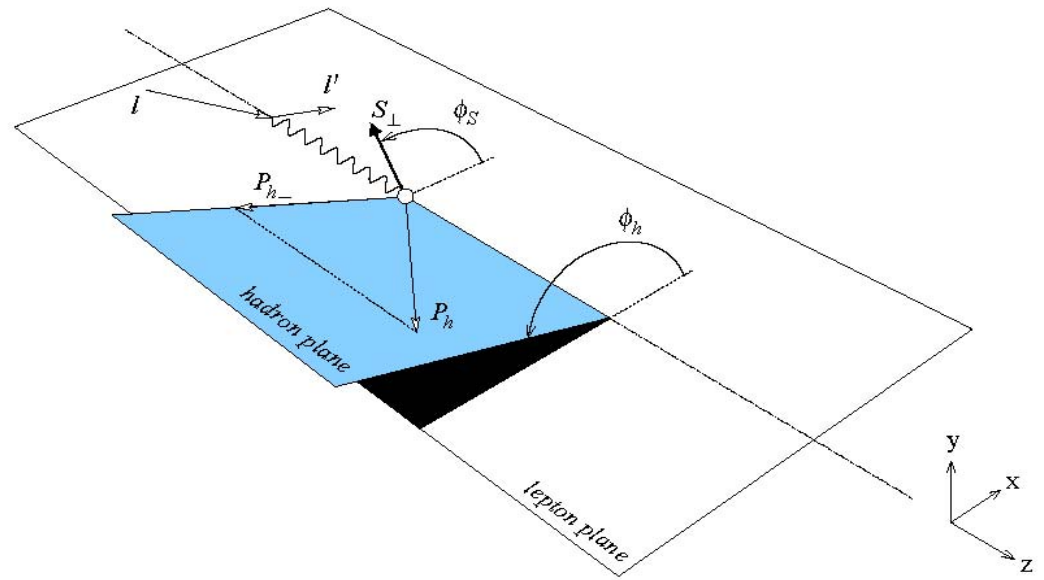


PDF	chiral even	chiral-odd
	q non pol.	q [↑]
H non pol.	f ₁	h ₁ [⊥]
H [→] = L		g _{1L}
H [↑] = T	f _{1T} [⊥]	g _{1T}

$$h_1 = h_{1T} + h_{1T}^\perp \frac{\vec{p}_T^2}{M^2}$$

SIDIS $e p^\uparrow \rightarrow e' \pi X$

per avere l'effetto Collins
bisogna mantenere
la sez. d'urto
differenziale in $\mathbf{P}_{h\perp}$



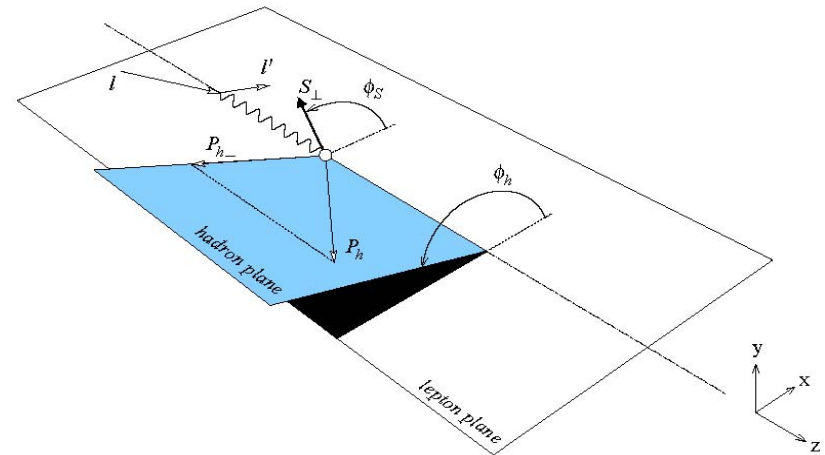
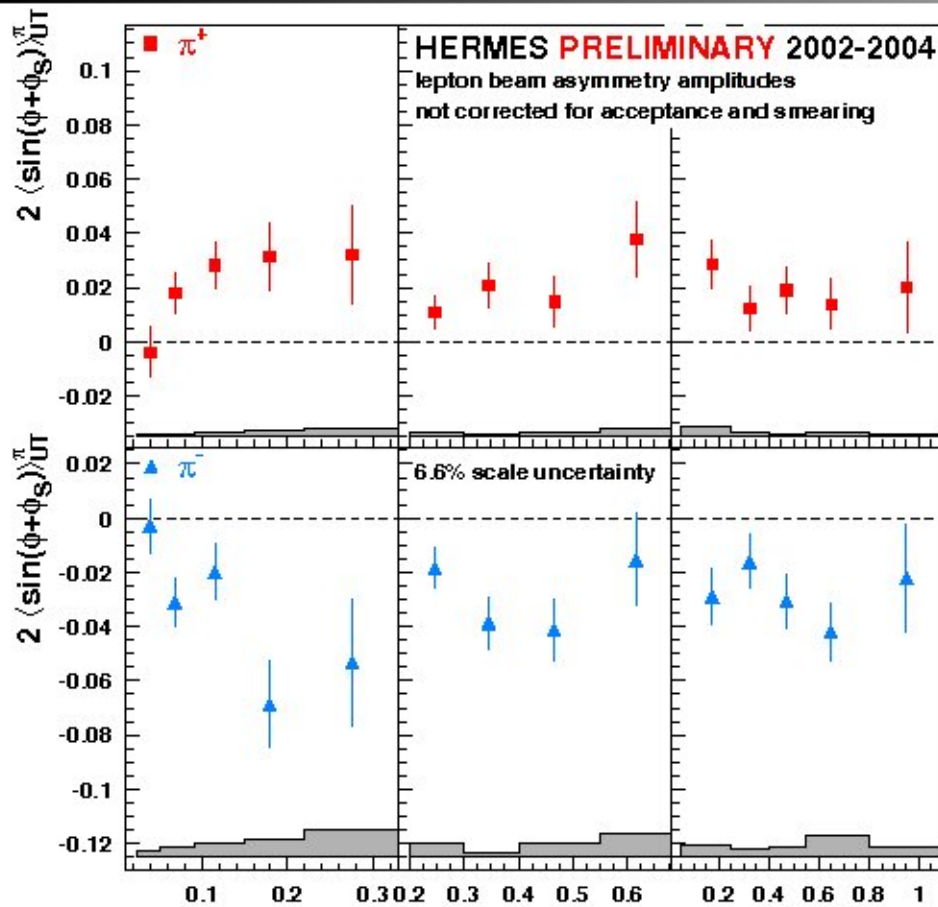
$$\frac{d^6 \sigma_{OT}}{dx dy dz d\phi_S d\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2} \sum_{f, \bar{f}} e_f^2 \left\{ A(y) \mathcal{F} \left[f_1^f(x, \mathbf{p}_T^2) D_1^f(z, \mathbf{P}_{hT}^2) \right] \right. \\ \left. + |\mathbf{S}_T| B(y) \underbrace{\sin(\phi_h + \phi_S)} \mathcal{F} \left[\frac{\mathbf{k}_T \cdot \hat{\mathbf{P}}_{h\perp}}{M_h} h_1^f(x, \mathbf{p}_T^2) H_1^{\perp f}(z, \mathbf{P}_{hT}^2) \right] \right\}$$



ϕ_C angolo di Collins

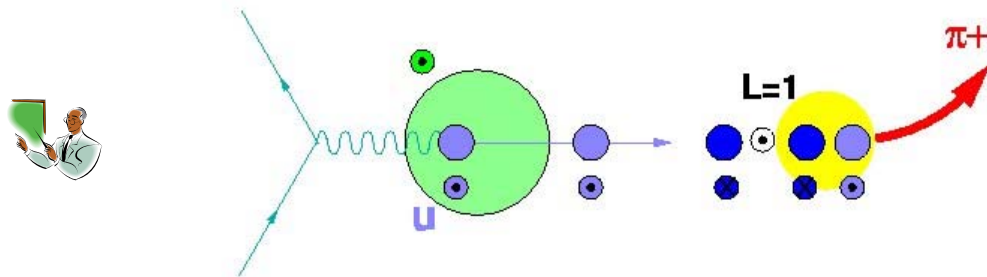
$$e p^{\uparrow} \rightarrow e' \pi^{\pm} X$$

Airapetian *et al.*, HERMES
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$$N^{\uparrow} = p^{\uparrow} = \{u^{\uparrow} d^{\downarrow} u^{\uparrow}\}$$

$$\vec{k}(\parallel \hat{z}) \times \vec{P}_h(\parallel \hat{x}) \cdot \vec{S}_T(\parallel \hat{y}) \rightarrow \sin \phi_C > 0$$



$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = 0 \end{array} \right\} \sin(\phi + \phi_S) > 0$$

$$u \text{ dominance} \rightarrow h_1^u H_1^{\perp(1)u} > 0$$

