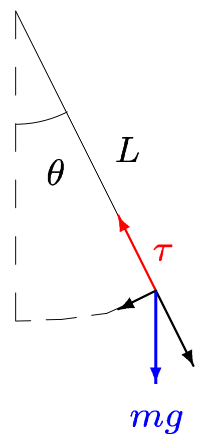
Invarianti adiabatici

Paul Ehrenfest (1880–1933)

per un oscillatore armonico (p.es. pendolo semplice):



$$\theta = \theta_0 \cos(\omega t + \phi), \qquad \omega = 2\pi\nu$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{L}}, \qquad E = \tfrac{1}{2} m g L \theta_0^2$$

$$\tau - mg\cos\theta = mL\dot{\theta}^2$$

i.e.

$$\begin{aligned} \tau &\simeq mg(1 - \tfrac{1}{2}\theta^2) + mL\dot{\theta}^2 \\ &= mg - mg\theta_0^2\left[\tfrac{1}{2}\cos^2(\omega t + \phi) - \sin^2(\omega t + \phi)\right] \end{aligned}$$

tensione media: $\overline{\tau} = mg + \frac{1}{4} mg\theta_0^2$

lavoro per accorciare di δL:

$$\begin{aligned} \delta W &= -\overline{\tau}\,\delta L \\ &= -mg\,\delta L - \tfrac{1}{4} mg\theta_0^2\,\delta L \end{aligned}$$

aumenta energia gravitazionale — aumenta energia di oscillazione

$$\Rightarrow \quad \frac{\delta E}{E} = -\frac{1}{2}\frac{\delta L}{L}$$

ma $\quad \delta\nu = -\frac{1}{2}\sqrt{\frac{g}{L}}\frac{\delta L}{L} = -\nu\frac{1}{2}\frac{\delta L}{L}$

$$\Rightarrow \quad \boxed{\frac{E}{\nu} = \text{costante}}$$