rappresentazione degli impulsi

- base dell'operatore impulso $\boldsymbol{p}$

$$\boldsymbol{p}\,|p\rangle = p\,|p\rangle, \qquad \langle p|p'\rangle = \delta(p-p')$$

$$\int |p\rangle\, dp\, \langle p| = \mathrm{I}$$

- stato $|f\rangle \Rightarrow$ rappresentativo $\langle p|f\rangle$

$$\langle p|f\rangle = \int \langle p|x\rangle\, dx\, \langle x|f\rangle = \frac{1}{(2\pi\hbar)^{1/2}} \int dx\, e^{-ikx}\, f(x)$$

trasformata di Fourier !

- operatore posizione $\boldsymbol{x}$:

$$\begin{aligned}
\boldsymbol{x}\,|f\rangle \to \langle p|\,\boldsymbol{x}\,|f\rangle \quad &= \int \langle p|\,\boldsymbol{x}\,|p'\rangle\, dp'\, \langle p'|f\rangle \\
&= \int \langle p|\,\boldsymbol{x}\,|x\rangle\, dx\, \langle x|f\rangle = \int dx\, x\, \langle p|x\rangle\langle x|f\rangle \\
&= i\frac{\partial}{\partial k}\, \frac{1}{(2\pi\hbar)^{1/2}} \int dx\, e^{-ikx}\, f(x) \\
&= i\frac{\partial}{\partial k}\, \langle p|f\rangle
\end{aligned}$$

$$\Rightarrow \quad \boxed{\langle p|\,\boldsymbol{x}\,|p'\rangle = i\frac{\partial}{\partial k}\,\delta(p-p')} \qquad \boxed{\boldsymbol{x} \to i\frac{\partial}{\partial k}}$$