

# Commutatore

$$[A, B] = AB - BA$$

p.es.  $[x, p]f(x) = x \left(-i\hbar \frac{d}{dx}\right) f(x) - \left(-i\hbar \frac{d}{dx}\right) x f(x) = i\hbar f(x)$

i.e.  $[x, p] = i\hbar$

proprietà:

- $[a, A] = 0$ ,  $a =$  numero complesso
- $[aA + bB, C] = a[A, C] + b[B, C]$ ,  $a, b =$  numeri complessi
- $[AB, C] = A[B, C] + [A, C]B$
- $[F(A), B] = \frac{dF(A)}{dA}[A, B]$  se  $[A, B] =$  c-numero

p.es.  $[A^n, B] = n A^{n-1}[A, B]$

- $[[A, B], C] + [[B, C], A] + [[C, A], B] = 0$   
(identità di Jacobi)

p.es.

$$H = \frac{p^2}{2m} + V(x, y, z) \rightarrow [H, x] = -\frac{i\hbar}{m} p_x \quad [H, p_x] = i\hbar \frac{\partial V}{\partial x}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \rightarrow [L_i, L_j] = i\hbar e_{ijk} L_k \quad [L^2, L_i] = 0$$