Vettori e operatori

- data base $\{|\boldsymbol{e}_i\rangle\} \in \mathcal{H}$,

$$|\boldsymbol{e}_1\rangle = \begin{vmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{vmatrix}, \quad |\boldsymbol{e}_2\rangle = \begin{vmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{vmatrix}, \quad \ldots, \quad |\boldsymbol{e}_n\rangle = \begin{vmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{vmatrix},$$

$$\forall\, |\boldsymbol{f}\rangle \in \mathcal{H}\colon\ |\boldsymbol{f}\rangle = \sum_i v_i\, |\boldsymbol{e}_i\rangle = \begin{vmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{vmatrix}$$

- operatore A:

$$A = \begin{vmatrix} A_{11} & A_{12} & \ldots & A_{1n} \\ A_{21} & A_{22} & \ldots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \ldots & A_{nn} \end{vmatrix}$$

- $|\boldsymbol{g}\rangle = A\, |\boldsymbol{f}\rangle$

$$\begin{vmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{vmatrix} = \begin{vmatrix} A_{11} & A_{12} & \ldots & A_{1n} \\ A_{21} & A_{22} & \ldots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \ldots & A_{nn} \end{vmatrix} \begin{vmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{vmatrix} \quad \Rightarrow \quad g_i = \sum_k A_{ik} f_k$$

- prodotto scalare: $\langle \boldsymbol{g} | \boldsymbol{f} \rangle = \sum_i g_i^*\, f_i$

$$= |g_1^* \quad g_2^* \quad \ldots \quad g_n^*| \cdot \begin{vmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{vmatrix}$$