Cambiamento di base e matrici unitarie

- $$|\boldsymbol{e}'_i\rangle = \sum_k U_{ki}\,|\boldsymbol{e}_k\rangle, \qquad U_{ki} = \langle \boldsymbol{e}_k|\boldsymbol{e}'_i\rangle$$

$$\forall|\boldsymbol{f}\rangle \in \mathcal{H}:\ |\boldsymbol{f}\rangle = \sum_k f_k\,|\boldsymbol{e}_k\rangle = \sum_i f'_i\,|\boldsymbol{e}'_i\rangle$$

$$f_k = \sum_i U_{ki}\,f'_i, \qquad f'_i = \sum_k U^\dagger_{ik}\,f_k$$

$$\boxed{U^\dagger = U^{-1}}$$

- $$U = e^{iA}, \qquad A^\dagger = A$$

infatti: $U^\dagger U = \mathrm{I} \rightarrow \sum_k U^\dagger_{ik}\,U_{kj} = \delta_{ij}$

nella rappresentazione in cui U è diagonale

$U_{ij} = a_i\,\delta_{ij}, \qquad U^\dagger_{ij} = a^*_i\,\delta_{ij}$

$\sum_k a^*_i\,\delta_{ik}\,a_j\,\delta_{kj} = a^*_i\,a_j\,\delta_{ij} \ \Rightarrow\ a^*_i\,a_i = 1$, i.e. $a_i = e^{i\alpha_i}$

con $\quad A\,|\boldsymbol{\alpha}_i\rangle = \alpha_i\,|\boldsymbol{\alpha}_i\rangle$

- $$A' = U^{-1}AU$$

$$\begin{aligned} A'_{ik} = \langle \boldsymbol{e}'_i|\,A\,|\boldsymbol{e}'_k\rangle &= \left\langle \textstyle\sum_l U_{li}\,\boldsymbol{e}_l\right|\,A\,\left|\textstyle\sum_{l'} U_{l'k}\,\boldsymbol{e}_{l'}\right\rangle \\ &= \sum_{l,l'} U^*_{li}\,\langle \boldsymbol{e}_l|\,A\,|\boldsymbol{e}_{l'}\rangle\,U_{l'k} \\ &= \sum_{l,l'} (U^\dagger)_{il}\,A_{ll'}\,U_{l'k} \end{aligned}$$