Diagonalizzazione di una matrice

- $A \to A', \quad A'_{ik} = \alpha_i \, \delta_{ik}$

$$A' = U^{-1} A U \;\; \Rightarrow \;\; (U^{-1} A U)_{ik} = \alpha_i \, \delta_{ik}$$

$$\underbrace{\sum_i U_{li}} \sum_{j,j'} \underbrace{(U^{-1})_{ij}} A_{jj'} \, U_{j'k} = \underbrace{\sum_i U_{li}} \alpha_i \, \underbrace{\delta_{ik}}$$

$$\sum_{j,j'} \delta_{lj} \, A_{jj'} \, U_{j'k} = \alpha_k \, U_{lk}$$

$$\sum_{j'} A_{lj'} \, U_{j'k} = \alpha_k \, U_{lk}$$

$$\sum_j \left(A_{lj} - \alpha_k \, \delta_{lj}\right) U_{jk} = 0$$

- condizione di solubilità: $\boxed{\det(A - \alpha \, \mathrm{I}) = 0}$

per ogni $\alpha \to \alpha_k$

$\Rightarrow \; A \, |\boldsymbol{\alpha}_k\rangle = \alpha_k \, |\boldsymbol{\alpha}_k\rangle$

$\Rightarrow \; U_{jk} = \langle \boldsymbol{e}_j | \boldsymbol{\alpha}_k \rangle$

$$U = \begin{vmatrix} \langle \boldsymbol{e}_1|\boldsymbol{\alpha}_1\rangle & \dots & \langle \boldsymbol{e}_1|\boldsymbol{\alpha}_k\rangle & \dots & \langle \boldsymbol{e}_1|\boldsymbol{\alpha}_n\rangle \\ \langle \boldsymbol{e}_2|\boldsymbol{\alpha}_1\rangle & \dots & \langle \boldsymbol{e}_2|\boldsymbol{\alpha}_k\rangle & \dots & \langle \boldsymbol{e}_2|\boldsymbol{\alpha}_n\rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \langle \boldsymbol{e}_n|\boldsymbol{\alpha}_1\rangle & \dots & \langle \boldsymbol{e}_n|\boldsymbol{\alpha}_k\rangle & \dots & \langle \boldsymbol{e}_n|\boldsymbol{\alpha}_n\rangle \end{vmatrix}$$