equazione di continuità

$\Psi^* \times$ (eq. S.) $- \Psi \times$ (eq. S.)*

$\Rightarrow \quad i\hbar \frac{\partial}{\partial t}|\Psi|^2 = -\frac{\hbar^2}{2m}(\Psi^* \nabla^2 \Psi - \Psi \nabla^2 \Psi^*)$

ma $\quad \boldsymbol{\nabla} \cdot (a \boldsymbol{\nabla} b) = \boldsymbol{\nabla} a \cdot \boldsymbol{\nabla} b + a \nabla^2 b$

$$\Rightarrow \quad \boxed{\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0}$$

$$\boxed{\rho = |\Psi|^2, \quad \boldsymbol{j} = -\frac{i\hbar}{2m}(\Psi^* \boldsymbol{\nabla} \Psi - \Psi \boldsymbol{\nabla} \Psi^*)}$$

normalizzazione

se $\lim_{r \to \infty} \Psi(\boldsymbol{r}, t) = 0$

$\Rightarrow \quad \frac{d}{dt} \int d\boldsymbol{r} |\Psi(\boldsymbol{r}, t)|^2 = 0$

$$\text{i.e.} \quad \boxed{\int d\boldsymbol{r} |\Psi(\boldsymbol{r}, t)|^2 = N}$$

$\Psi(\boldsymbol{r}, t) \in \mathcal{L}^2(R^3)$ funzione a quadrato sommabile

opportuno porre $N = 1 \Rightarrow \quad \Psi$ normalizzata