Verso l'equazione di Schrödinger

- particella libera

approssimazione non relativistica: $E \simeq m_0 c^2 + \frac{p^2}{2m}$

$$\omega = \frac{E}{\hbar} = \frac{m_0 c^2}{\hbar} + \frac{\hbar k^2}{2m} \equiv \frac{m_0 c^2}{\hbar} + \omega' \qquad (\omega' \ll m_0 c^2)$$

$$\Rightarrow \Psi(\boldsymbol{r}, t) = e^{-i m_0 c^2 t/\hbar} \underbrace{\int d\boldsymbol{k}\, A(\boldsymbol{k})\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega' t)}}_{\Psi'(\boldsymbol{r}, t)}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi(\boldsymbol{r}, t) = e^{-i m_0 c^2 t/\hbar} \underbrace{\frac{m_0^2 c^2}{\hbar^2}}\Psi'(\boldsymbol{r}, t)$$

ma anche

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi(\boldsymbol{r}, t)$$

$$= e^{-i m_0 c^2 t/\hbar}\left(\nabla^2 + \underbrace{\frac{m_0^2 c^2}{\hbar^2}} + \frac{2 i m_0}{\hbar}\frac{\partial}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi'(\boldsymbol{r}, t)$$

$$\Rightarrow \quad \boxed{\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\right)\Psi(\boldsymbol{r}, t) = 0}$$

velocità di fase: $v_f = \frac{\omega'}{k} = \frac{\hbar k}{2m} \neq v = \frac{\hbar k}{m}$

velocità di gruppo: $v_g = \frac{\partial \omega'}{\partial k} = \frac{\hbar k}{m} \equiv v$