Inclusive radiative B decays: an update

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Rare B decays: an express way to New Physics in the Flavour sector

Rare processes are interesting when their suppression is associated to some conservation law

Flavor Changing Neutral Currents have no tree level SM sensitive to new degrees of freedom (e.g., $H^+$) and to new sources of flavor violation (e.g., gluino FCNC)

Inclusive rare B decays allow precision tests (OPE+RGE improved pert theory) of the SM and its extensions
The present situation

- Exp: $\text{BR}(B \rightarrow X_s \gamma) = (3.55 \pm 0.24 \pm 0.09 \pm 0.03) \times 10^{-4}$
  
  latest HFAG $E_\gamma > 1.6$ GeV

- Present SM prediction at NLO + leading EW, nonpert
  
  $\text{BR}(B \rightarrow X_s \gamma, E_\gamma > 1.6$ GeV$) = (3.60 \pm 0.30) \times 10^{-4}$  PG,Misiak

- Need to match $\sim 5\%$ exp error at end of B factories:
  
  NNLO calculation under way
b → s transitions

\[ \Lambda_{\text{QCD}} \ll m_b < M_W \]

Large \( L = \log \frac{m_b}{M_W} \) must be resummed.

LO: \( \alpha_s^n L^n \), NLO: \( \alpha_s^n L^{n-1} \)

Tower of local ops

OPE

But many more operators appear adding gluons

\[ m_b \ll M_W \]

Inclusive decays are described by OPE (except charm loop contributions!)

The current is not conserved and runs between \( M_W \) and \( m_b \)

We have AT LEAST 3 scales

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Why inclusive?

$\Lambda_{QCD} \ll m_b$: inclusive decays admit systematic expansion in $\Lambda_{QCD}/m_b$
Non-pert corrections are generally small and can be controlled

Hadronization probability = 1 because we sum over all states
Approximately insensitive to details of meson structure as $\Lambda_{QCD} \ll m_b$
The main ingredients

Process independent:

• The Wilson coefficients $C_i$ (encode the short distance information, initial conditions)
• The Anomalous Dimension Matrix (mixing among operators, determines the evolution of the coefficients, allowing to resum large logs)

Process dependent: matrix elements

$B \rightarrow X_s \gamma$: NLO QCD calculation completed, all results checked, EW, power corrections

$B \rightarrow X_s \ll$: NNLO & EW calculation just completed, power corrections
Error anatomy of $BR_{\gamma}$

$$BR \left[ \bar{B} \to X_s \gamma \right]_{E_\gamma > 1.6 \text{ GeV}} = (3.61 \pm 0.30) \times 10^{-4},$$

$$= 3.61 \times 10^{-4} \left( 1 \pm 0.06 \left( \frac{m_c}{m_b} \text{ in } K_c \right) \pm 0.04 \text{ (other NNLO)} \right)$$

$$\pm 0.01 \text{ (pert C)} \pm 0.02 \lambda_1 \pm 0.02 \Delta$$

$$\pm 0.02 \alpha_s(M_Z) \pm 0.02 BR(\text{semilept})_{\text{exp}} \pm 0.01 m_t$$

Total error 8% dominated by charm mass
Can be partially resolved by NNLO

Misiak, PG 2001
The charm mass problem

$m_c$ enters the phase factor due to normalization

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma[\bar{B} \to X_{ce\bar{v}}]}{\Gamma[\bar{B} \to X_{ue\bar{v}}]} = 0.581 \pm 0.017$$

Misiak & PG

and the NLO matrix elements

As LO diagrams vanish, the definition of $m_c$ is a NNLO issue. Numerically very important because these are large NLO contributions:

$m_c(m_c) = 1.25 \pm 0.10 \text{ GeV} \quad m_c(m_b) = 0.85 \pm 0.11 \text{ GeV} \quad m_c(\text{pole}) \sim 1.5 \text{ GeV}$

But pole mass has nothing to do with these loops

Changing $m_c/m_b$ from 0.29 (pole) to 0.22 (MSbar) increases $BR_\gamma$ by 11%

0.22 ±0.04 gives DOMINANT 6% theory error
Photon spectrum vs total BR

The OPE does not predict the spectrum, only its global properties: the higher the cut the higher the uncertainty.

Conversely, constraining the HQE parameters constrains the possible shape functions.

Possible subleading shape functions effects in $V_{ub}$ applications.

The shape function gets renormalized by perturbative effects: some implications may be better understood in SCET (Bauer & Manohar, Neubert et al).
Universality: spectrum of $B \to X_s \gamma$

Motion of $b$ quark inside $B$ and gluon radiation smear the spike at $m_b/2$

The photon spectrum is very insensitive to new physics, can be used to study the $B$ meson structure

$$<E_\gamma> = m_b/2 + \ldots \quad \text{var}<E_\gamma> = \mu^2/12 + \ldots$$

Importance of extending to $E_\gamma^{\text{min}} \sim 1.8 \text{ GeV}$ or less for the determination of both the BR AND the HQE parameters

Bigi Uraltsev

Belle: lower cut at 1.8 GeV

Info from radiative spectrum compatible with semileptonic moments

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Global fit to $|V_{cb}|$, $\text{BR}_{sl}$, HQE

Result of fit to all moment measurements:

$|V_{cb}| @ 2\%$

$m_b < 1\%$

$m_c @ 5\%$

In MS scheme:

$m_b(m_b) = 4.20 \pm 0.04 \text{ GeV}$

$m_c(m_c) = 1.24 \pm 0.07 \text{ GeV}$

$m_c(\mu)/m_b(\mu) = 0.235 \pm 0.012$

courtesy of N.Uraltsev

Good agreement with other similar analyses:

Bauer et al. hep-ph/0408002
DELPHI hep-ex/0510024
More cuts problems

\[ \mu_h \sim m_b \]
\[ \mu_i \sim \sqrt{\Delta m_b} \]
\[ \mu_0 \sim \Delta = m_b - 2E_{\text{cut}} \]

The lower photon energy cut \( E_{\text{cut}} \) introduces two new scales

Need to disentangle 3 scales \( \Rightarrow \) MultiScaleOPE

QCD \( \Rightarrow \) SCET \( \Rightarrow \) HQET \( \Rightarrow \) local OPE

EVEN when local OPE works fine: terms \( \alpha_s(\Delta) \) could be large
The NNLO spectrum (dominant part)

NNLO calculation very close to BLM

Non-BLM corrections change BR$_{\gamma}$ by 0.5%

Situation seems under control

NNLO calculation very close to BLM
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\[ z = \frac{2E_{\gamma}}{m_b} \]

Melnikov & Mitov 2005
Contributions to the NNLO analysis:

Three-loop matching for $O_7$ and $O_8$:

Three-loop mixing in the $(O_1,\ldots,O_6)$ and $(O_7, O_8)$ sectors:

Four-loop mixing $(O_1,\ldots,O_6) \rightarrow (O_7, O_8)$:
M. Czakon, U. Haisch, MM, in progress

Two-loop matrix elements of $O_7$ and $O_8$ (and bremsstrahlung):
H.M. Asatrian, T. Ewerth, C. Greub, T. Hurth, A. Hovhannisyan, V. Poghosyan, to be published
A. Feroglia, P. Gambino, in progress

Three loop matrix elements of $O_1$ and $O_2$ (and bremsstrahlung):
M. Steinhauser, MM, to be published (interpolation in $m_c$)
Theoretical accuracy beyond SM

• Status of theory predictions in models of new physics not always satisfactory

• New enhancement factors in higher orders, eg $\tan \beta$,

• complete NLO results available in the 2HDMs

• in MSSM only partial NLO results so far, mostly because of new sources of flavor violation
Flavor mixing in the squark sector

6x6 squark mass matrices in the super-CKM basis

\[
\mathcal{M}_{\tilde{u}}^2 = \begin{pmatrix}
V_{\text{CKM}} \hat{m}_{\tilde{Q}}^2 V_{\text{CKM}}^\dagger + m_u^2 + D_{\tilde{u}_{LL}} & v_2 \hat{T}_U - \mu^* m_u \cot \beta \\
v_2 \hat{T}_U - \mu m_u \cot \beta & \hat{m}_{\tilde{u}}^2 + m_u^2 + D_{\tilde{u}_{RR}}
\end{pmatrix}
\]

\[
\mathcal{M}_{\tilde{d}}^2 = \begin{pmatrix}
\hat{m}_{\tilde{Q}}^2 + m_d^2 + D_{\tilde{d}_{LL}} & v_1 \hat{T}_D - \mu^* m_d \tan \beta \\
v_1 \hat{T}_D - \mu m_d \tan \beta & \hat{m}_{\tilde{d}}^2 + m_d^2 + D_{\tilde{d}_{RR}}
\end{pmatrix}
\]

New flavor violation can arise in the **soft SUSY-breaking terms**

A disalignment induces **gluino** and **neutralino** flavor-changing tree-level interactions
Minimal Flavor Violation

Under **Minimal Flavor Violation** (MFV) the only source of flavor violation is the CKM matrix.

- the squark mass matrices are flavor diagonal in the super-CKM basis
- gluino and neutralino interactions conserve flavor
- alignment is spoiled by EW corrections
- alignment can be imposed only at a scale $\mu_{\text{MFV}}$
LO contributions in the MSSM
NLO corrections in the MSSM with MFV

MSSM contributions encoded in the Wilson coefficients of effective operators $Q_{7,8}$ etc

NLO $O(\alpha_s)$ corrections to the Wilson coeffs:

- gluonic corrections to $H^+$ loops Ciuchini et al, Borzumati&Greub
- gluonic corrections to $\chi^+$ loops Ciuchini et al, Bobeth et al
- $\tan \beta$ enhanced corrections for heavy gluino DGG, Carena et al
- $\tan \beta$ enhanced corrections to $H^+$ loops Borzumati et al
- Log $m_W/M_{\text{susy}}$ enhanced contributions DGG

New calculation of remaining gluino contributions for arbitrary values of superpartner masses

Degrassi, PG, Slavich hep-ph/0601135

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Two-loop diagrams with gluino and H$^+$ or W

goes beyond leading tan$\beta$ and beyond heavy gluino
Two-loop diagrams with gluino and chargino

New NLO $\chi$ contributions prop to $\tan \beta$
The calculation

- NDR scheme, background gauge, first 2 gen degenerate
- standard non-FC $O(\alpha_s)$ renormalization (masses, LR mixing)
- EW renormalization of the FC gluino-down quark-squark couplings and external legs:
  MSbar subtraction introduces $\mu_{\text{MFV}}$, “on-shell” subtraction is scale independent and corresponds approx to MFV at $M_{\text{susy}}$
- several analytic and numerical checks
Numerical results

Assume MFV in MSbar at $\mu_{MFV} = 500$ GeV. Two sets:

(I) $m_Q = 230$ GeV, $m_T = 210$ GeV, $m_B = 260$ GeV, $A_t = -70$ GeV, $A_b = 0$, $m_{H^\pm} = 350$ GeV, $m_\tilde{g} = M_2 = 200$ GeV, $\mu = 250$ GeV, $\tan \beta = 30$

(II) $m_Q = 480$ GeV, $m_T = 390$ GeV, $m_B = 510$ GeV, $A_t = -560$ GeV, $A_b = -960$, $m_{H^\pm} = 430$ GeV, $m_\tilde{g} = 600$ GeV, $M_2 = 190$ GeV, $\mu = 390$ GeV, $\tan \beta = 10$

To study decoupling we rescale all SUSY masses by common increasing factor
Full calculation vs effective lagrangian (I)
Full calculation vs effective Lagrangian (II)
Dependence on the MFV scale (II)

Large logs must be resummed!
Realistic setting

In a realistic MSSM setting alignment holds at a very high scale $\mu_{\text{MFV}}$. If RGE induced $b\to s$ flavor violation is small:

1. start from flavor diagonal squark mass matrices at $\mu_{\text{MFV}}$
2. evolve quark and squark matrices down to $\mu_{\text{SUSY}}$: flavor mixing is generated
3. diagonalize mass matrices at low scale $\mu_{\text{SUSY}}$
4. compute LO gluino and neutralino $b\to s\gamma$
5. add LO+NLO SM,2HDM,chargino contributions using MFV at $\mu_{\text{SUSY}}$. This ensures proper resummation,neglecting QCD corrections to FC gluino contributions
Summary

• NNLO calculation of inclusive radiative B decays is necessary and under way

• NLO calculation of SUSY-QCD contributions to $b \rightarrow s\gamma$ completed assuming alignment at scale $\mu_{MFV}$

• for relatively light superpartners results may differ significantly from effective lagrangian approach

• public code soon to be available
**b-->sl^+l^- : a more complicated case**

This decay mode is sensitive to different operators, hence to different new physics.

Here large logs are generated even without QCD: LO $\alpha_s^n L^{n+1}$, NLO $\alpha_s^n L^n$, ...

However, numerically the leading log is subdominant, yielding an awkward series:

in BR $1 + 0.7 (\alpha_s) + 5.5 (\alpha_s^2) + ...$
Electroweak corrections to $b \rightarrow s l^+ l^-$

This decay is suppressed by two e.w. couplings wrt $bs\gamma$:
$$\text{BR} \sim \alpha(\mu)^2 \quad \mu = m_b \text{ or } M_W? \quad \text{Difference is 8%!}$$

Lowest order diagrams

Two sources of large QED logs:
- running of $\alpha$
- and of operators

LO and NLO EW effects studied, New $O(\alpha), O(\alpha\alpha_s)$ and $O(\alpha_s)$ ADM required
**Error Anatomy for $BR_{ll}$**

\[
BR_{\ell\ell} \left( 1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2 \right) = \\
\left[ 1.574 \pm^{0.106}_{0.100} \left| M_t \right| \pm^{0.059}_{0.075} \text{scale} \pm^{0.045}_{0.035} C \pm^{0.072}_{0.067} BR_{sl} \pm^{0.001}_{0.013} \left| m_b \right| \pm^{0.001}_{0.013} \left| m_c \right| \right] \times 10^{-6}
\]

Bobeth, PG, Gorbahn, Haisch

- $M_{top}$ dominant error 7%
- scale uncertainty 5%
- $m_b^{pole} = 4.80 \pm 0.15$ GeV $\rightarrow 5$
- phase space factor 3%
- No $m_c$ issue as charm enters at LO

**TOTAL ERROR $\sim 10\%$**

**BUT:** bottom uncertainty is not a fundamental limitation

$\delta m_b^{short \ distance} \approx 30-50 \text{ MeV}$

simply change scheme!

**EXP:** only inclusive rate,
Belle (140 fb$^{-1}$): $(4.4\pm0.8\pm0.8) \times 10^{-6}$
Babar (80 fb$^{-1}$): $(5.6\pm1.5\pm1.3) \times 10^{-6}$

We get $(4.6\pm0.8) \times 10^{-6}$ ($m_{ll} > 0.2 \text{ GeV}$)
Model independent constraints on new physics from $b \to s$ incl decays

Haisch, Misiak, PG, PRL 2005

\[ \text{BR}_\gamma \sim |\tilde{C}^{\text{eff}}_7(\mu_b)|^2 \]

To have large NP contributions, reverse sign $C_7$

\[ \frac{d\Gamma[B \to X_s l^+ l^-]}{d\hat{s}} \sim (1 - \hat{s})^2 \left\{ (1 + 2\hat{s}) \left ( |\tilde{C}^{\text{eff}}_9|^2 + |\tilde{C}^{\text{eff}}_{10}|^2 \right ) + \left ( 4 + \frac{8}{\hat{s}} \right ) |\tilde{C}^{\text{eff}}_7|^2 + 12 \text{Re} \left ( \tilde{C}^{\text{eff}}_7 \tilde{C}^{\text{eff}*}_9 \right ) \right \} \]

$C_{9,10}$ related to $(s_L \gamma_\alpha b_L)(l \gamma^\alpha l) (s_L \gamma_\alpha b_L)(l \gamma^\alpha \gamma_5 l)$

If $C_{9,10}$ get small non-standard contributns (large $\tan\beta$, large $(\delta_{23} d)_{LR}$ ...) $\text{BR}_{bsll}$ fixes sign $C_7$

\[ \text{BR}(B \to X_s ll) \text{ in the low } q^2 \text{ window (1-6GeV}^2) \]
state of the art NNLO calculations, Bobeth et al

Exp(wa) : $1.60 \pm 0.51 \times 10^{-6}$

Theory:

$\text{SM } C_7: 1.57 \pm 0.16 \times 10^{-6}$

$\text{-SM } C_7: 3.30 \pm 0.25 \times 10^{-6}$

Non standard $C_7$ sign disfavored at $3\sigma$

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90\% CL constraints on $C_{9,10}$ from $B\to X_s\gamma$ and $B\to X_s\ell\ell$

Typical MFV susy contributions make up for the difference Ali et al

Different $BR_\gamma$ values

Rare decays place important constraints on MSSM paramtrns, relevant for neutralino dark matter studies

Likely implications (large stop mixing disfavored)
Extracting $C_9$ and $C_{10}$ from $B \to K^* \ell^+\ell^-$

- Precise determination of $C_9$ and $C_{10}$ is possible
- $\Delta C_9/C_9 \sim 11\%$, $\Delta C_{10}/C_{10} \sim 13\%$ at 5 $ab^{-1}$, $C_7$ fixed from $b \to s\gamma$
- Current branching fraction / background extrapolated
- Fit to 2-dim $q^2$ vs angular distribution, not simple $A_{FB}$
- Systematic error is neglected

![Graph showing Belle MC, 5 $ab^{-1}$](image)
B→X_sγ: a new physics killer

It is the best measured rare decay
Good agreement with SM strongly constrains most new physics models

\[ \text{Exp: } B(B \to X_s \gamma) = (3.55 \pm 0.24) \times 10^{-4} \]

\[ \text{SM: } B(B \to X_s \gamma) = (3.61 \pm 0.30) \times 10^{-4} \quad (E_{\gamma} > 1.6 \text{ GeV}) \]

charged Higgs mass bounds in type II 2HDM

Does not carry over to MSSM!
But very strong bounds there too