

Riassunto della lezione precedente

- verifica di QPM in reazioni elettrodeboli, regole di somma :
 1. Gottfried sum rule \rightarrow $SU_f(3)$ rotta per i quark del mare
 2. Momentum sum rule \rightarrow gluoni portano metà del momento del nucleone
- verifica fattorizzazione tra sez. d'urto elementare e densità partoniche
universalità di densità partoniche \rightarrow esplorare nuovi processi fondamentali
- Drell-Yan (DY) : cinematica, formule generali; QPM picture \rightarrow test $SU(3)_c$
- test sperimentali del QPM in DY :
scaling di sezione d'urto in s ; rapporto di carica su nuclei isoscalari
distribuzione angolare della coppia leptonica; violazione Lam-Tung sum rule
K factor
- e^+e^- inclusivo: formalismo e interpretazione in QPM
scaling della sezione d'urto totale; rapporto $R \rightarrow$ test di $SU_c(3)$ e $SU_f(N_f)$

e^+e^- semi-inclusivo adronico

crossing con DIS inclusivo

particelle ultrarelativistiche

$$k^\pm{}^\mu = (E, 0, 0, \mp E)$$

$$q^\mu = (2E, \mathbf{0})$$

$$P_h^\mu = (E_h, E_h \sin \theta, 0, E_h \cos \theta)$$

invarianti

$$z = \frac{2P_h \cdot q}{q^2} \sim \frac{4EE_h}{4E^2} = \frac{E_h}{E}$$

\sim “ x_B^{-1} ”, misura elasticità

$$E_h \sim 0 \quad z \rightarrow 0$$

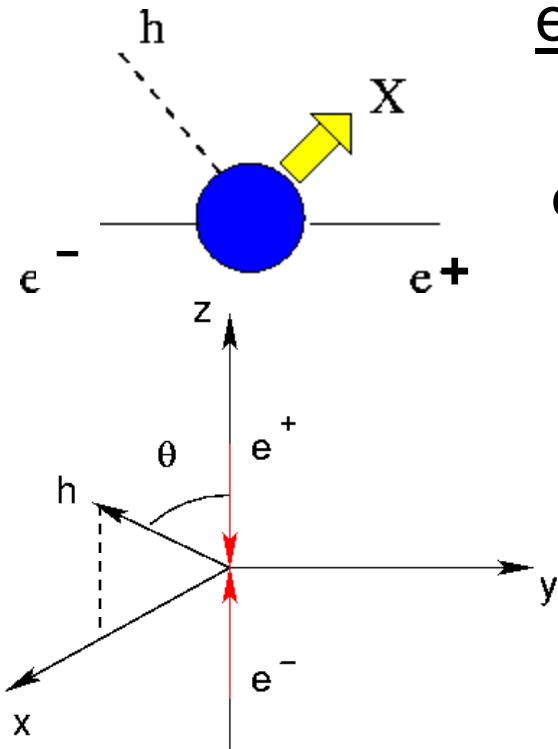
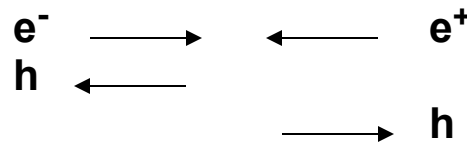
$$E_h \sim E \quad z \rightarrow 1$$

processo
elastico

$$y = \frac{P_h \cdot k'}{P_h \cdot q} \sim \frac{EE_h}{2EE_h} (1 + \cos \theta) = \frac{1}{2} (1 + \cos \theta) \sim \text{rapidity}$$

$$y = 0 \rightarrow \theta = \pi$$

$$y = 1 \rightarrow \theta = 0$$



e^+e^- semi-inclusivo adronico (continua)

$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 dR \quad \mathcal{F} = 4\sqrt{(k \cdot k')^2 - k^2 k'^2} \stackrel{\text{TRF}}{=} 2Q^2 \equiv 2s$$

$$dR = (2\pi)^4 \delta(q - P_X - P_h) \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \frac{d\mathbf{P}_h}{(2\pi)^3 2E_h}$$

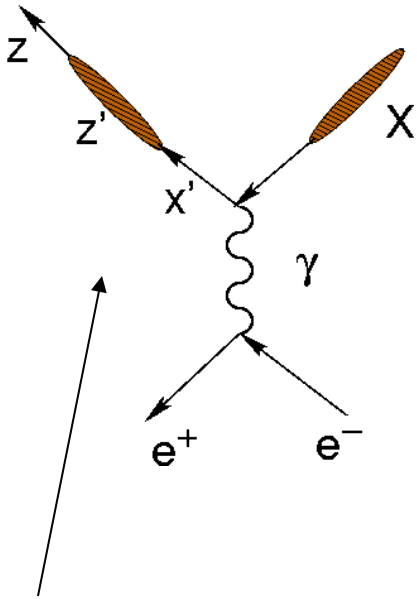
$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H_{1PI}^{\mu\nu}$$

$$\begin{aligned} 2E_h \frac{d\sigma}{d\mathbf{P}_h} &= \frac{1}{2} \frac{1}{2Q^2} \frac{1}{(2\pi)^2} \frac{e^4}{Q^4} L_{\mu\nu} \frac{1}{2\pi} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(q - P_X - P_h) H_{1PI}^{\mu\nu} \\ &= \frac{\alpha^2}{Q^6} L_{\mu\nu} W_{1PI}^{\mu\nu} \end{aligned}$$

N.B. cross-check $\sigma = \int \frac{d\mathbf{P}_h}{2E_h} 2E_h \frac{d\sigma}{d\mathbf{P}_h} = \frac{4\pi^2 \alpha^2}{Q^6} L_{\mu\nu} W^{\mu\nu}$



QPM picture



regime DIS : $Q^2 \rightarrow \infty$ con $z = \frac{2P_h \cdot q}{Q^2}$ finito

$$\begin{aligned} \frac{d\sigma}{dydz} &= N_c \sum_{f=q,\bar{q}} \int_0^1 dx' dz' \frac{d\sigma^{el}}{dx'dy} (e^+ e^- \rightarrow q\bar{q}) \\ &\quad \times D_f(z') \delta(z'x' - z) \\ &= N_c \sum_{f=q,\bar{q}} e_f^2 \int_0^1 dx' dz' \frac{d\sigma^{el}}{dy} (e^+ e^- \rightarrow \mu^+ \mu^-) \\ &\quad \times \delta(1 - x') D_f(z') \delta(z'x' - z) \end{aligned}$$

elastico \nearrow



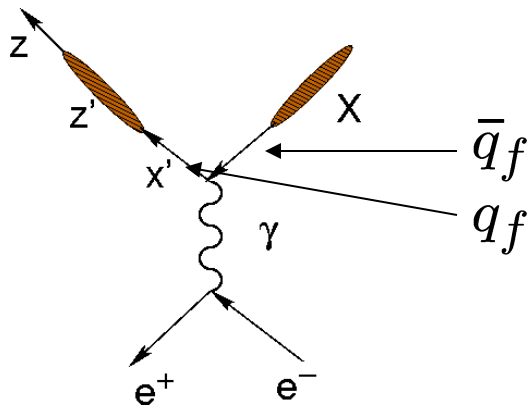
$$= N_c \frac{\pi\alpha^2}{Q^2} (1 + \cos^2 \theta) \sum_{f=q,\bar{q}} e_f^2 D_f(z)$$

probabilità di trovare adrone con frazione z' del momento x' del partone.

somma su tutte le combinazioni $x' z' \rightarrow$ adrone con frazione z dell' energia disponibile

commenti





conservazione del sapore nel vertice
 → stesso f per (anti)quark \Leftarrow Born approx.

correzioni ad ordini superiori da, e.g., g che decade in coppie → f' di antiquark $\neq f$ di quark

$$\frac{d\sigma}{dydz} = N_c \frac{\pi\alpha^2}{Q^2} (1 + \cos^2 \theta) \sum_{f=q,\bar{q}} e_f^2 D_f(z)$$

evidenza del colore

nuova incognita
 fattorizzazione
 → estrarre info da
 confronto con i dati

distribuzione angolare
 dell'adrone è data da
 una sez. d'urto di QED
 ($e^+e^- \rightarrow \mu^+\mu^-$) !

Fenomenologia e^+e^- semi-inclusivo

$$\sigma_{incl} = \sigma(e^+e^- \rightarrow X) = N_c \frac{4\pi\alpha^2}{3Q^2} \sum_f e_f^2$$

$$\sigma_{\mu\mu} = \int_0^1 dy \frac{d\sigma}{dy}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{\pi\alpha^2}{Q^2} \int_0^1 dy (1 + \cos^2 \theta) = \frac{4\pi\alpha^2}{3Q^2}$$

$$\frac{d\sigma}{dz} = \int_0^1 dy \frac{d\sigma}{dzdy}(e^+e^- \rightarrow hX) = N_c \frac{4\pi\alpha^2}{3Q^2} \sum_{f=q,\bar{q}} e_f^2 D_f(z)$$

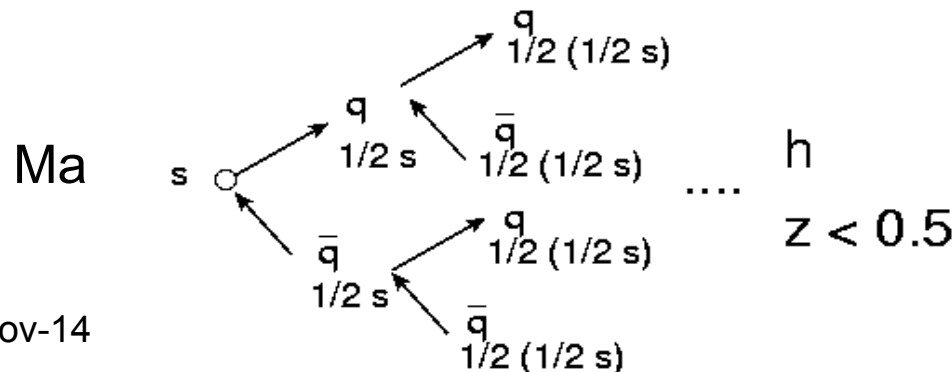


$$\frac{1}{\sigma_{\mu\mu}} \frac{d\sigma}{dz} = N_c \sum_f e_f^2 (D_f(z) + D_{\bar{f}}(z))$$

info su frammentazione

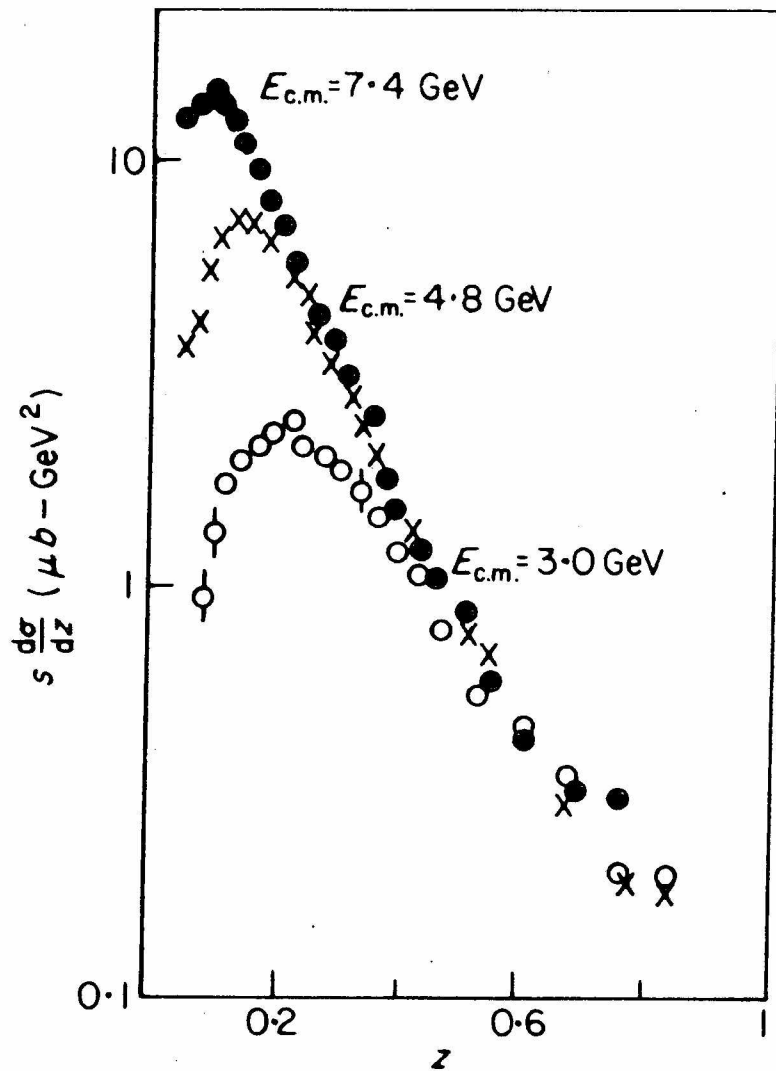
$$Q^2 \frac{d\sigma}{dz} = \frac{4\pi\alpha^2}{3} N_c \sum_f e_f^2 (D_f(z) + D_{\bar{f}}(z))$$

scaling in $z \forall Q^2 \equiv s$



violazione per $z \leq 0.5$

$$s \frac{d\sigma}{dz}(e^+e^- \rightarrow hX)$$



Schwitters *et al.*,
P.R.L. **35** 1320 (75)

FIG. 12.9. $s(d\sigma/dz)$ at 3, 4.8, and 7.4 GeV e^+e^- centre of mass energies.

e^+e^- semi-inclusivo \leftarrow fattorizzazione \rightarrow DIS ?

$$\frac{d\sigma}{dydz} = N_c \frac{\pi\alpha^2}{Q^2} (1 + \cos^2 \theta) \sum_f e_f^2 (D_f(z) + D_{\bar{f}}(z))$$

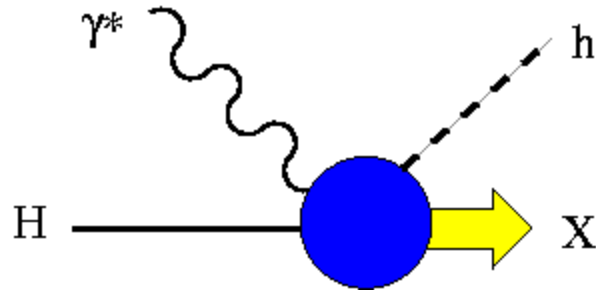


nuova incognita

fattorizzazione \rightarrow estrarre info da confronto con i
dati anche in altri processi

\Rightarrow DIS semi-inclusivo (SIDIS)

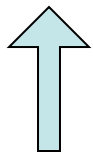
Semi-inclusive DIS (SIDIS)



stesse definizioni del caso inclusivo per cinematica e invarianti con in più

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

$z > 0$ P_h equiverso a $P \rightarrow h$ viene da frammentazione di partone



current fragmentation region

$z < 0$ P_h opposto a $P \rightarrow h$ viene da frammentazione del bersaglio H
target fragmentation region

N.B. in TRF $z \sim \frac{ME_h}{M\nu} = \frac{E_h}{\nu}$

SIDIS

$$\mathcal{F} = 4\sqrt{(P \cdot k)^2 - P^2 k^2} \stackrel{\text{TRF}}{=} 4ME \equiv 2s$$

$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 dR$$

$$dR = (2\pi)^4 \delta(k + P - k' - P_X - P_h) \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \frac{d\mathbf{k}}{(2\pi)^3 2E'} \frac{d\mathbf{P}_h}{(2\pi)^3 2E_h}$$

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H_{1PI}^{\mu\nu} \begin{cases} L_{\mu\nu} = 2(k_\mu k'_\nu + k'_\mu k_\nu - k \cdot k' g_{\mu\nu}) \\ H_{1PI}^{\mu\nu} = \frac{1}{2} \sum_{SS_h} \langle PS | J^\mu | P_X, P_h S_h \rangle \langle P_X, P_h S_h | J^\nu | PS \rangle \end{cases}$$

$$\begin{aligned} 2E_h \frac{d\sigma}{d\mathbf{P}_h dE' d\Omega} &= \frac{1}{4ME} \frac{E'}{16\pi^3} \frac{e^4}{Q^4} L_{\mu\nu} \frac{1}{(2\pi)^3} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(k + P - k' - P_X - P_h) H_{1PI}^{\mu\nu} \\ &= \frac{\alpha^2}{Q^4} \frac{E'}{E} L_{\mu\nu} \frac{1}{2M} 2M W_{1PI}^{\mu\nu} \end{aligned}$$



N.B. cross-check $\frac{d\sigma}{dE' d\Omega} = \int \frac{d\mathbf{P}_h}{2E_h} 2E_h \frac{d\sigma}{d\mathbf{P}_h dE' d\Omega} = \frac{\alpha^2}{Q^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}$

SIDIS (continua)

$$\frac{d\mathbf{k}'}{(2\pi)^3 2E'} = \frac{E' dE' d\Omega}{16\pi^3} = \frac{\pi y dx_B dy}{16\pi^3}$$

$$\frac{d\mathbf{P}_h}{2E_h} \sim d\mathbf{P}_{h\perp} \frac{dE_h}{2E_h} \sim d\mathbf{P}_{h\perp} \frac{dz}{2z}$$

$$\frac{d\sigma}{d\mathbf{P}_{h\perp} dx_B dy dz} = \frac{\pi\alpha^2}{Q^4} \frac{y}{2z} L_{\mu\nu} 2M W_{1PI}^{\mu\nu}$$



$$\frac{d\sigma}{dx_B dy dz} = \frac{\pi\alpha^2}{Q^4} \frac{y}{2z} L_{\mu\nu} 2M \int d\mathbf{P}_{h\perp} W_{1PI}^{\mu\nu}$$



cruciale per fattorizzazione tra
distribuzione e frammentazione

QPM picture

$$\begin{aligned} \frac{d\sigma}{dx_B dy dz} &= \sum_{f=q, \bar{q}} \int_0^1 dx' dz' \phi_f(x') \frac{d\sigma^{(e^- q \rightarrow e^- q)}}{dx' dy} \left(\frac{x'}{x_B} \right) D_f(z') \delta \left(z - z' \frac{x'}{x_B} \right) \\ &= \frac{4\pi\alpha^2 s}{Q^4} \left(\frac{y^2}{2} + 1 - y \right) x_B \sum_{f=q, \bar{q}} e_f^2 \phi_f(x_B) D_f(z) \end{aligned}$$



N.B. cross-check

$$\begin{aligned} \frac{d\sigma}{dx_B dy} &= \int dz \frac{d\sigma}{dx_B dy dz} \Big|_{D_f(z)=\delta(1-z)} \\ &= \frac{4\pi\alpha^2 s}{Q^4} \left(\frac{y^2}{2} + 1 - y \right) x_B \sum_{f=q, \bar{q}} e_f^2 \phi_f(x_B) = \frac{4\pi\alpha^2 s}{Q^4} \left(1 - y + \frac{y^2}{2} \right) F_2 \end{aligned}$$

Callan-Gross
 $F_2 = 2 x_B F_1$

DIS inclusivo

SIDIS

$$e^- p \rightarrow e'^- h^\pm X$$

$$\frac{1}{\sigma_{incl}} \frac{d\sigma}{dx_B dy dz} = \frac{\sum_f e_f^2 \phi_f(x_B) D_f(z)}{\sum_f e_f^2 \phi_f(x_B)} \sim D_u^h$$

e^+e^- semi-inclusivo

$$e^+ e^- \rightarrow h^\pm X$$

$$\frac{1}{\sigma_{incl}} \frac{d\sigma}{dz} = \frac{\sum_f e_f^2 D_f(z)}{\sum_f e_f^2} \sim D_u^h + D_{\bar{u}}^h$$

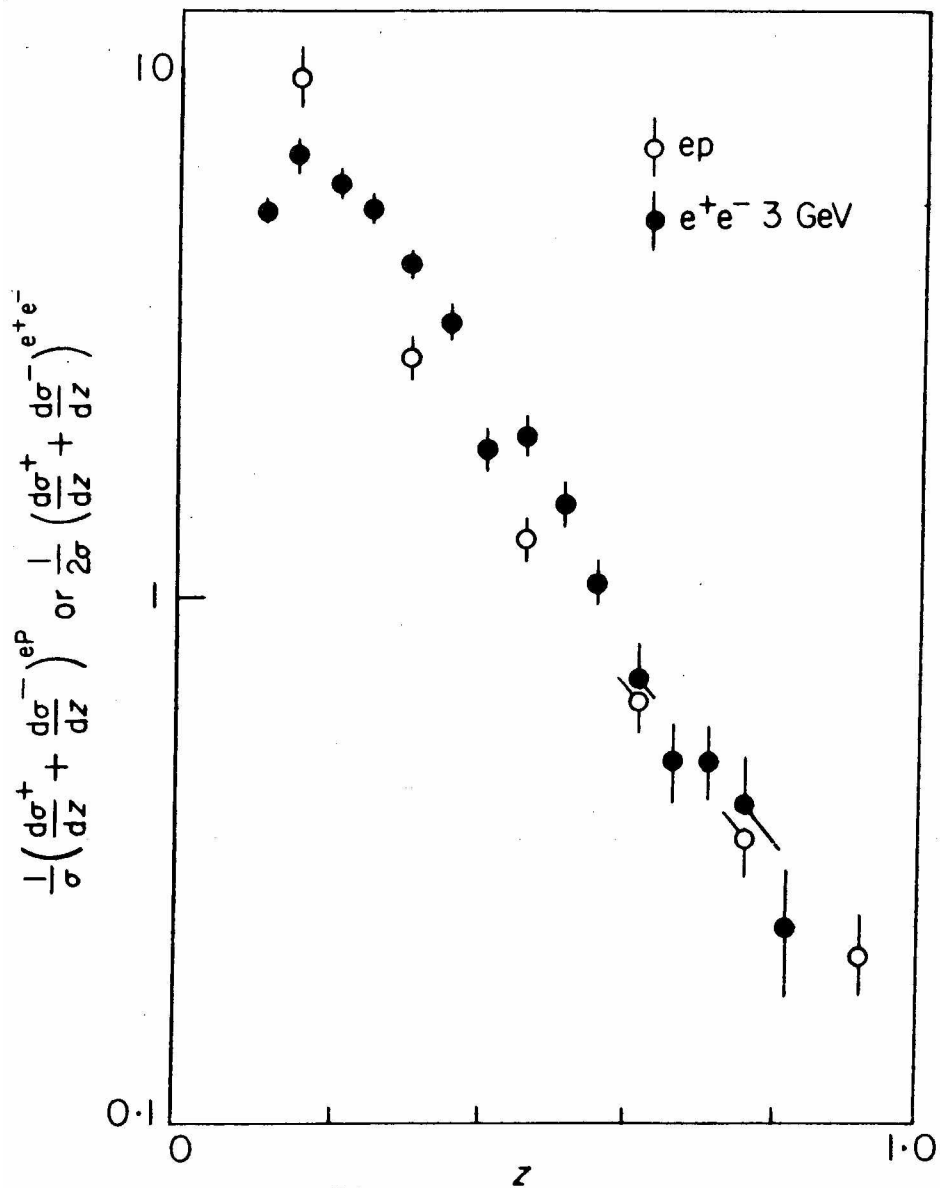
dominanza u quark

quindi

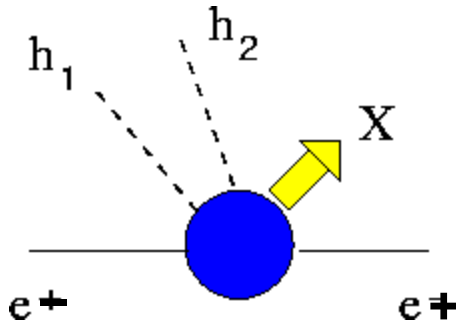
$$\frac{1}{\sigma_{incl}} \left(\frac{d\sigma^{h^+}}{dx_B dy dz} + \frac{d\sigma^{h^-}}{dx_B dy dz} \right) \Leftrightarrow \frac{1}{\sigma_{incl}} \frac{1}{2} \left(\frac{d\sigma^{h^+}}{dz} + \frac{d\sigma^{h^-}}{dz} \right) \sim D_u^{h^+} + D_{\bar{u}}^{h^-}$$



Gilman, Int. Symp. on lepton
and photon interactions
at high energies,
SLAC (75)



e^+e^- semi-inclusivo in due adroni



$$z_1 = \frac{2P_1 \cdot q}{q^2} \quad z_2 = \frac{2P_2 \cdot q}{q^2}$$

$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 dR \quad |\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H_{2PI}^{\mu\nu}$$

$$\mathcal{F} \stackrel{\text{TRF}}{=} 2Q^2 \equiv 2s$$

$$dR = (2\pi)^4 \delta(q - P_X - P_1 - P_2) \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \frac{d\mathbf{P}_1}{(2\pi)^3 2E_1} \frac{d\mathbf{P}_2}{(2\pi)^3 2E_2}$$

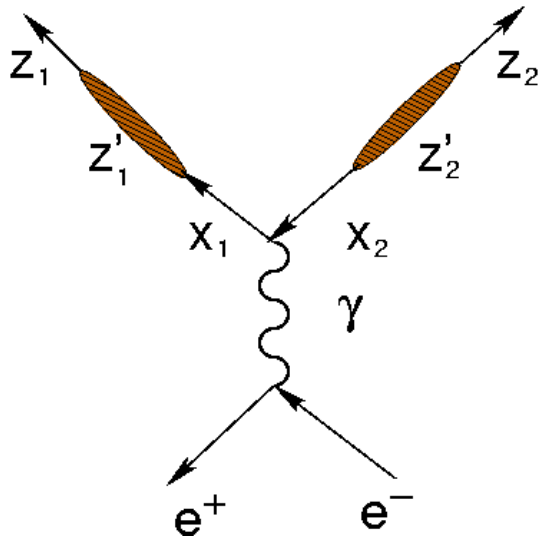
$$\begin{aligned} 2E_1 2E_2 \frac{d\sigma}{d\mathbf{P}_1 d\mathbf{P}_2} &= \frac{1}{2} \frac{1}{2Q^2} \frac{1}{(2\pi)^2} \frac{e^4}{Q^4} L_{\mu\nu} \\ &\quad \times \frac{1}{(2\pi)^4} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(q - P_X - P_1 - P_2) H_{2PI}^{\mu\nu} \\ &= \frac{\alpha^2}{Q^6} L_{\mu\nu} W_{2PI}^{\mu\nu} \end{aligned}$$



N.B. cross-check $\int \frac{d\mathbf{P}_2}{2E_2} 2E_1 2E_2 \frac{d\sigma}{d\mathbf{P}_1 d\mathbf{P}_2} = 2E_1 \frac{d\sigma}{d\mathbf{P}_1} = \dots = \frac{\alpha^2}{Q^6} L_{\mu\nu} W_{1PI}^{\mu\nu}$

QPM picture

$$\begin{aligned}
 \frac{d\sigma}{dydz_1dz_2} &= N_c \sum_{f=q,\bar{q}} \int_0^1 dx'_1 dx'_2 dz'_1 dz'_2 \frac{d\sigma^{el}}{dx'_1 dx'_2 dy} (e^+ e^- \rightarrow q\bar{q}) \\
 &\quad \times D_f(z'_1) \delta(z_1 - z'_1 x'_1) D_f(z'_2) \delta(z_2 - z'_2 x'_2) \\
 &= N_c \sum_f e_f^2 \int_0^1 dx'_1 dx'_2 dz'_1 dz'_2 \frac{d\sigma^{el}}{dy} (e^+ e^- \rightarrow \mu^+ \mu^-) \\
 &\quad \times \delta(1 - x'_1) D_f(z'_1) \delta(z_1 - z'_1 x'_1) \\
 &\quad \times \delta(1 - x'_2) D_f(z'_2) \delta(z_2 - z'_2 x'_2) \\
 &= N_c \frac{\pi\alpha^2}{Q^2} (1 + \cos^2 \theta) \sum_{f=q,\bar{q}} e_f^2 D_f(z_1) D_f(z_2)
 \end{aligned}$$



N.B. cross-check $D_f(z_2) = \delta(1 - z_2)$ secondo adrone \equiv jet adronico

$$\int dz_2 \frac{d\sigma}{dydz_1dz_2} \Big|_{D_f(z_2)=\delta(1-z_2)} = \frac{d\sigma}{dydz} = N_c \frac{\pi\alpha^2}{Q^2} (1 + \cos^2 \theta) \sum_f e_f^2 D_f(z)$$

e⁺e⁻ semi-inclusivo

Adesso $D_f(z_1) = \delta(1 - z_1)$ anche primo adrone \equiv jet adronico

$$\int dz \frac{d\sigma}{dydz} \Big|_{D_f(z)=\delta(1-z)} = \frac{d\sigma^{jet}}{dy} = N_c \frac{\pi\alpha^2}{Q^2} (1 + \cos^2 \theta) \sum_f e_f^2$$

distribuzione angolare di tutti gli adroni
nello stato finale
=
sezione d'urto di jet

$$\frac{d\sigma^{el}}{dy}(e^+e^- \rightarrow q\bar{q}) = \frac{\pi\alpha^2}{Q^2} \underline{(1 + \cos^2 \theta)} e_f^2$$

$$\frac{d\sigma}{dydz}(e^+e^- \rightarrow hX) = N_c \frac{\pi\alpha^2}{Q^2} \underline{(1 + \cos^2 \theta)} \sum_f e_f^2 D_f(z)$$

$$\frac{d\sigma}{dy}(e^+e^- \rightarrow \text{jets}) = N_c \frac{\pi\alpha^2}{Q^2} \underline{(1 + \cos^2 \theta)} \sum_f e_f^2$$

gli adroni sono “frammenti” dei partoni a spin $\frac{1}{2}$ del processo elementare
 eventi a molti adroni = gruppi di adroni con p_T limitato rispetto ad un certo asse

dato asse θ , sfericità $S = \frac{3 \sum_i p_{Ti}^2}{2 \sum_i p_i^2} \begin{cases} S = 1 & \text{sfera} \\ S = 0 & \text{jet} \end{cases} \quad S \xrightarrow{s \rightarrow \infty} 0$

adrone in stato finale con $0 \leq z \leq 1$ si muove in un jet che rappresenta la direzione θ del quark di frammentazione rispetto all'asse z

la direzione del jet è data da processo elementare di QED !