

Riassunto della lezione precedente

- OPE su DIS inclusivo; operatore di correlazione q - q Φ al leading twist coinvolge tre strutture indipendenti: Parton Distribution Functions (PDF); interpretazione probabilistica; impossibilità di estensione al subleading twist
- nuova PDF: densità di polarizzazione trasversa, o trasversità; caratteristiche molto differenti da elicità: chiral-odd, evoluzione disgiunta dai gluoni, carica tensoriale scala con Q^2 ed ottenuta da elemento di matrice di operatore locale C-odd
- trasversità è PDF chiral-odd \rightarrow soppressa in DIS inclusivo
strategia per estrazione dai dati: serve partner chiral-odd a leading twist
 - Drell-Yan completamente polarizzato \rightarrow soppresso
 - SIDIS con π finali in cinematica collineare \rightarrow soppresso
 - SIDIS con Λ finali " " " \rightarrow quale meccanismo?
 - SIDIS con π finali e misura del P_T
- dipendenza da momenti trasversi e naïve T-rev. odd
 \rightarrow nuove strutture in Φ

Decomposizione di Δ al leading twist

Base di matrici di Dirac $\{\mathbf{1}, \gamma^\mu, \gamma^\mu \gamma_5, i\gamma_5, i\sigma^{\mu\nu} \gamma_5\}$

$$\Delta(k, P_h, S_h) = \frac{1}{2} [S \mathbf{1} + V_\mu \gamma^\mu + A_\mu \gamma^\mu \gamma_5 + iP \gamma_5 + iT_{\mu\nu} \sigma^{\mu\nu} \gamma_5]$$

$$S = \frac{1}{2} \text{Tr}(\Delta) = C_1(k^2, k \cdot P_h)$$

$$V^\mu = \frac{1}{2} \text{Tr}(\gamma^\mu \Delta) = C_2 P_h^\mu + C_3 k^\mu + C_{10} \epsilon_{\mu\nu\rho\sigma} S_h^\nu P_h^\rho k^\sigma$$

$$A^\mu = \frac{1}{2} \text{Tr}(\gamma^\mu \gamma_5 \Delta) = C_4 S_h^\mu + C_5 p \cdot S_h P_h^\mu + C_6 P_h \cdot S_h k^\mu$$

$$P_5 = \frac{1}{2i} \text{Tr}(\gamma_5 \Delta) = C_{11} k \cdot S_h$$

$$T^{\mu\nu} = \frac{1}{2i} \text{Tr}(\sigma^{\mu\nu} \Delta) = C_7 P_h^{[\mu} S_h^{\nu]} + C_8 k^{[\mu} S_h^{\nu]} + C_9 k \cdot S_h P_h^{[\mu} k^{\nu]} + C_{12} \epsilon_{\mu\nu\rho\sigma} P_h^\rho k^\sigma$$

$$\text{Tr} [\dots \gamma^-] \rightarrow \Delta[\gamma^-] = \sum_X \int \frac{d\zeta^+}{2\pi} e^{iP_h^- \zeta^+ / z} \langle 0 | \psi_f(\zeta^+) | P_h S_h, X \rangle \langle P_h S_h, X | \bar{\psi}_f(0) \gamma^- | 0 \rangle$$

$$\text{Tr} [\dots \gamma^- \gamma_5] \rightarrow \Delta[\gamma^- \gamma_5] = \sum_X \int \frac{d\zeta^+}{2\pi} e^{iP_h^- \zeta^+ / z} \langle 0 | \psi_f(\zeta^+) | P_h S_h, X \rangle \langle P_h S_h, X | \bar{\psi}_f(0) \gamma^- \gamma_5 | 0 \rangle$$

$$\text{Tr} [\dots \gamma^- \gamma^i \gamma_5] \rightarrow \Delta[i\sigma^{i-} \gamma_5] = \sum_X \int \frac{d\zeta^+}{2\pi} e^{iP_h^- \zeta^+ / z} \langle 0 | \psi_f(\zeta^+) | P_h S_h, X \rangle$$

Correlatore con momento trasverso intrinseco

$$\Delta^{[\Gamma]}(z, \mathbf{P}_{hT}, S_h) = \frac{1}{4z} \int dk^+ \text{Tr} [\Delta(k, P_h, S_h) \Gamma] \Big|_{k^- = P_h^- / z}$$



Proiezioni al leading twist

$$D_1 = \bullet \longrightarrow \bigcirc \qquad D_{1T}^\perp = \left(\bullet \longrightarrow \bigcirc \xrightarrow{\uparrow} - \bullet \longrightarrow \bigcirc \xrightarrow{\uparrow} \right)$$

$$\Delta^{[\gamma^-]}(z, \mathbf{P}_{hT}, S_h) = D_1(z, \mathbf{P}_{hT}^2) + \frac{(\mathbf{k}_T \times \mathbf{S}_{hT})_z}{M_h} D_{1T}^\perp(z, \mathbf{P}_{hT}^2)$$

$$G_{1L} = \left(\bullet \xrightarrow{\rightarrow} \bigcirc \xrightarrow{\rightarrow} \right) - \left(\bullet \xrightarrow{\leftarrow} \bigcirc \xrightarrow{\rightarrow} \right) \qquad G_{1T} = \left(\bullet \xrightarrow{\rightarrow} \bigcirc \xrightarrow{\uparrow} \right) - \left(\bullet \xrightarrow{\leftarrow} \bigcirc \xrightarrow{\uparrow} \right)$$

$$\Delta^{[\gamma^- \gamma_5]}(z, \mathbf{P}_{hT}, S_h) = \lambda_h G_{1L}(z, \mathbf{P}_{hT}^2) + \frac{\mathbf{k}_T \cdot \mathbf{S}_{hT}}{M_h} G_{1T}^\perp(z, \mathbf{P}_{hT}^2)$$

$$H_{1T} = \left(\bullet \xrightarrow{\uparrow} \bigcirc \xrightarrow{\uparrow} \right) - \left(\bullet \xrightarrow{\downarrow} \bigcirc \xrightarrow{\uparrow} \right) \qquad H_1 = \left(\bullet \xrightarrow{\uparrow} \bigcirc \right) - \left(\bullet \xrightarrow{\downarrow} \bigcirc \right)$$

$$\Delta^{[i\sigma^{i-} \gamma_5]}(z, \mathbf{P}_{hT}, S_h) = S_{hT}^i H_{1T}(z, \mathbf{P}_{hT}^2) + \frac{(\hat{\mathbf{n}}_T \times \mathbf{k}_T)_i}{M_h} H_1^\perp(z, \mathbf{P}_{hT}^2)$$

$$+ \frac{k_T^i}{M_h} \left[\lambda_h H_{1L}^\perp(z, \mathbf{P}_{hT}^2) + \frac{\mathbf{k}_T \cdot \mathbf{S}_{hT}}{M_h} H_{1T}^\perp(z, \mathbf{P}_{hT}^2) \right]$$

$$H_{1L}^\perp = \left(\bullet \xrightarrow{\uparrow} \bigcirc \xrightarrow{\rightarrow} \right) - \left(\bullet \xrightarrow{\downarrow} \bigcirc \xrightarrow{\rightarrow} \right) \qquad H_{1T}^\perp = \left(\bullet \xrightarrow{\uparrow} \bigcirc \xrightarrow{\rightarrow} \right) - \left(\bullet \xrightarrow{\downarrow} \bigcirc \xrightarrow{\rightarrow} \right)$$

Decomposizione di Φ al leading twist

Base di matrici di Dirac $\{\mathbf{1}, \gamma^\mu, \gamma^\mu \gamma_5, i\gamma_5, i\sigma^{\mu\nu} \gamma_5\}$

$$\Phi(p, P, S) = \frac{1}{2} [S \mathbf{1} + V_\mu \gamma^\mu + A_\mu \gamma^\mu \gamma_5 + iP \gamma_5 + iT_{\mu\nu} \sigma^{\mu\nu} \gamma_5]$$

$$S = \frac{1}{2} \text{Tr}(\Phi) = C_1(p^2, p \cdot P)$$

$$V^\mu = \frac{1}{2} \text{Tr}(\gamma^\mu \Phi) = C_2 P^\mu + C_3 p^\mu + C_{10} \epsilon_{\mu\nu\rho\sigma} S^\nu P^\rho p^\sigma$$

$$A^\mu = \frac{1}{2} \text{Tr}(\gamma^\mu \gamma_5 \Phi) = C_4 S^\mu + C_5 p \cdot S P^\mu + C_6 P \cdot S p^\mu$$

$$P_5 = \frac{1}{2i} \text{Tr}(\gamma_5 \Phi) = C_{11} p \cdot S$$

$$T^{\mu\nu} = \frac{1}{2i} \text{Tr}(\sigma^{\mu\nu} \Phi) = C_7 P^{[\mu} S^{\nu]} + C_8 p^{[\mu} S^{\nu]} + C_9 p \cdot S P^{[\mu} p^{\nu]} + C_{12} \epsilon_{\mu\nu\rho\sigma} P^\rho p^\sigma$$

C_{10} è vincolato da T-rev. invar. e $\epsilon_{\mu\nu\rho\sigma} S^\nu P^\rho p^\sigma$: $\mu, \rho = +/- \Rightarrow \nu, \sigma = i (=1,2)$

\Rightarrow coinvolge momenti trasversi partonici (\mathbf{p}_\perp)

idem per $C_{12} \epsilon_{\mu\nu\rho\sigma} P^\rho p^\sigma$

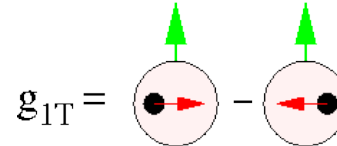
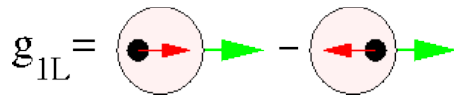
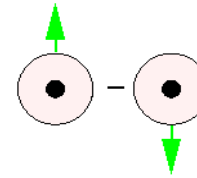
PDF dipendenti da momento trasverso intrinseco

Proiezioni al leading twist

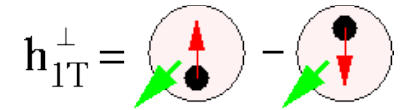
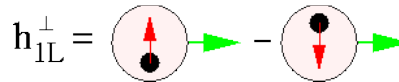
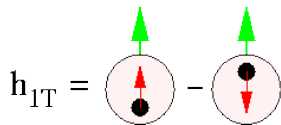
$$\Phi^{[\Gamma]}(x, \mathbf{p}_T, S) = \int dp^- \text{Tr} [\Phi(p, P, S) \Gamma] \Big|_{p^+ = xP^+}$$

$$f_1 = \text{diagram}$$

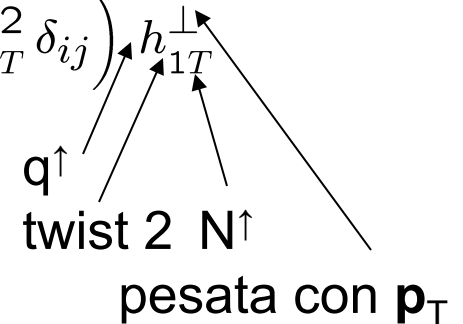
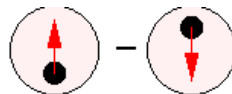
$$\Phi^{[\gamma^+]}(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{\mathbf{p}_T \times \mathbf{S}_T \cdot \hat{\mathbf{P}}}{M} f_{1T}^\perp(x, \mathbf{p}_T^2)$$



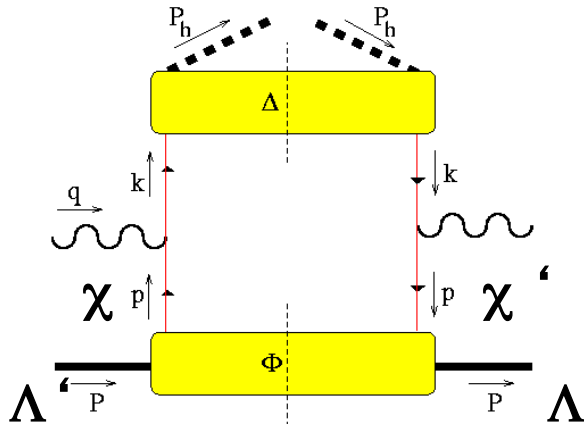
$$\Phi^{[\gamma^+ \gamma_5]}(x, \mathbf{p}_T, S) = \lambda g_{1L}(x, \mathbf{p}_T^2) + \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{p}_T^2)$$



$$\Phi^{[i\sigma^i + \gamma_5]}(x, \mathbf{p}_T, S) = S_T^i h_1 + \frac{p_T^i}{M} \lambda h_{1L}^\perp + \frac{S_T^i}{M^2} \left(p_T^i p_T^j - \frac{1}{2} \mathbf{p}_T^2 \delta_{ij} \right) h_{1T}^\perp + \frac{(\mathbf{p}_T \times \hat{\mathbf{P}})_i}{M} h_1^\perp$$



Rappresentazione di elicità di $\Phi(x, \mathbf{p}_T, S)$



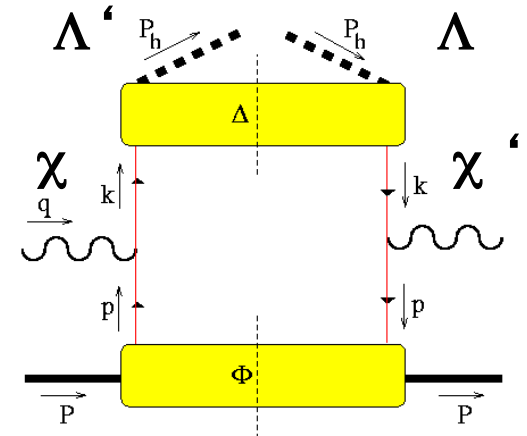
| PDF | chiral even | | chiral-odd |
|-----------------------|------------------|-------------------|-----------------------|
| | q non pol. | q^{\rightarrow} | q^{\uparrow} |
| H non pol. | f_1 | | h_1^{\perp} |
| $H^{\rightarrow} = L$ | | g_{1L} | h_{1L}^{\perp} |
| $H^{\uparrow} = T$ | f_{1T}^{\perp} | g_{1T} | h_1, h_{1T}^{\perp} |

naïve T-even

naïve T-odd

$$h_1 = h_{1T} + h_{1T}^{\perp} \frac{\vec{p}_T^2}{M^2}$$

Rappresentazione di elicità di $\Delta(z, \mathbf{P}_{hT}, S_h)$

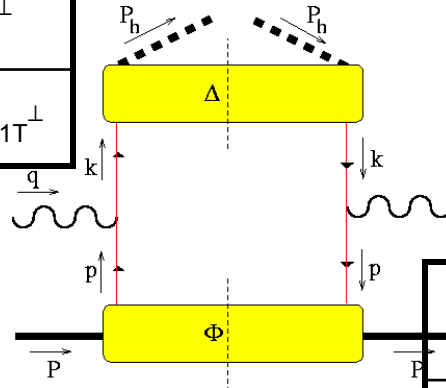


| PFF | chiral even | | chiral-odd |
|-----------------------|---------------|-------------------|--------------------|
| | q non pol. | q^{\rightarrow} | q^{\uparrow} |
| H non pol. | D_1 | | $H_{1\perp}$ |
| $H^{\rightarrow} = L$ | | G_{1L} | $H_{1L\perp}$ |
| $H^{\uparrow} = T$ | $D_{1T\perp}$ | G_{1T} | $H_1, H_{1T\perp}$ |

naïve T-even

naïve T-odd

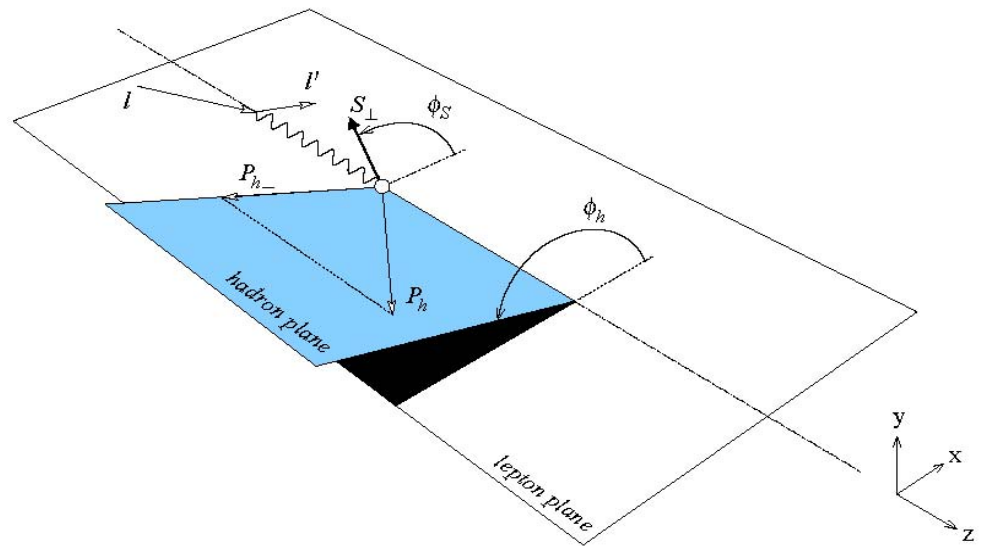
| PDF | chiral even | chiral-odd |
|---------------------|----------------|---------------------|
| | q non pol. | q^\uparrow |
| H non pol. | D_1 | H_1^\perp |
| $H^\rightarrow = L$ | | H_{1L}^\perp |
| $H^\uparrow = T$ | D_{1T}^\perp | H_1, H_{1T}^\perp |



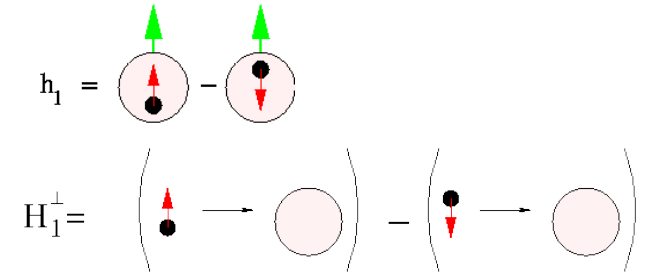
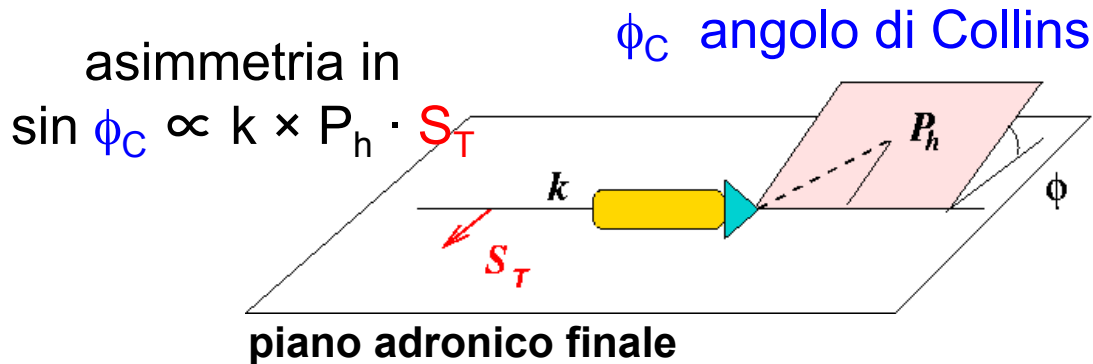
| PDF | chiral even | chiral-odd |
|---------------------|----------------|---------------------|
| | q non pol. | q^\uparrow |
| H non pol. | f_1 | h_1^\perp |
| $H^\rightarrow = L$ | | h_{1L}^\perp |
| $H^\uparrow = T$ | f_{1T}^\perp | h_1, h_{1T}^\perp |



$$e p^\uparrow \rightarrow e' \pi^\pm X$$

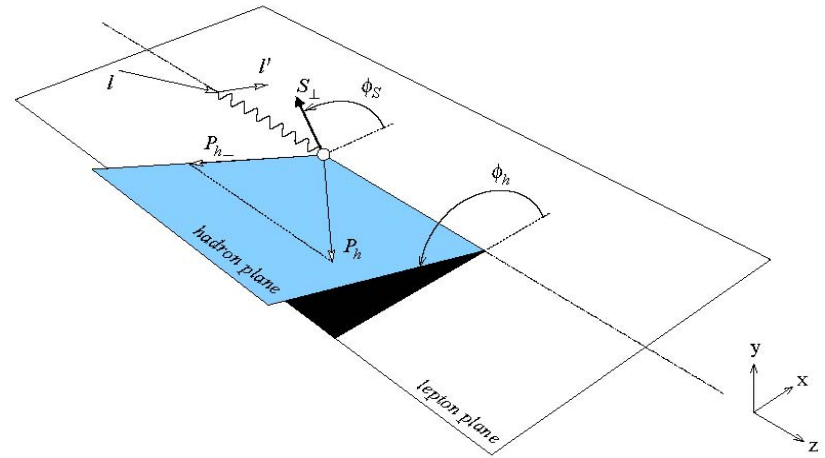


$$\frac{d^6 \sigma_{OT}}{dx dy dz d\phi_S d\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2} \sum_{f, \bar{f}} e_f^2 \left\{ A(y) \mathcal{F} \left[f_1^f(x, \mathbf{p}_T^2) D_1^f(z, \mathbf{P}_{hT}^2) \right] \right. \\ \left. + |\mathbf{S}_T| B(y) \underbrace{\sin(\phi_h + \phi_S)}_{\phi_C} \mathcal{F} \left[\frac{\mathbf{k}_T \cdot \hat{\mathbf{P}}_{h\perp}}{M_h} h_1^f(x, \mathbf{p}_T^2) H_1^{\perp f}(z, \mathbf{P}_{hT}^2) \right] \right\}$$



$$e p^\uparrow \rightarrow e' \pi^\pm X$$

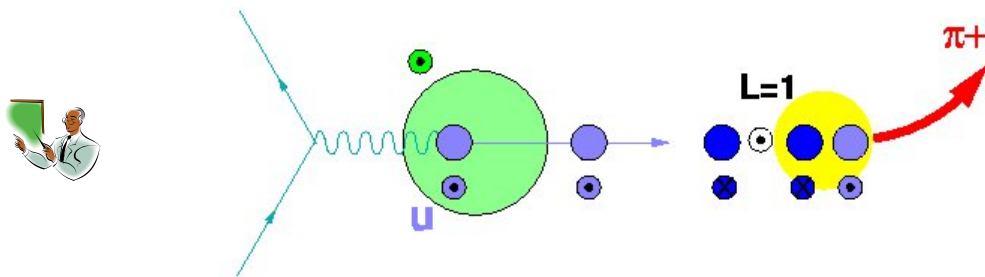
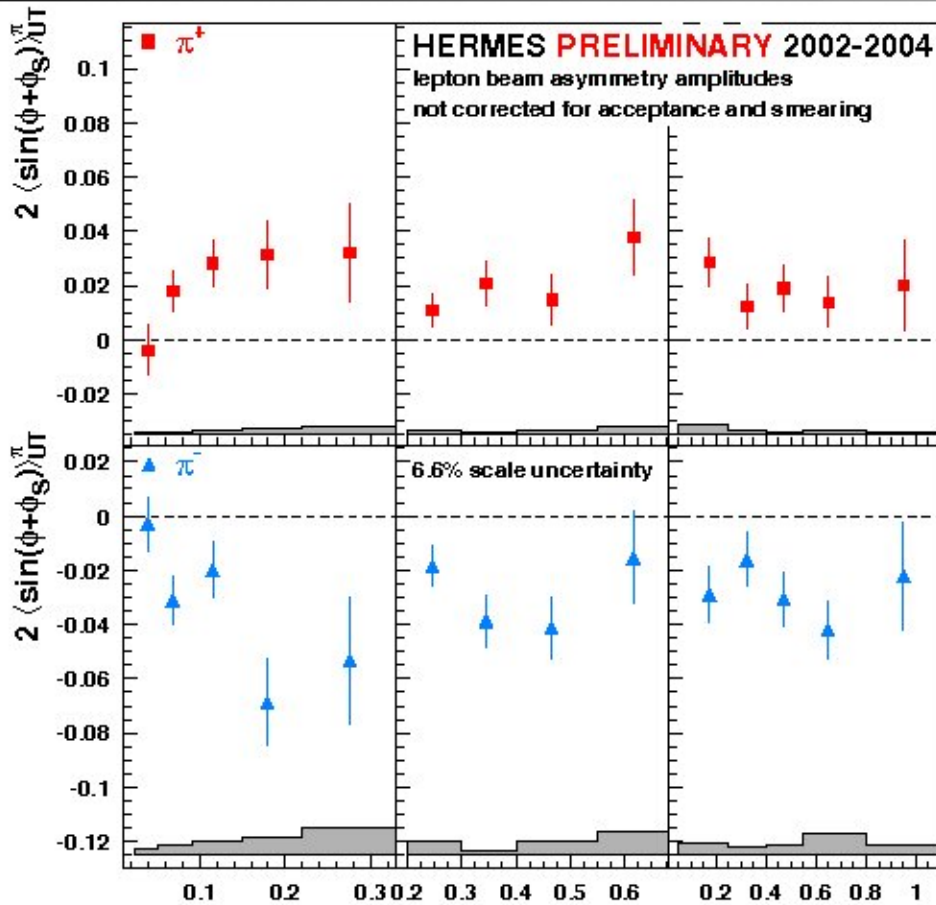
Airapetian *et al.*, HERMES
P.R.L. **94** (05) 012002



$$N^\uparrow = p^\uparrow = \{u^\uparrow d^\downarrow u^\uparrow\}$$

$$\vec{k}(\parallel \hat{z}) \times \vec{P}_h(\parallel \hat{x}) \cdot \vec{S}_T(\parallel \hat{y}) \rightarrow \sin \phi_C > 0$$

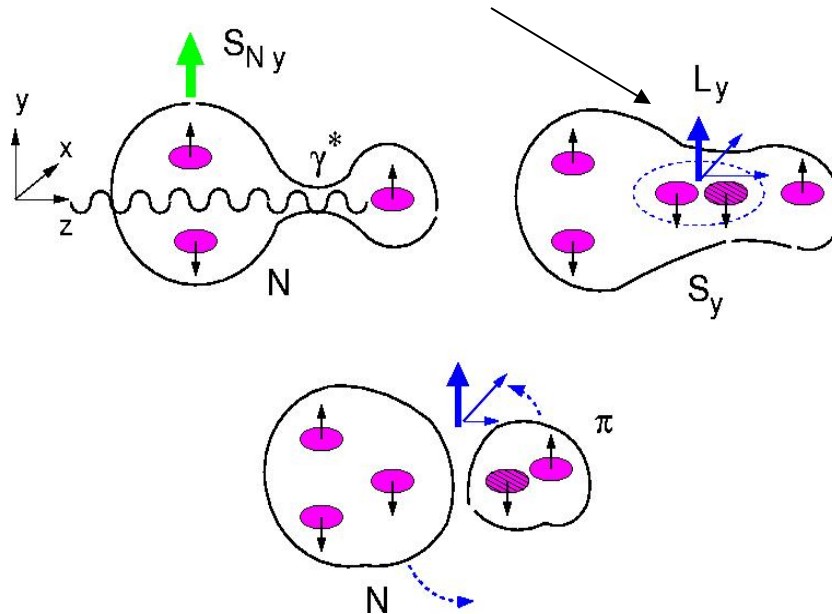
$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = 0 \end{array} \right\} \sin(\phi + \phi_S) > 0$$



Congettura semi-classica : poichè γ^* colpisce $q\uparrow$ si forma una stringa di forza di colore; quando la stringa si rompe, si forma un quarkonio con spin 1 e momento angolare orbitale opposto; tale momento angolare orbitale e determina l'asimmetria azimutale nell' emissione dell' adrone finale

(Artru , hep-ph/9310323)

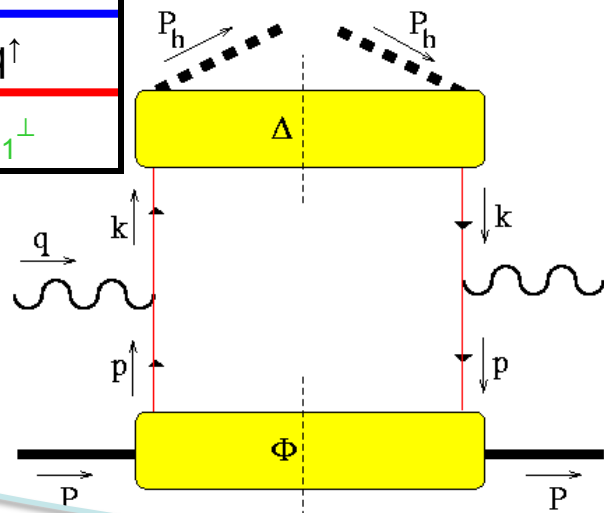
quantici vuoto $J^{PC} = 0^{++}$
 quarkonio ha $S=1 \rightarrow 2S+1L_J = 3P_0$



K. Bruhnel - TJNAF - 2003

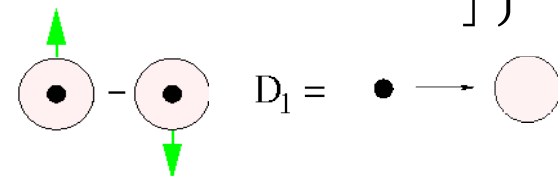
effetto Sivers in SIDIS $e p^\uparrow \rightarrow e' \pi^\pm X$

| Δ | chiral | even | chiral-odd |
|----------|--------|-----------------|--------------|
| | q | q \rightarrow | q \uparrow |
| π | D_1 | | H_1^\perp |



| Φ | chiral | even | chiral-odd |
|------------------|----------------|-----------------|---------------------|
| | q | q \rightarrow | q \uparrow |
| p | f_1 | | h_1^\perp |
| p \uparrow = T | f_{1T}^\perp | g_{1T} | h_1, h_{1T}^\perp |

$$\frac{d^6 \sigma_{OT}}{dx dy dz d\phi_S d\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2} \sum_{f, \bar{f}} e_f^2 \left\{ A(y) \mathcal{F} \left[f_1^f(x, \mathbf{p}_T^2) D_1^f(z, \mathbf{P}_{hT}^2) \right] - |\mathbf{S}_T| B(y) \sin(\phi_h - \phi_S) \mathcal{F} \left[\frac{\mathbf{p}_T \cdot \hat{\mathbf{P}}_{h\perp}}{M} f_{1T}^{\perp f}(x, \mathbf{p}_T^2) D_1^f(z, \mathbf{P}_{hT}^2) \right] \right\}$$

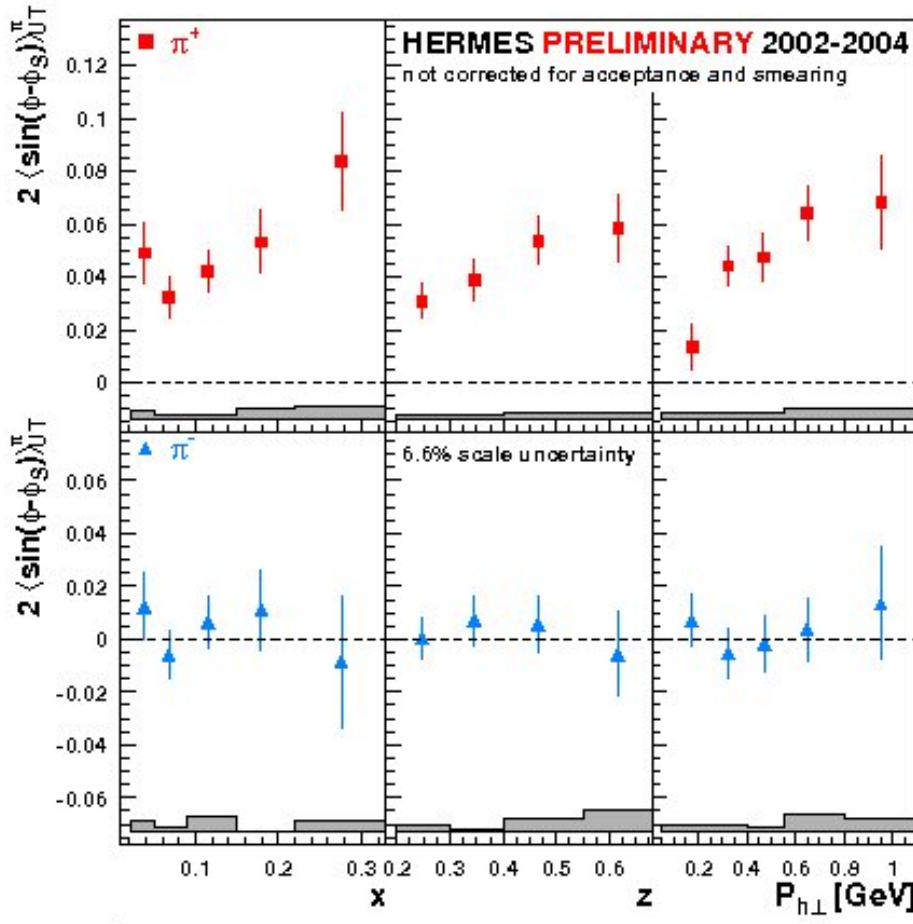


$$A_{UT} = \left[\int d\phi_h d\phi_S \sin(\phi_h - \phi_S) [d\sigma^\uparrow - d\sigma^\downarrow] \right] / \left[\int d\phi_h d\phi_S [d\sigma^\uparrow + d\sigma^\downarrow] \right]$$

effetto Sivers e relativa **S**ingle **S**pin **A**symmetry

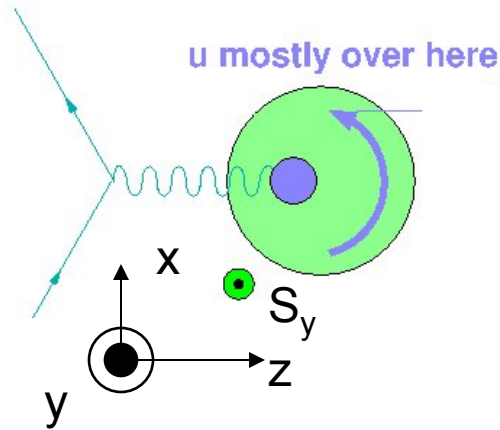
$$\frac{\int d\phi_h \sin(\phi_h - \phi_S) (d\sigma^\uparrow - d\sigma^\downarrow)}{\int d\phi_h (d\sigma^\uparrow + d\sigma^\downarrow)} \propto -|\mathbf{S}_T| \frac{B(y)}{A(y)} \frac{\sum_{f\bar{f}} e_f^2 f_{1T}^{\perp f(1)}(x) D_1^f(z)}{\sum_{f\bar{f}} e_f^2 f_1^f(x) D_1^f(z)}$$

$$f_{1T}^{\perp(n)}(x) = \int d\mathbf{p}_T \left(\frac{\mathbf{p}_T^2}{2M^2} \right)^n f_{1T}^{\perp}(x, \mathbf{p}_T)$$



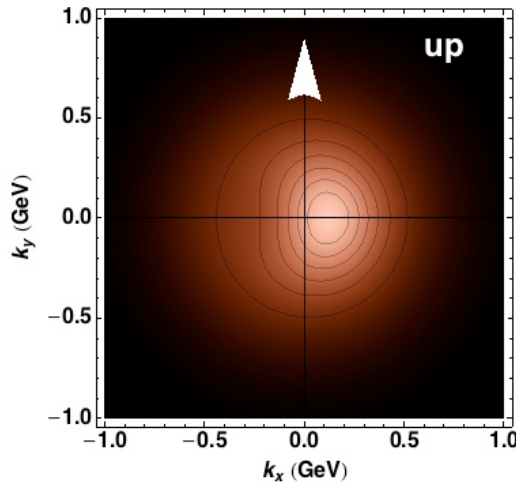
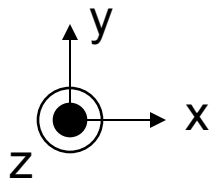
π^+ positivo $\rightarrow f_{1T}^{\perp u}$ negativa
 $f_{1T}^{\perp d}$ positiva
 (piccola)

(continua)

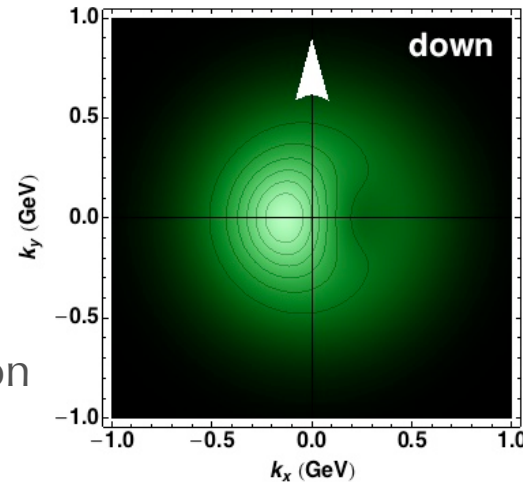


possibile interpretazione: * $N^\uparrow \rightarrow$ distribuzione asimmetrica nel piano (xy):
 u va a $x > 0$ e d va a $x < 0$ perché $S_y \neq 0 \rightarrow L_q \neq 0$

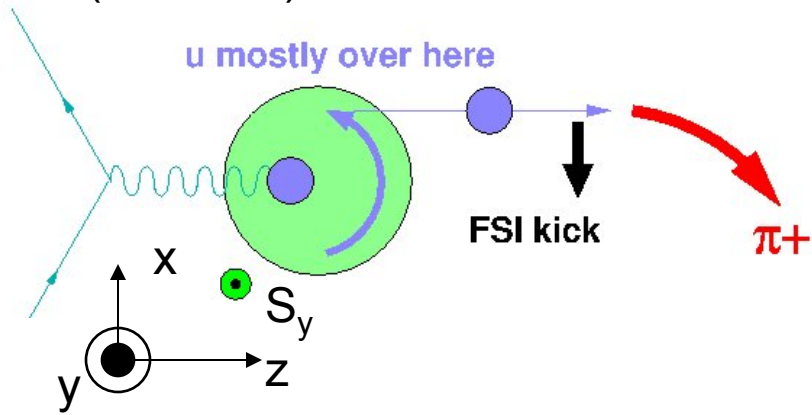
effetto diretto del momento angolare orbitale dei quark



\uparrow
 S_y
polarization



(continua)



$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = \pi \end{array} \right\} \sin(\phi - \phi_S) > 0$$

possibile interpretazione: * $N^\uparrow \rightarrow$ distribuzione asimmetrica nel piano (xy):
 u va a $x > 0$ e d va a $x < 0$ perché $S_y \neq 0 \rightarrow L_q \neq 0$
 * γ colpisce u che viene deflesso a $x < 0$ per confinamento (forza colore attrattiva)

opposto per d

(Burkardt, Phys. Rev. D**66** ('02) 114005)

esempio di deflessione
per quark d a $x > 0$



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Cenni sulla Regola di somma di spin

"Spin Crisis" : l'elicità dei quark di valenza giustifica solo circa il 30% dello spin 1/2 del nucleone (a $Q^2 \sim 1-10 \text{ GeV}^2$)

correzioni radiative e scaling con Q^2
non modificano sostanzialmente la situazione:
dove sta il resto dello spin?
Contributo dei gluoni?
Moto orbitale dei partoni?

elicità (PDF) definite in IFM \neq spin dei partoni di valenza
punto di partenza: definizione gauge invariante degli operatori in gioco

R.L. Jaffe e A. Manohar, N.P. **B337** (90) 509
X. Ji, J. Tang e P. Hoodboy, P.R.L. **76** (96) 740

Cenni sulla Regola di somma di spin

densità d'energia-impulso (operatore locale) $T^{\mu\nu} = \frac{\partial \mathcal{L}_{QCD}}{\partial \partial_\mu q} \partial_\nu q - \delta^{\mu\nu} \mathcal{L}_{QCD} \quad \partial_\mu \equiv \frac{\partial}{\partial x^\mu}$

$$= \sum_q \bar{q} \gamma^\mu i D^\nu q + G^{\mu\alpha} G_\alpha^\nu + \frac{1}{4} g^{\mu\nu} G^{\alpha\beta} G_{\alpha\beta}$$

$$= T_q^{\mu\nu} + T_G^{\mu\nu}$$

quark gluoni

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow P^\nu = \int d\mathbf{x} T^{0\nu} \quad \text{conservato}$$

densità momento angolare $M^{\lambda\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} \quad \text{non locale}$

operatore momento angolare $J^i = \epsilon^{ijk} \int d\mathbf{x} M^{0jk}$

$$\partial_\mu M^{\mu\nu\lambda} = 0 \Rightarrow J_q^i, J_G^i \quad \text{conservato}$$

(continua)

spin del protone \rightarrow elemento di matrice della componente lungo z
di operatore momento angolare su stati di protone
con 3^a componente dello spin = 1/2 lungo z

$$\frac{1}{2} = \langle P \frac{1}{2} | \hat{J}_z | P \frac{1}{2} \rangle \equiv \langle P \frac{1}{2} | \hat{J}_3 | P \frac{1}{2} \rangle = \int d\mathbf{x} \langle P \frac{1}{2} | M^{012}(x) | P \frac{1}{2} \rangle$$

$$= \int d\mathbf{x} \langle P \frac{1}{2} | x_1 T^{02}(x) - x_2 T^{01}(x) | P \frac{1}{2} \rangle$$

$$= \int d\mathbf{x} \left[x_1 \langle P \frac{1}{2} | T^{02}(0) | P \frac{1}{2} \rangle - x_2 \langle P \frac{1}{2} | T^{01}(0) | P \frac{1}{2} \rangle \right]$$

$$\partial_\mu T^{\mu\nu} = 0$$

invarianza traslazionale

$T^{\mu\nu}$ operatore locale a twist=2 \rightarrow decomposizione OPE

$$\langle P' | J^\mu | P \rangle = \bar{u}(P') \gamma^\mu u(P) F_1(t) + \bar{u}(P') \frac{i\sigma^{\mu\alpha} \Delta_\alpha}{2M} u(P) F_2(t) \quad t = (P' - P)^2$$

generalizzazione

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{u}(P') \gamma^\mu u(P) A_{2,0}(t) \bar{P}^\nu + \bar{u}(P') \frac{i\sigma^{\mu\alpha} \Delta_\alpha}{2M} u(P) B_{2,0}(t) \bar{P}^\nu$$

sia per q che G

$$\bar{P}^\mu = \frac{(P + P')^\mu}{2}$$

(continua)

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{u}(P') \gamma^\mu u(P) A_{2,0}(t) \bar{P}^\nu + \bar{u}(P') \frac{i\sigma^{\mu\alpha} \Delta_\alpha}{2M} u(P) B_{2,0}(t) \bar{P}^\nu$$

sia per q che G

$$\begin{aligned} \frac{1}{2} &= \langle P_{\frac{1}{2}} | \hat{J}_z | P_{\frac{1}{2}} \rangle = \frac{1}{2} [A_{20}(0) + B_{20}(0)] \equiv \frac{1}{2} [A(0) + B(0)] \\ &= \sum_q \frac{1}{2} [A_q(0) + B_q(0)] + \frac{1}{2} [A_G(0) + B_G(0)] \\ &= \sum_q \langle \hat{J}_z^q \rangle + \langle \hat{J}_z^G \rangle \end{aligned}$$



- A_{20}, B_{20} fattori di forma generalizzati rispettivamente di F_1, F_2
- $T^{\mu\nu}$ fornisce la frazione di spin del protone portata da quark & gluoni
- ma non esiste esperimento per “rivelare” $T^{\mu\nu}$
(tensore sensibile a sonda tensoriale: il gravitone?)
- come misurare $A(0)$ e $B(0)$, e quindi $\langle J_z \rangle$ dei costituenti?
serve nuovo strumento: Generalized Parton Distributions (GPD)

Decomposizione dello spin

si dimostra la seguente decomposizione gauge invariante: Ji, P.R.L. 78 ('97) 610

$$\hat{J}_z = \int d\mathbf{x} M^{012} = \epsilon^{ij3} \int d\mathbf{x} [x_i T_q^{0j} + x_i T_G^{0j}] = \hat{J}_z^q + \hat{J}_z^G$$

$$\begin{aligned} \hat{J}_z^q &= \int d\mathbf{x} \left[\frac{1}{2} \psi^\dagger \gamma_0 \gamma_3 \gamma_5 \psi + \epsilon^{ij3} \psi^\dagger x_i \times (-iD_j) \psi \right] \\ &= \int d\mathbf{x} \left[\psi^\dagger \frac{\Sigma_3}{2} \psi + \epsilon^{ij3} \psi^\dagger x_i \times (-iD_j) \psi \right] \equiv \hat{S}_z^q + \hat{L}_z^q \end{aligned}$$

$\Sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$
operatore elicità

$$\hat{J}_z^G = \epsilon^{ij3} \int d\mathbf{x} x_i \times \underbrace{(\mathbf{E} \times \mathbf{B})_j}_{\text{vettore di Poynting del campo di colore = densità di momento della radiazione}}$$

$E^i = F^{i0}$
 $B^i = -\frac{1}{2} \epsilon^{ilm} F_{lm}$
 $\mathbf{D} = \nabla + ig\mathbf{A}$

densità di momento angolare della radiazione

Prendendo el. di matrice degli operatori di cui sopra su stati $|P^{1/2}\rangle$ di protone con $P_z=P$ e $S_z=1/2$ alla scala di rinormalizzazione μ^2 abbiamo

$$\frac{1}{2} = \langle \hat{J}_z \rangle \Big|_{\mu^2} = \sum_q \langle \hat{J}_z^q \rangle \Big|_{\mu^2} + \langle \hat{J}_z^G \rangle \Big|_{\mu^2} = \frac{1}{2} \Delta\Sigma(\mu^2) + \sum_q L_z^q(\mu^2) + J_z^G(\mu^2)$$

Regola di somma di spin?

$$\frac{1}{2} = \langle \hat{J}_z \rangle \Big|_{\mu^2} = \sum_q \langle \hat{J}_z^q \rangle \Big|_{\mu^2} + \langle \hat{J}_z^G \rangle \Big|_{\mu^2} = \frac{1}{2} \Delta\Sigma(\mu^2) + \sum_q L_z^q(\mu^2) + J_z^G(\mu^2)$$

- anomalia $U_A(1)$ mischia elicità dei q ($\Delta\Sigma$) con quella dei G a NLO
 → separazione dipendente da schema oltre che da μ^2
- la relazione operatoriale (da cui scende la regola di cui sopra) è una regola di somma se gli elementi di matrice di operatori locali su stati $|P^{1/2}\rangle$ possono essere rappresentati come momenti di Mellin di distribuzioni misurate in processi anelastici sugli stessi stati $|P^{1/2}\rangle$ e con gli stessi operatori

Ad esempio:

$$\Delta\Sigma(\mu^2) = \sum_q \int dx g_1^q(x, \mu^2) = \sum_q \langle PS | \bar{\psi}^q(0) \gamma^+ \gamma_5 \psi^q(0) | PS \rangle \Big|_{\mu^2}$$

$$\int dx \int d\xi^- e^{-ixP^+ \xi^-} \langle PS | \bar{\psi}^q(\xi^-) \gamma^+ \gamma_5 \psi^q(0) | PS \rangle \Big|_{\mu^2}$$

ma per L_z chi è $L_z^q(x)$?

Inoltre non esiste una decomposizione univoca e gauge invariante.

Analogamente non si sa decomporre J^G in L^G e ΔG . Work in progress...