Seveso 1976, Chernobyl 1986: fractal description of two ecological disasters

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1 Introduction

Towards the end of the last Century, several ecological accidents occurred in
the World; among other a chemical accident in the town of Meda, few miles
north of Milan (Italy), which heavily contaminated the town of Seveso and
the surroundings with a super toxic chemical compound (thus known as the
Seveso accident) and the major nuclear accident in the town of Chernobyl, known to be caused by a human mistake in handling a power nuclear
reactor, which contaminated a large part of all Europe with several radioactive
materials. An additional chemical accident occurred in Bophal (India)
but there a lethal gas was distributed in the atmosphere, and no systematic
measurements were performed which might be used for a statistical analysis
of the event. The Three Miles Island (Pennsylvania, U.S.A.) nuclear accident
released a negligible amount of radioactive nuclides and cannot be considered
as a major event.

This paper is applicative in nature as it describes the fractal properties
shown by the pollutant’s distribution in the atmosphere or on the soil around
the location of the unexpected (and undesired) events, in an attempt to learn
as much as possible, the severe lecture delivered by History. The interesting
aspect of the present analysis consists in the comparison of two rare events:
the first involving a limited geographical region (thus microclimate), the sec-
ond involving a large proportion of all the World (thus macroclimate) which
both appear to be described in the frame of a single Universal Multifractal
Model.
After a brief account of the accidents for the benefit of the reader we shall address the major points which deserve particular attention, before performing the statistical analysis of the data: points which are their quality and, in many aspects, their relatively limited statistical significance.

2 The Seveso accident

In the small town of Meda[1], about 15 km north of the City of Milan, a small company, Icmesa, was producing pharmaceutical compounds, among others, 2,4,5-TriChloroPhenol (TCP), a non flammable compound used as a basis for the synthesis of herbicides. The company was a minor section of Givaudan, a branch of the colossal swiss company Hoffmann La Roche. The fabrication of TCP is obtained by mean of an esothermic reaction at 150-160 °C. At much higher temperature a new reaction can be started producing in large concentration 2,3,7,8-TetraChloro-Dibenzo-p-Dioxin (TCDD or simply dioxin). The very symmetric positions of the chlorine atoms in the chemical formula shown in fig. 1, as well as the presence of the two oxygen bindings, are responsible for its extreme toxicity.

![Figure 1: The structure formula of the dioxin TCDD.](image)

On july 10th, 1976, an anomalous pressure caused by an exothermic reaction of tri-chloro-phenol, caused the brake of a disk in the chemical reactor; at 250 °C an unwanted production of TCDD took place that was released in the atmosphere due to the fact that the reactor itself was not equipped with a safety expansion chamber. The release lasted several hours followed by simple evaporation till the final cooling of the whole system.

Unusual for that period of the year, a wind was blowing at about 5 m/sec; thus the white toxic powder was deposited along a rather linear path in the south-east direction for as long as about 7 km (the topography of the region is depicted in fig.s 2 and 3a).
It has been estimated that at the instant of the accident, about 3 tons of product were present inside the reactor. Contradictory estimates of the emission in the atmosphere range from 300 to 3000 kilos. It must be stressed that at that time of the accident, dioxin was an almost unknown chemical compound. No human beings was killed, many small animals died and childrens suffered of chloracne, a skin disease.

The contaminated area was divided into three zones[3] (called A, B and R) depending upon the detected contamination, as shown in fig. 3a and the zones were subjected to systematic campaign of measurements of the TCDD deposition on the soil (in µg/m²). The boundaries of the zones are, by all means, of geo-political nature as they follow exactly the boundaries of the different townships (which are indicated by the dashed lines in fig. 3a). After the accident the shed hosting the chemical reactor was demolished (as shown in fig. 3b).

3 The Chernobyl accident

In the small town of Chernobyl, north of Kiev, Ukraina (see fig. 4a), the ex-U.S.S.R. built one of the major nuclear plants of the Country, probably the most powerfull plant of the World.

At 1.24 am of april 26th 1986[5], two explosions occurred, a power equiv-
alent to one ton of TNT, blowing out the core of reactor n. 4 of the nuclear plant, together with the roof of the building (contrary to the western technology, the USSR technology of that time did not constrain the active nuclear reactors with appropriate super thick and heavy concrete containers). In spite of the fact that immediate action by emergency units suffocated the fire before 5.00 am the same day, the result was the emission of several tons of uranium-di-oxide into the atmosphere, together with a miriad of radioactive nuclides, among others $^{137}Cs, ^{134}Cs, ^{131}I, ^{132}I, ^{140}Ba, ^{140}La, ^{132}Te, ^{99}Mo$, plus more short lived radioactive nuclides and white hot graphite.

The radioactive release, according to the local authorities, was of no less than 50 megacuries of radionuclides plus no less than 50 megacuries of radioactive chemical gases.

For sake of information in Table 1 we summarize the most relevant radiological units which are not always commonly known. In order to have an idea of the health situation, it is worth mentioning that the dangerous dose for humans is 50 rem/y. In the village of Lenev, 19 km lee-wards from Chernobyl, over 250 rem of $^{131}I$ were measured in the thyroid of childrens\textsuperscript{1}, i.e. more than 5 times the dangerous yearly dose in few days only by the iodine nuclide.

\textsuperscript{1}Iodine is strongly absorbed in the thyroid; for this reason, in case of radioactive contamination iodine pills are taken in order to saturate the thyroid absorption capability.
The pinnacle went up to about 5 km in the atmosphere for some days, thus contaminating also several European countries, from Scandinavia to Greece and Turkey. The immediate cost of the accident was of 31 persons killed, 203 persons hospitalized for severe radiation dependent symptoms plus an additional 500 persons hospitalized for different reasons. Finally a total of over 115,000 population was evacuated.

The topography of the region around Chernobyl is depicted in Fig. 4a and what was left after the disaster is shown in Fig. 4b. It is not appropriate here to describe the detailed reasons which contributed to the occurrence of the accident[5]. The fact is that radioactive materials were spread all over Europe carried by the winds blowing depending upon the meteorological evolution of those days.

4 A caveat: the quality of the data

The nature of the two episodes is drastically different. In one case a well defined heavy solid substance was released; it covered an area of about 4 km x 7km; it was elevated to a relatively low altitude; it was dominated by a microclimate. It appears as a small scale phenomenon. In the second case, radioactivity is not a substance but rather a physical status of matter; all sort of radioactive nuclides were thrown into the air which are very light.
Table 1: Reminder of the major radiological units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity</td>
<td>Becquerel (Bq)</td>
<td>1 Bq= 1 decay/second (1 Hz!)</td>
</tr>
<tr>
<td></td>
<td>Curie (Ci)</td>
<td>1 Ci = $3.710^{10}$ Bq</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1 nCi= 37 Bq)</td>
</tr>
<tr>
<td>Absorbed dose</td>
<td>Gray (Gy)</td>
<td>1 Gy= 1 joule/kg</td>
</tr>
<tr>
<td></td>
<td>Rad (rad)</td>
<td>1 rad= 100 erg/g=0.01 Gy</td>
</tr>
<tr>
<td>Equivalent dose</td>
<td>Sievert (Sv)</td>
<td>1 Sv= QF Gy = 1 joule/kg</td>
</tr>
<tr>
<td></td>
<td>QF=quality factor</td>
<td>Relative biological Effectiveness</td>
</tr>
<tr>
<td></td>
<td>Rad. Equiv. Man (Rem)</td>
<td>1 rem = 0.01 Sv</td>
</tr>
<tr>
<td></td>
<td>Roentgen (R)</td>
<td>1 R = charge/mass</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= $2.58\times10^{-4}$ coulomb/kg</td>
</tr>
</tbody>
</table>

mostly inorganic chemical elements. The toxicants covered an area of many thousand squared km; they were shot at high altitude. The phenomenon is to be seen on a large scale and dominated by a macroclimate. Nonetheless, both events spread undesired toxic material in the air and on the soil. Thus, the events become statistical in nature and subject to all laws of atmospheric phenomena, either large or small scale.

However, contrary to all climate studies and effects, in the case of these two accidents, the data (amount of dioxin deposited on the soil or radioactivity concentration in air or left on the ground) are not directly acquired, but are rather the result of sophisticated manipulations performed by a very aetherogeneous ensemble of different peoples.

For these reasons a short discussion of the measuring methods is appropriate.
4.1 Seveso data

In the Seveso case, samples of contaminated soil were collected in the form of cylindrical cores, 7 cm thick and 7 cm in diameter. The cores were largely unhomogeneous: they might contain stones, grass, anything. The measuring protocol is pictorially sketched in fig. 5a; the end of it is the gaschromatographic search for the molecular spectral emission lines. The crucial points are the steps solvents-extraction-purification. The measurements were performed essentially by one single laboratory in the City of Milano but the number of extractions varied and the purification methods improved with the years from 1976 to 1981. Thus for us the measuring method is a black box providing numbers without a precise accuracy attached to them. An additional problem was given by the several samples classified as nv, i.e. no value that, to us, sounded as below the measuring threshold. To make it brief,

![Figure 5](image)

Figure 5: a- Sketch of TCDD extraction protocol: 1- soil sample pickup; 2-solvent’s addition and sample homogeneization; 3- TCDD sample enrichment and product extraction; 4- chemical purification and dose preparation for gaschromatography; b- map of measurement points around Seveso; c- Distribution of the measurements (µg/m²) in all zones around Seveso.

the amount of TCDD extracted in the 1976/77 campaigns was about half the product extracted in the 1980/81 campaigns[6] (a posteriori, the sensitivities were 0.75µg/m² in 1977 and 0.25µg/m² in 1984). Systematic investigations were performed[6] to correct and renormalize the measurements of the different campaigns prior to using them in the fractal analysis. However the role of the nv measurements, which are strictly linked to the sensitivity of the measuring black box is more subtle and crucial when fractal models and data are compared (see §6). The measuring campaigns were exploited following
regular grids according to the several zones. The map of the locations in which measurements were taken is shown in fig. 5b.

4.2 Chernobyl data

The radioactivity measurements after the Chernobyl explosions were done by all sort of Institutions: Research Centers, Universities, Hospitals, Meteo Stations, Welfare Offices and many many other subjects. Concentration in air was measured using filters of many different types. You may name thousand of other situations. Radioactivity levels were measured in vegetables, in water, in air, in small dead animals, deposited on ground etc, for several nuclides. Not all the collected information was usefull for comparison to fractal models. Also in this case, the measuring procedure must be considered as a black box in the sense that no calibrations were known so that no normalization was possible. The measured quantity was the radioactive contamination in Becquerel. There was an advantage though in the Chernobyl case: the Joint Research Center of the European Community in Ispra (Varese, Italy) took over the job of collecting the global data base. Our group then was able to scan the values and send back for due verification all apparently anomalous measurements (the most part was due to trivial misprints in the data base). Here the data set had a major problem: the location associated to the measurement in Bq was the name of the closest major Town. This, of course produced a severe loss of geometrical resolution in the position of the measured quantity. However those were the conditions and nothing could be done about them.

5 Simulation by Fractal Sum of Pulses

5.1 FSP simulation of Seveso data

The overall data distribution of the Seveso concentration of TCDD in $\mu g/m^2$ is collected in fig. 6a; as an example. The first “emergy map”, shown in fig. 6b, was a simple isopleche level curve in which the wild fluctuations appear very clearly. Apparent are also regions (in white) where no measurements were possible. The overall ground distribution of $^{137}Cs$ all over Europe, provided by the IAEA and the ukrainian Agencies[13], is collected in fig. 6c. It can
be noted that the Countries running nuclear power programs did provide the least number of measurements. Clear is the difference in the amount of data available in the two cases. At least in Italy, the local political authorities were pressing, at the time of the Seveso accident, to estimate the TCDD contamination in location were the measurements could not be done and at the time of the Chernobyl accident, the pressure was strong for the same reasons.

In 1977 fractal geometry was just in the very early stage and, among the physicists, not a well known technique for simulation of statistically fluctu-
ating events. Thus at that time\cite{ref2} we interpolated the data by means of Tchebitchev polynomials as it was a current procedure to simulate atmospheric events. The result, obtained using 52 parameters, known in Europe as the \textit{Seveso ghost} is reproduced in fig. 7a.

We immediately realized that the intrinsic principle of \textit{interpolation} implied the cutting of peaks and the filling of holes in the reproduction of the experimental data. However, the aim of the study was to make predictions and estimates of contamination in areas in which no measurements were possible neither at that time nor in the future. For this reason we addressed our attention to alternative approaches, ending up to the use of fractal geometry\cite{ref8}.

In the early eighties, using the Fractal Sum of Pulses (FSP) naive technique we simulated the TCDD distribution in all zones around Seveso. The fractal dimension of the distribution in $\mu g/m^2$ was directly measured obtaining $D_A = 1.69248$ for zone A and $D_{A+B+R} = 1.69498$ for all zones (thus for us $D = 1.69$ is more than enough). The FSP model assumed bubbles for pulse shape; a probability distribution $Pr(V > V^*) = 1/V$ to guarantee scaling; finally for the selection of the pulse intensities $I$, a law $\Delta I = \pm V^{1/D}$. The result is shown in fig. 7b. More activity is evident in the simulation. To check its validity we compared the results of the model to the values collected \textit{along the line of maximal contamination}\cite{ref3} obtained by fitting broad gaussian distributions at different values of the coordinates (either x or y) used in fig. 6a (see fig. 8a). The comparison is shown in fig 8b: black triangles

Figure 8: a- Fit of the wind direction; b- comparison between simulation and data
are the simulation, gray squares are the data (the coordinates in fig. 8b are increasing numbers located along the fitted line if fig. 8a in arbitrary units).

A number of comments are proper here:

a- first of all a renormalization of the data was done, when possible, to merge 1977 data with 1980 data[6];

b- close to the factory several areas without measurements were present where contamination was high (shaded area visible in fig. 8b);

c- zone B is very narrow (see fig. 6b) and possibly missed by the line of maximal contamination;

d- we already mentioned in §4 that in zone R (away from the factory) the number of nu measurements was very large and therefore only measurements above the sensitivity level of the measuring method were quoted. Thus it is obvious that, in the terminal region, the simulation (shaded area in fig. 8b) provides a lower contamination level compared to the data. Note that the horizontal line drawn in fig. 8b is at the level of 1\(\mu g/m^2\).

The simulation was then considered very good by the local authorities.

5.2 FSP for \(^{137}Cs\) air concentration in Northern Italy

A good lot of data were collected for the major long living nuclides released by the Chernobyl accident.

FSP simulation of the \(^{137}Cs\) air concentration (in \(Bq/m^3\)) in Northern Italy and its time dependence required a lot more data handling.

Out of 120,453 measurements, 25,422 were on \(^{131}I\), 4717 on \(^{132}I\), 22,737 on \(^{134}Cs\) and 25,600 on \(^{137}Cs\). These numbers unfortunately distributed during about 15 days and distributed over several European Countries. The \(^{137}Cs\) nuclide has an average lifetime of over 20 years so that it can be considered as stable for all practical purposes in the simulation.

Italy is politically organized in regions; the regions are organized in provinces and the provinces are organized in towns, cities and villages. Only 8 provinces have measured the radioactivity concentration in air for all the 4 major nuclides, i.e.: \(^{131}I\), \(^{132}I\), \(^{134}Cs\) and \(^{137}Cs\). The locations where these measurements were done are shown in fig. 9a. It was then clear that simulations would be impossible in southern Italy, due to the scanty measurements there. It is here evident also the very poor geometrical resolution. In addition, most of the data came from three daily collections of pollutant nuclides
by means of ordinary air filters. In order to include also daily measurements we must assume as temporal uncertainty one day. When more than one measurement per day were present in one location, we could assign an estimated uncertainty. The time dependence of the radioactive phenomenon was described\[8\] by an analytical shape given by the following formula:

\[ Y(t) = K + e^{\left[-\frac{\lambda}{\tau} t + B\right]|1-e^{-\tau t}|} \]  \hspace{1cm} (1)

where: \( t \) stand for the time (days) past the accident (04-26-1986); \( \tau \) is the decay mean lifetime of the given nuclide; \( A \) stands for the effective fading of the nuclide concentration (depending also upon the meteo condition of the day); \( B \) is a parameter linked to the intensity of the phenomenon (for different nuclides \( R = B_{Cs}/B_{\text{nuclide}} = \text{constant} \)); \( C \) \((>0)\) is a calibration parameter linked to the arrival time of the radioactive cloud in the given location and finally \( K \) is the background level at the given location. \( A, B, C, \) and \( K \) are the free parameters of the fits. The values for \( K \), when possible, are compared for consistency, to the level of natural radioactivity reported in the literature. Some examples of the fitted functions are reproduced in fig. 10.

Due to the poor coverage of the territory, we needed to recover as much as possible information also from measurements on other nuclides\[10\] . To do that we fitted equation (1) to all available nuclides (i.e. \(^{103}\)Ru, \(^{141}\)Ba, \(^{140}\)La,
\(^{132}\)Te). The parameter A is assumed to be independent of the provinces; C is connected to the arrival time\([4]\) of the cloud \((t_{\text{arr}} = \sqrt{C}\) with very good approximation\) which is nuclide independent and therefore can be used whatever the analyzed nuclide is in that particular location; finally B can be recovered by the constant ratio mentioned above.

In this way we could add *pseudomeasurements* in the provinces of Bologna, Genova, Padova, Pisa, Trieste to the available data for the provinces of Alessandria, Milano, Pavia, Piacenza, and Vercelli, thus doubling the amount of information to start the FSP process. The simulation was similar to the one adopted for the Seveso case, using the same parameters. Here, northern Italy was subdivided in 10 *domains* on the basis of a *best vicinity* criterion as shown in fig. 9b; as a starting point, the value of the air concentration in the 10 domains is considered flat corresponding to the value derived by eq. (1) for that location. Then the procedure is unchanged, compared to the Seveso case. As an example, the results are shown for simulations at 5, 8 and 11 days after the accident in fig. 11.

The simulation for the 10 provinces has been successfully checked comparing it to the data set in the different locations where the measurements were actually performed. The *uncertainty* was obtained by circularly neglecting one province at a time and repeating the simulation on the basis of 9 provinces only. In fig. 12 the simulation is given for 15 provinces (from top left to bottom right: Cuneo, Imperia, Belluno, Florence, Macerata, Novara,
Figure 11: Full simulation of $^{137}\text{Cs}$ air concentration in northern Italy. From top: 5, 8 and 11 days after the accident.

Savona, Venice, Leghorn, Ravenna, Turin, La Spezia, Verona, Pisa, Pesaro) in which, excluding Pisa, no measurement were done.

It is worth to underline that the vertical scales are not the same, the maximal values ranging from about 1.3 Bq (Ravenna and La Spezia), 2.5-3.0 Bq (Macerata, Novara and Verona) as well as the time evolutions of the phenomenon that turn out to be rather different in the different locations.

The simulation was certified by the J. R.C. in Ispra where the simulation data were satisfactorily compared to a set of measurements exploited every 20 minutes and considered adequate[9] No other simulation was able -at least at that time, to the best of our knowledge- to give a better agreement with the Ispra data set conserved by the Laboratory as reference set.
The $^{137}$Cs cumulative soil deposition was analyzed in several European countries [12] but the results are not shown in this paper. In Table 2 the data sets collected by the JRC-Ispra are collected. In particular it is to be noted (last column) that the average distance between the locations of the measurement points are enormous. Thus only countries with at least 100 usable measurements could be considered. The simulations have been done [8] but are not shown here due to the limited interest in the field of atmospheric physics.

The results are of interest in the field of sociology. It can be noted again that countries running intense nuclear programs did not provide a large amount of data.

Fig. 13 shows, as examples, two PDMS distributions (see later) for the data from Greece (left, 1108 datapoints) and Austria (right, 104 data points).

Here we prefer now to concentrate on the interpretation of the meteoratmospheric phenomena in terms of statistical-physical parameters; i.e. in terms of Universal Multifractals of the two cumulative soil depositions thus comparing the results of a microclimate phenomenon to that of a macroclimate phenomenon. Indeed, the Seveso episode emitted heavy molecules and invested an area of order few ten kilometer square, while the Chernobyl
Figure 13: Multifractal behaviour of the measurements done in Austria and Greece: PDMS plot according to equation (2).

episode emitted relatively light fragments and covered almost one fifth of the whole planet.

For this comparison we used all the data provided by the Ukrainian Atomic Energy Authority [13], collected in 148 cities within 100 Km from Chernobyl.

The Universal Multifractal algorithm is straightforward. We define:
1- \( \lambda \) as the grid resolution;
2- \( \epsilon(\lambda) \) as the amount of pollutant at resolution \( \lambda \), at the position \( \mathbf{x} \) on the geographical grid;
3- we consider \( \epsilon(\lambda) \) as a random variable, so that a set of measurements is a single realization of the stochastic process;
4- from multifractal fields we use the Probability Distribution Multiple Scaling PDMS law:

\[
Pr(\epsilon > \lambda^\gamma) \propto \lambda^{-c(\gamma)}
\]  

(2)

where: \( \gamma \) is the order of singularity and \( c(\gamma) \) is the codimension function.

5- calculate \( c(\gamma) \) using the PDMS technique (eq. 2), i.e.:

\[
Pr(\epsilon > \lambda^\gamma) = Pr \left( \frac{\log(\epsilon)}{\log(\lambda)} > \gamma \right) = \frac{N_{\lambda}(\gamma)}{N_{\lambda}} \propto \lambda^{-c(\gamma)}
\]  

(3)

where \( N_{\lambda}(\gamma) \) is the number of boxes (resolution \( \lambda \)), where \( \frac{\log(\epsilon)}{\log(\lambda)} > \gamma \) holds and \( N_{\lambda} \) is the total number of boxes (resolution \( \lambda \));
Table 2: $^{137}$Cs data deposited on ground in European Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Sym.</th>
<th>Original N. of measurements</th>
<th>Usable N. of measurements</th>
<th>min (Bq)</th>
<th>Max (Bq)</th>
<th>av. distance (Km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>A</td>
<td>104</td>
<td>97</td>
<td>.704</td>
<td>518.1</td>
<td>135</td>
</tr>
<tr>
<td>Belgium</td>
<td>B</td>
<td>21</td>
<td>11</td>
<td>.275</td>
<td>3.9</td>
<td>-</td>
</tr>
<tr>
<td>Germany</td>
<td>D</td>
<td>301</td>
<td>293</td>
<td>.45</td>
<td>44.1</td>
<td>334</td>
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<tr>
<td>Denmark</td>
<td>DK</td>
<td>30</td>
<td>16</td>
<td>.61</td>
<td>5.3</td>
<td>-</td>
</tr>
<tr>
<td>England</td>
<td>GB</td>
<td>70</td>
<td>46</td>
<td>.06</td>
<td>19.0</td>
<td>-</td>
</tr>
<tr>
<td>Greece</td>
<td>GR</td>
<td>1469</td>
<td>1108</td>
<td>0.1</td>
<td>149.2</td>
<td>186</td>
</tr>
<tr>
<td>Hungary</td>
<td>H</td>
<td>19</td>
<td>18</td>
<td>.58</td>
<td>13.0</td>
<td>-</td>
</tr>
<tr>
<td>Italy</td>
<td>I</td>
<td>404</td>
<td>383</td>
<td>.01</td>
<td>490.7</td>
<td>333</td>
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<td>101</td>
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<td>450</td>
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<td>693</td>
<td>349</td>
<td>.79</td>
<td>82.9</td>
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<td>200</td>
<td>.43</td>
<td>54.6</td>
<td>122</td>
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<td>4</td>
<td>.25</td>
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<tr>
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<td>22</td>
<td>20</td>
<td>.07</td>
<td>35.7</td>
<td>-</td>
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<tr>
<td>ex-USSR</td>
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<td>109</td>
<td>109</td>
<td>19.0</td>
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<td>Yugoslavia</td>
<td>YU</td>
<td>7</td>
<td>6</td>
<td>1.00</td>
<td>10.1</td>
<td>-</td>
</tr>
</tbody>
</table>

6- from (2) follows:

$$\log \left( \frac{N_\lambda(\gamma)}{N_\lambda} \right) = -c(\gamma)\log(\lambda) + \text{const.} \quad (4)$$

7- we perform linear fits for different values of $\gamma$ and $\lambda$.

With these assumptions and definitions the Universal Multifractal law is written as:

$$c(\gamma) = C_1 \left( \frac{\gamma}{\alpha' C_1} + \frac{1}{\alpha} \right)^{\alpha'} \quad (5)$$

with: $0 < \alpha < 2$ and $\frac{1}{\alpha} + \frac{1}{\alpha'} = 1$; $C_1$ codimension of the mean field; $\alpha$ increasing degree of multifractality.
8. \( C_1 \) and \( \alpha \) are finally numerically determined. In the Seveso case, by making use of the Double Trace Moments (DTM), defined as an extension of the Trace Moments \( TM(\epsilon^\eta) = \int_{A_\lambda} \epsilon^\eta \, d^D x \), that is as trace moments of \( \epsilon^\eta \): the \( \eta \) power of the field \( \epsilon \), the most robust statistical estimators of the two parameters. DTM are defined as:

\[
DTM^{(\eta)}(\epsilon^\eta) = \int_{A_\lambda} \epsilon^\eta \, d^D x \tag{6}
\]

that is the flux of \( \epsilon^\eta \) through a subset \( A_\lambda \) of a \( D \)-dimensional support space \( A \).

In doing this, we may use the easy generalization of the statistical moments \( K(q) \) as:

\[
K(q, \eta) = \eta^q K(q, 1) \tag{7}
\]
The linear logarithmic relationship:

\[ \log K(q, \eta) = \alpha \log \eta + \log K(q, 1) \]  

allows a direct estimate of \( \alpha \) from the slope of the behaviour of \( \log K(q, \eta) \) vs. \( \eta \).

The numerical estimates of the two parameters \( \alpha \) and \( C_1 \) for the Seveso case were done by means of the data in fig. 14 for the two measurement campaigns 1976/77 (upper part) and 1980/81 (bottom part). On the left hand side the plots of \( c(\gamma) \) vs. \( \gamma \); on the right hand side the plot \( \log K(q, \eta) \) vs. \( \eta \) for different values of \( \alpha \).

The results is for both campaigns:

\[ \alpha \approx 0.4 \quad ; \quad C_1 \approx 1.0 \]  

As already mentioned, the measuring procedures used had very different sensitivities so that merging all data in a unique sample can easily generate some distortion.

Figure 15: Codimension dependence on \( \gamma \), \( c(\gamma) \) vs \( \gamma \) for \(^{137}\)Cs ground deposition in 148 towns within 100 km from Chernobyl.

The estimate of the universal multifractal parameters for 148 towns within 100 km from Chernobyl was done by Daniel Schertzer and Yulia Chirginskaya[13]
Fig. 15 shows the $\gamma$ dependence of the codimension function $c(\gamma)$ for the data of the $^{137}$Cs cumulative soil deposition in an area within 100 km from Chernobyl.

The values of the parameters $\alpha$ and $C_1$ are:

$$\alpha \approx 1.5 \quad ; \quad C_1 \approx 0.4$$

It is easy to conclude that the numerical values of the universal multifractal parameters of the Seveso accident are very well compatible with those of any rainfall atmospheric episode and that the numerical values of the universal multifractal parameters of the Chernobyl accident are very well compatible with those of cloud formation.

In essence, the two accidents are two aspects of an unique meteorological game: the former attaining to phenomena in which heavy particles are involved, the second dominated by very light particulates suspended in the atmosphere.

References


