Compton Scattering

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An experiment is set up in order to observe the angular dependency of the energy of scattered \( \gamma \)-rays and the differential cross section for Compton scatterings off electrons in a polymethyl methacrylate (PMMA) target. The energy data is fitted to Compton’s energy shift equation, and the electron mass is found to be \( 512 \pm 18 \text{keV} \). Discrepancies between the intensity data and the theory are found. Follows a discussion on the efficiency of particle detectors and data selection for analysis. The data is corrected and fitted to the Klein-Nishina equation.

The Compton Effect is one of the traditional examples of the Quantization of electromagnetic radiation. Compton prepared a beam of high energy photons traveling in the x-y plane through a graphite target, and measured the intensity of the beam as a function of wavelength at different angles from the original beam path, noticing that there were two peaks of intensity for \( \theta > 0 \): the first was the original wavelength of the beam and the second was a larger wavelength. By assuming that a ballistic collision between a photon and an electron caused the photons to change their energy, Compton found that the shift in energy is given by:

\[
\frac{1}{E_S} = \frac{1}{E_0} + \frac{1 - \cos \theta}{m_ec^2}
\]

(1)

Where \( E_0 \) is the original energy of the beam and \( E_S \) is the energy of the photon after scattering. [1]

The probability that a given photon in the beam will strike an electron and be scattered is proportional to the Compton scattering cross section, \( \sigma_{\text{comp}} \). For a beam incident on the target with \( N_0 \) photons, the change in the number of photons at position \( x \), \( dN(x) \), is proportional to the number of photons \( N(x) \) in the beam, to the number of electrons in the path \( (N_e = n_e \cdot V = n \cdot Adx) \) and to the probability of collision \( (\frac{\sigma_{\text{comp}}}{A}) \):

\[
dN(x) = -N(x) \cdot (n_e \cdot Adx) \cdot \left( \frac{\sigma_{\text{comp}}}{A} \right)
\]

(2)

\[
\Rightarrow N(x) = N_0 \cdot e^{-n_e \cdot \sigma_{\text{comp}} \cdot dx}
\]

(3)

where \( n_e \) is the density of electrons. The quantity \( \mu = n_e \sigma_{\text{comp}} \) is called the linear attenuation coefficient, and the attenuation length is defined as the thickness of the material required to attenuate the intensity of the beam by a factor of \( e^{-1} \): \( \Lambda = 1/\mu \). The Compton scattering cross section can be rewritten as:

\[
\sigma_{\text{comp}} = \frac{\mu}{n_e}
\]

(4)

The number of scattered particles is typically measured in terms of the differential cross section \( \frac{dN}{d\Omega} \) [1]:

\[
dN = \frac{d\sigma}{d\Omega} \cdot N_0 \cdot n_e \cdot d\Omega
\]

(5)

The differential cross section is given by the Klein-Nishina equation [2]:

\[
\frac{d\sigma}{d\Omega} = \frac{r_0}{2} \left( 1 + \cos \theta^2 + \frac{\gamma^2(1 - \cos \theta)^2}{(1 + \gamma(1 - \cos \theta))^2} \right)
\]

(6)

where \( \gamma = \frac{E}{m_ec^2} \).

The Klein-Nishina formula can be verified by measuring the intensity of scattered photons for multiple angles. The experiment is done with: (a) a source of photons, in this case, \( \gamma \)-rays, (b) a scattering target - a polymethyl methacrylate (PMMA) scintillator; (c) and a detector - the sodium iodide (NaI) scintillator. The source for this experiment is Cesium (\(^{137}\)Cs), which emits \( \gamma \)-rays at 661.65KeV.

A scintillator is an organic or inorganic crystal that gives off a flash of light when struck by a particle or radiation. The incoming \( \gamma \)-ray knocks an electron off a nucleus when it enters the scintillator either through photoelectric effect or Compton scattering. The electron excites impurities inside the crystal, which give off light when returning to the ground state. The scintillators typically have high index of refraction so that the light produced
inside is reflected on the surface back into the crystal and redirected towards a Photomultiplier Tube (PMT), which converts a flash of light into an electric pulse. When photons enter the PMT, they hit a "photocathode" and produce electrons through photoelectric effect. The electrons are then accelerated toward a series of dynodes, where they strike again and again creating an increasingly large cascade of electrons that are collected into an anode, where a current can be detected.

Using a scintillator as the target allows for the use of a coincidence circuit to filter out non-scattering events. The detector only registers a detected event when target scintillator also detects an event. The materials for each scintillator are chosen according to their function. The scintillator also detects an event. The materials for each anode, where a current can be detected.

A plot of the number of events detected (the intensity) versus energy will reveal two major features - a sharp, tall peak for high energy events and a wide distribution with low energy events (See Figure 2). The sharper peak corresponds to events detected through Photoelectric absorption (the "photo-peak"), and the low energy spectrum corresponds to events detected through Compton scattering inside the NaI detector. The detector PMT measures the energy of the electron knocked off by the incoming γ-ray after it has been scattered in the PMMA target. When the electron is knocked off by a Photoelectric effect, it will have the energy of the incoming photon. When the electron is knocked off by a Compton scattering, the photon will keep a large portion of its initial energy, so the electron will have a very low energy. Therefore, nuclei with high Z are desirable for the detector scintillator, since it increases the relative number of Photoelectric absorptions.

The linear attenuation of the beam when passing through PMMA is measured by removing the target from the beam path and placing PMMA bricks in front of the hole in the lead encasing. The number of events is recorded for different thicknesses of PMMA, and the linear attenuation coefficient is found to be \( \mu = 9.79 \text{m}^{-1} \), which corresponds to an attenuation length \( \Lambda = 10.2 \text{cm} \). Using Equation (4) to a collisional cross section \( \sigma_{\text{comp}} = 25.5 \text{fermi}^2 \), very close to the theoretical value \( \sigma_{\text{comp}} = 25.6 \text{fermi}^2 \).

For our target, which is 2 inches in diameter, about 40% of the incoming γ-rays will be scattered. The photopeak energy and the counting rate at the peak are measured for several angles between 0° and 160°. Each point is measured for 3 hours to ensure enough events are recorded to obtain accurate scatter rates for all angles.

A plot of the energy vs. angle will closely resemble Compton’s energy shift equation. By using a curve-fitting software and leaving the mass of the electron as a free parameter, a very approximation is attained (See Figure 3) and the mass of the electron is found to be \( m_e c^2 = 513 \pm 18.4 \text{keV} \).

\[
C_{\text{measured}}(\theta) = C_{\text{correct}}(\theta) \cdot \alpha(\theta) \cdot \beta(\theta) \tag{7}
\]

where \( \alpha(\theta) \) accounts for efficiency issues and \( \beta(\theta) \) for multiple scatterings. Each of these factors affects the counting rate in such way that

FIG. 2: Sample Data. Events detected vs. Channel #, as detected by the software Maestro. Note that the sharp peak corresponds to events detected through Photoelectric absorption and the low energy spectrum corresponds to events detected through Compton scatterings.

FIG. 3: Energy distribution for the Compton scattering of γ-rays (\( E = 661.65 \text{keV} \)). The line is a fit to Equation (1). The rest mass of the electron is found to be \( m_e c^2 = 513 \pm 18.4 \text{keV} \).
20° should be discarded. At very small angles, the beam coming directly from the source is swamping the detector with more γ-rays that it can handle. The detector has a finite time resolution - when an event is detected, it has to wait a certain time before it is capable of detecting another event. The time it takes to become active again is called “dead time” and it was too large for small angles, which means that it was not able to count the number of events correctly. Another point to consider is that not all γ-rays scattered by the target at angle θ will be detected. As pointed out before, the NaI scintillator only detects an incoming γ-ray through Photoelectric absorption or Compton scattering. Therefore, we call α(θ) the probability that a scattered γ-ray will be detected - that is, absorbed or scattered in the NaI. This probability is 1 minus the probability that it will go unscathed, which is given in Equation (3):

\[ \alpha(\theta) = 1 - e^{-\mu L} \]  

where L is the length of the scintillator (2 inches). The attenuation coefficient for NaI is obtained from the literature and it is given by:

\[ \mu(E) = 5.18 \cdot E^{-2.85} + 0.539 \cdot E^{-0.45} \text{cm}^{-1} \]  

for energies between 100keV and 700keV (where E is measured in units of 100keV). The first term corresponds to Photoelectric absorptions, and the second to Compton scatterings. Because only events in the photo-peak are counted, γ-rays detected through Compton scattering inside the NaI are to be ignored, and the second term is dropped:

\[ \mu(E) = 5.18 \cdot E^{-2.85} \text{cm}^{-1} \]  

The second correction to the count rate is the possibility of multiple scatterings in the target by a single photon. If an incident photon first scatters at an angle θ and then it scatters again, it will not reach the detector placed at θ. Let’s call the distance traveled by an incident photon before being scattered x, the distance traveled after the scattering y, and the scattering position P. The probability that it will reach P without being scattered is \( e^{-\mu x} \) and the probability that it will reach the detector after being scattered is \( e^{-\mu y} \), where \( \mu \) is the linear attenuation coefficient for PMMA for the incident photon and \( \mu_s \) for the scattered photon (the have different coefficients because they have different energies, \( \mu = \mu(E) \)). Thus, the probability that a photon incident on the target of volume V (assuming P could be anywhere in V), and scattered at an angle θ, will reach the detector is:

\[ \beta(\theta) = \frac{\int e^{-\mu x} \cdot e^{-\mu y} dV}{\int e^{\mu y} dV} \]  

There is also the possibility that a photon initially scattered at an angle \( \phi_0 \) will scatter again at an angle \( \phi_1 \) such that \( \phi_0 + \phi_1 = \theta \), and the photon is detected. Although the precise formula is too complicated, we can make a crude approximation in modifying \( \beta(\theta) \). We can argue that the decrease in the number of incident photons traveling towards the point P is comparable to the number of photons traveling towards P after being scattered from other points, and then scattering at angle θ. In that case, we just have to drop the term \( e^{-\mu x} \) from the integral, since we are arguing that roughly the same number of photons is reaching P as if there was no attenuation. If this is true, then we have:

\[ \beta(\theta) = \frac{1}{V} \int e^{-\mu y} dV \]  

where the linear attenuation coefficient of PMMA for the scattered photon, \( \mu_s \), is given by:

\[ \mu(E) = 0.205 \cdot E^{-0.365} \text{cm}^{-1} \]
To understand this approximation consider that Compton scattering is peaked at very small angles (see a plot for Klein-Nishina equation to verify this). So, let’s say N photons are traveling towards P when they enter the target. If they are scattered before reaching P, it’s likely that they still are traveling towards P, so that the number of photons reaching P does not change much because of previous scatterings. The approximation is crude but good enough for the purpose of this paper, and therefore we are justified in using the new $\beta(\theta)$.

While $\alpha(\theta)$ ranges from 0.1 to 0.9, $\beta(\theta)$ goes from 0.81 to 0.72 - its range is much smaller, and its correction is not as significant. Efficiency considerations play a bigger role in correcting the data than the problem of multiple scatterings. It is, therefore, crucial to understand the limitations of the detection equipment in order to analyze the data correctly. After the corrections have been applied, the measured differential cross section is:

$$\frac{d\sigma}{d\Omega} = A \cdot C_{correct}(\theta)$$

(14)

where A is a normalization constant given by:

$$A = \frac{\sigma_{comp}}{\int C_{corr}(\theta)d\Omega} = \frac{\sigma_{comp}}{\int C_{corr}(\theta) \cdot 2\pi \cdot \sin \theta d\theta}$$

(15)

Comparing the measured differential cross section to the plot of the Klein-Nishina equation one can see that the results for this experiment were satisfactory. It is important to note that the data used to measure the differential cross section is just a small fraction of the total data gathered in the experiment. By only taking the energy and the intensity of the photo-peak, count also the events in the Compton scattering spectrum. The suggested method, although richer in data amount, does not surpass the method used in quality. From the plot of intensity vs energy (See Figure 2), one notices that the intensity is initially low and climbs to a maximum in the Compton scattering region. This is a problematic behavior: the intensity should be a maximum for low energies, because most scattering will be for very small angles. The PMT is not sensitive to low energies, and the intensity data is cut off at some arbitrary energy minimum. The amount of data lost is a changing fraction of the actual number of events, so the data for detections through scatterings in NaI is unreliable.

This experiment embodies one of the landmark experiments in Quantum Mechanics, but still present challenges for the Physics student. The careful analysis of the setup used in the experiment has lead to leaps in the quality of the data. The experiment provides an excellent for the student chance to get acquainted, albeit superficially, with particle detection techniques. Finally, it shows that only by understanding what the data really shows one can process it correctly to extract the information necessary to understand the physics behind it.

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